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Letters

Mixed H_∞ and passivity based state estimation for fuzzy neural networks with Markovian-type estimator gain change ☆

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ABSTRACT

This paper is concerned with the mixed H_∞ and passivity based state estimation for a class of discrete-time fuzzy neural networks with the estimator gain change, where a discrete-time homogeneous Markov chain taking value in a finite set $\Gamma = \{0, 1\}$ is introduced to model this phenomenon. Based on the Markovian system approach and linear matrix inequality technique, a new sufficient condition has been derived such that the estimation error system is exponentially stable in the mean square sense and achieves a prescribed mixed H_∞ and passivity performance level. The estimator parameter is then determined by solving a set of linear matrix inequalities (LMIs). A numerical example is presented to show the effectiveness of the proposed design method.

1. Introduction

In the past two decades, artificial neural networks have received considerable attention due to the extensive applications in signal processing, pattern recognition, static image processing, associative memory and combinatorial optimization [1,2]. It is well recognized that the practical applicability of neural networks depends on the existence and stability of the equilibrium point of the neural networks, and a great number of papers have been reported on the stability of various neural networks, see [3–7] and the references therein. For example, the sufficient conditions for absolute stability (global asymptotic stability) were obtained by using the M-matrix theory and qualitative property of the integro-differential inequalities [4]. In [7], the exponential stability and passivity analysis were dealt with for a class of discrete-time switched neural networks with discrete and distributed delays by using the average dwell time approach and the linear matrix inequality technique.

On the other hand, the well known Takagi–Sugeno (T–S) fuzzy model is an efficient tool in approximating the nonlinear systems [8]. During the last decades, increasing research attention has been paid on the stability analysis and control synthesis of T–S fuzzy systems, see [9–14] and the references therein. Based on traditional cellular neural networks, the authors in [15] first introduced the fuzzy cellular neural networks that integrate fuzzy logic into the structure of traditional cellular neural networks and maintain local connectedness among cells. The fuzzy neural network is a very useful tool for image processing problems [16], and since then fruitful results have been reported on the stability analysis of the fuzzy neural networks [17,18]. For example, the global asymptotic stability problem was considered for a class of T–S Fuzzy neural networks with Markovian jumping parameters, and a novel LMI-based stability criterion was obtained by using Lyapunov functional theory [18].

Meanwhile, in relatively large-scale neural networks, it is often that only partial information about the neuron states is available in the network outputs, in this scenario, the neuron state estimation problem becomes important for the applications that need to utilize the estimated neuron state. The state estimation for neural networks was first studied in [19], and later a delay-dependent estimator design result was presented in [20]. Recently, the authors studied the state estimation problem for a class of fuzzy Hopfield neural networks with time delay in [21], and the estimator gain was determined by solving a set of linear matrices inequalities. In [23], attention has focused on the design of a state estimator to estimate the neuron states by using the delay partition technique to reduce the possible conservatism. In our earlier work [25], the estimator design for discrete-time switched neural networks with time delay was

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investigated, where asynchronous switching between the mode of neural networks and the mode of estimator was addressed. Explicit relation between the maximal asynchronous time and the estimation performance has been established. Recent advances in the state estimation for various neural networks are referred to [21–26].

It is worth pointing out that all the above state estimation results are based on an implicit assumption that estimator can be implemented exactly. However, inaccuracies or uncertainties do occur in filter implementation, and as a result the estimation error may diverge [27]. Such uncertainties can be due to the roundoff errors in numerical computation during the filter implementation and the need to provide practicing engineers with safe-tuning margins [27]. So a significant issue is how to design an estimator for a given plant such that the estimator is insensitive to some amount of errors with respect to its gain, i.e., the designed estimator is non-fragile. This issue has received considerable attention from the control community, and many results have been reported in the last decades [27,28]. In [28], the authors studied the non-fragile state estimation problem for a class of fuzzy systems, and the additive estimator gain perturbation case was addressed. They showed that the optimal estimator gain can be determined by solving a set of linear matrix inequalities. Note that the existing results on the non-fragile estimation problem assumed that the uncertainty in the estimator gain occurs in a deterministic way, which may not be realistic as the estimator can be implemented for most of time, but the uncertain perturbation only occur at some random time instant. To the best of the authors' knowledge, few results have been reported on the state estimation of the delayed neural networks with the estimator gain perturbation, especially the stochastic perturbation case, which motivates the present study.

In this paper, the state estimation for a class of fuzzy neural networks with time delay is investigated, where the stochastic estimator gain perturbation is first studied from the Markovian system point of view. Based on the Lyapunov stability theory and linear matrix inequality technique, a sufficient condition is obtained such that the estimation error system is exponentially stable in the mean-square sense and achieves a prescribed mixed H_∞ and passivity performance level. The estimator design is determined by solving a set of linear matrix inequalities. Finally, a numerical example is given to show the effectiveness of the proposed design.

The main contributions of this paper are summarized as follows:

- The first attempt has been made to study the state estimator design for neural networks with estimator gain perturbation, and a novel modelling approach is proposed from the Markovian system point of view.
- A new estimation performance, namely, the mixed H_∞ and passivity performance index is first introduced to study the state estimation of neural networks. Relations among the weighing factor in the performance, maximal delay bound and the estimation performance level are established.
- Based on the Lyapunov stability theory and linear matrix inequality technique, a sufficient condition is obtained such that the estimation error system is exponentially stable in the mean-square sense and achieves a prescribed mixed H_∞ and passivity performance level. Our estimator gain is determined by solving a set of LMIs, which is numerical efficient.

