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On synchronization criterion for coupled discrete-time neural networks with interval time-varying delays

M.J. Park [a], O.M. Kwon [a,*], Ju H. Park [b], S.M. Lee [c], E.J. Cha [d]

[a] School of Electrical Engineering, Chungbuk National University, 52 Naesudong-ro, Heungduk-gu, Cheongju 361-763, Republic of Korea
[b] Department of Electrical Engineering, Yeungnam University, 214-1 Dae-Dong, Kyongsan 712-749, Republic of Korea
[c] School of Electronic Engineering, Daegu University, Gyungsan 712-714, Republic of Korea
[d] Department of Biomedical Engineering, School of Medicine, Chungbuk National University, 52 Naesudong-ro, Heungduk-gu, Cheongju 361-763, Republic of Korea

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ABSTRACT

The purpose of this paper is to investigate the delay-dependent synchronization analysis for coupled discrete-time neural networks with interval time-varying delays in network couplings. Based on Lyapunov method, a new delay-dependent criterion for the synchronization of the networks is derived in terms of linear matrix inequalities (LMIs) by construction of a suitable Lyapunov–Krasovskii's functional and utilization of Finsler's lemma without free-weighting matrices. Two numerical examples are given to illustrate the effectiveness of the proposed method.

1. Introduction

Complex networks, which are a set of interconnected nodes with specific dynamics, have received increasing attention of researches from various fields of science and engineering such as the World Wide Web, social networks, electrical power grids, global economic markets, and so on. Many mathematical models were proposed to describe various complex networks [1,2]. Also, in the real applications of systems, there exists naturally time-delay due to the finite information processing speed and the finite switching speed of amplifiers. It is well known that time-delay often causes undesirable dynamic behaviors such as performance degradation, and instability of the systems. Therefore, recently, the problem of synchronization of coupled neural networks with time-delay which is one of hot research fields of complex networks has been a challenging issue due to its potential applications such as information science, biological systems and so on [3–5]. By use of Lyapunov functional method and Kronecker product properties, global synchronization for an array of delayed neural networks with hybrid coupling were proposed in [3]. In [4], the synchronization criteria were derived for a general array model of coupled delayed neural networks with hybrid coupling. Li et al. [5] presented two novel synchronization criteria for arrays of coupled delayed neural networks with both delayed coupling and one single delayed one. Moreover, the delayed neural networks also were addressed in [6–8].

On the other hand, these days, most systems use digital computers (usually microprocessor or microcontrollers) with the necessary input/output hardware to implement the systems. The fundamental character of the digital computer is that it takes compute answers at discrete steps. Therefore, discrete-time modeling with time-delay plays an important role in many fields of science and engineering applications [9–11]. In this regard, various approaches to synchronization stability criterion for coupled discrete-time neural networks with time-delay have been investigated in the literature [12–14]. Wang and Song [12] studied the problem of synchronization for an array of coupled stochastic discrete-time neural networks with mixed delays (discrete and distributed time-varying delays). In [13], by use of the novel Lyapunov–Krasovskii's functional and Kronecker product, synchronization and estimation conditions for discrete-time complex networks with distributed delays was presented. However, time-delay in only nodes was considered in [12,13]. Moreover, Yue and Li [14] derived the synchronization stability criteria for continuous/discrete complex dynamical networks with interval time-varying delays based on a piecewise analysis method and the convexity of matrix inequalities. In [14], the

* Corresponding author. Tel.: +82 43 261 2422; fax: +82 43 263 2419.
E-mail addresses: madwind@chungbuk.ac.kr (O.M. Kwon), jessie@ynu.ac.kr (J.H. Park), moony@daegu.ac.kr (S.M. Lee).

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synchronization problem of the discrete-time complex networks is transferred equally into the asymptotical stability problem of a group of uncorrelated delay functional difference equations. To derive the delay functional difference equations, an eigenvalue of outer-coupling matrix and a Jacobian at a solution of an isolated node of the complex dynamical networks are used. However, when we utilize Kronecker product properties, the transformations of complex networks are simple and can derive directly from the corresponding networks. Unfortunately, to the best of authors' knowledge, delay-dependent synchronization analysis of coupled discrete-time neural networks with time-varying delay in network couplings has not been investigated yet. It should be pointed out that delay-dependent analysis has been paid more attention than delay-independent one because the sufficient conditions for delay-dependent analysis make use of the information on the size of time-delay [15]. That is, the former is generally less conservative than the latter.