Notation: The notation used in this paper is standard. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the n-dimensional Euclidean space and the set of all $m\times n$ real matrices. $l_2[0,\infty)$ is the space of square summable sequences. A^T represents the transpose of the matrix A. I_n stands for the identity matrix with $n\times n$ dimension. The symbol diag$\{A\}$ indicates that A is a diagonal matrix. $\mathbb{E}\{\bullet\}$ denotes the mathematical expectation with respect to the given probability measure Pr, and the symbol "$*$" is used in some matrix expressions to represent the symmetric terms.

2. Problem formulation

In this paper, the neural network is described by the following T–S model:

Plant rule i: IF $\phi_1(k)$ is ψ_{i1} and $\phi_2(k)$ is $\psi_{i2}\cdots$ and $\phi_t(k)$ is ψ_{it}, THEN

$$\begin{cases} x(k+1)=A_ix(k)+W_{0i}f(x(k))+W_{1i}f(x(k-d(k)))+W_{2i}v(k)\\ y(k)=C_ix(k)+D_iv(k)\\ z(k)=L_ix(k)\\ x(s)=\vartheta(s), \quad s\in[-d_2,0]\end{cases} \tag{1}$$

where $\phi(k)=[\phi_1(k),\phi_2(k),\ldots,\phi_t(k)]$ is the premise variable vector, ψ_{ij} is the fuzzy set and r is the number of IF–THEN rules such that $i\in\Omega=\{1,2,\ldots,r\}$. $x(k)=[x_1(k),x_2(k),\ldots,x_n(k)]^T\in\mathbb{R}^n$ is the neuron state vector, $y\in\mathbb{R}^p$ is the network output measurement, $z\in\mathbb{R}^q$ is the signal to be estimated, and $v(k)\in\mathbb{R}^m$ is a disturbance signal belonging into $l_2[0,\infty)$, $f(x(k))=[f_1(x_1(k)),f_2(x_2(k)),\ldots,f_n(x_n(k))]^T\in\mathbb{R}^n$ is the neuron activation function, $A_i=\mathrm{diag}\{a_{1i},a_{2i},\ldots,a_{ni}\}$ with $a_{si}>0$ is the self-feedback matrix, W_{0i} is the connection weight matrix and W_{1i} is the delayed connection weight matrix, respectively. W_{2i},C_i,D_i and L_i are the constant matrices with appropriate dimensions. The time-varying delay $d(k)$ is assumed to satisfy $d_1\le d(k)\le d_2$, where d_1 and d_2 are constant scalars. The function $\vartheta(s)$ is the initial vector and is known on $s\in[-d_2,0]$. The activation functions $f_l(x_l(k))$ are assumed to be bounded and satisfy the following sector bound condition:

Assumption 1 (*Zhang et al. [25]*). Each activation function $f_l(s_1)$ in (1) is continuous and bounded, and for any scalars s_1 and s_2, $s_1\neq s_2$,

$$\varpi_l\le\frac{f_l(s_1)-f_l(s_2)}{s_1-s_2}\le\sigma_l, \quad l=1,2,\ldots,n \tag{2}$$

where ϖ_l and σ_l are known constant scalars.

By using a center-average defuzzifier, product fuzzy inference, and a singleton fuzzifier, (1) can be rewritten as

$$\begin{cases} x(k+1)=\sum_{i=1}^{r}h_i(\phi(k))[A_ix(k)+W_{0i}f(x(k))+W_{1i}f(x(k-d(k)))+W_{2i}v(k)]\\ y(k)=\sum_{i=1}^{r}h_i(\phi(k))[C_ix(k)+D_iv(k)]\\ z(k)=\sum_{i=1}^{r}h_i(\phi(k))[L_ix(k)]\\ x(s)=\psi(s), \quad s\in[-d_2,0]\end{cases} \tag{3}$$

where $h_i(\phi(k))=\omega_i(\phi(k))/\sum_{i=1}^{r}\omega_i(\phi(k))$ and $\omega_i(\phi(k))=\prod_{j=1}^{p}\psi_{ij}(\phi_j(k))$, with $\psi_{ij}(\phi_j(k))$ representing the grade of membership of $\phi_j(k)$ in ψ_{ij}. Usually, it is assumed that $\omega_i(\phi(k))\ge 0$ and $\sum_{i=1}^{r}\omega_i(\phi(k))>0$ for all $\phi(k)$.

Due to the fact that uncertain perturbation may occur in the implementation of estimator, in this paper we consider the following estimator:

$$\begin{cases} \hat{x}(k+1)=\sum_{i=1}^{r}h_i(\phi(k))\begin{bmatrix}A_i\hat{x}(k)+W_{0i}f(\hat{x}(k))+W_{1i}f(\hat{x}(k-d(k)))\\ +[K_i+r(k)\Delta K](y(k)-C_i\hat{x}(k))\end{bmatrix}\\ z_f(k)=\sum_{i=1}^{r}h_i(\phi(k))L_i\hat{x}(k)\\ \hat{x}(s)=\hat{\vartheta}(s), \quad s\in[-d_2,0]\end{cases} \tag{4}$$