Motivated by the above discussions, the problem of new delay-dependent synchronization criterion for coupled discrete-time neural networks with interval time-varying delays in network couplings is considered. The coupled discrete-time neural networks are represented as a simple mathematical model by use of Kronecker product technique. Then, by construction of a suitable Lyapunov–Krasovskii's functional and utilization of Finsler's lemma, a new synchronization criterion is derived in terms of LMIs which can be solved efficiently by standard convex optimization algorithms [16]. In order to utilize Finsler's lemma as a tool of getting less conservative synchronization criterion, it should be noted that a new zero equality from the constructed mathematical model is devised. The concept of scaling transformation matrix will be utilized in deriving zero equality of the method. Finally, two numerical examples are included to show the effectiveness of the proposed method.

Notation: $\mathbf{R}^n$ is the n-dimensional Euclidean space, and $\mathbf{R}^{m\times n}$ denotes the set of $m\times n$ real matrix. For symmetric matrices X and Y, $X>Y$ (respectively, $X\geq Y$) means that the matrix $X-Y$ is positive definite (respectively, nonnegative). $X^{\perp}$ denotes a basis for the null-space of X. I_n, 0_n and $0_{m\times n}$ denotes $n\times n$ identity matrix, $n\times n$ and $m\times n$ zero matrices, respectively. $\|\cdot\|$ refers to the Euclidean vector norm or the induced matrix norm. $diag\{\cdots\}$ denotes the block diagonal matrix. $\star$ represents the elements below the main diagonal of a symmetric matrix.

2. Problem statements

Consider the following discrete-time neural networks with interval time-varying delays:

$$y(k+1)=Ay(k)+W_1g(y(k))+W_2g(y(k-h(k)))+b, \tag{1}$$

where n denotes the number of neurons in a neural network, $y(\cdot)=[y_1(\cdot),\ldots,y_n(\cdot)]^T\in\mathbf{R}^n$ is the neuron state vector, $g(\cdot)=[g_1(\cdot),\ldots,g_n(\cdot)]^T\in\mathbf{R}^n$ denotes the neuron activation function vector, $b=[b_1,\ldots,b_n]^T\in\mathbf{R}^n$ means a constant external input vector, $A=diag\{a_1,\ldots,a_n\}\in\mathbf{R}^{n\times n}(0<a_q<1,q=1,\ldots,n)$ is the state feedback matrix, $W_q\in\mathbf{R}^{n\times n}(q=1,2)$ are the connection weight matrices, and $h(k)$ is interval time-varying delays satisfying

$$0<h_m\leq h(k)\leq h_M,$$

where h_m and h_M are positive integers.

In this paper, it is assumed that the activation functions satisfy the following assumption:

Assumption 1. The neurons activation functions, $g_a(y_a(\cdot))$ $(a=1,\ldots,n)$, are assumed to be nondecreasing, bounded and globally Lipschiz; that is

$$l_a^-\leq\frac{g_a(\xi_a)-g_a(\xi_b)}{\xi_a-\xi_b}\leq l_a^+,\quad\forall\xi_a,\xi_b\in\mathbf{R},\ \xi_a\neq\xi_b,$$

where l_a^- and l_a^+ are constant values.

For simplicity, in stability analysis of the network (1), the equilibrium point $y^*=[y_1^*,\ldots,y_n^*]^T$ is shifted to the origin by utilization of the transformation $x(\cdot)=y(\cdot)-y^*$, which leads the network (1) to the following form:

$$x(k+1)=Ax(k)+W_1f(x(k))+W_2f(x(k-h(k))),$$

where $x(\cdot)=[x_1(\cdot),\ldots,x_n(\cdot)]^T\in\mathbf{R}^n$ is the state vector of the transformed network, and $f(x(\cdot))=[f_1(x_1(\cdot)),\ldots,f_n(x_n(\cdot))]^T$ is the transformed neuron activation function vector with $f_b(x_b(\cdot))=g_b(x_b(\cdot)+y_b^*)-g_b(y_b^*)(b=1,\ldots,n)$ satisfies, from Assumption 1, $l_b^-\leq f_b(\xi_b)/\xi_b\leq l_b^+,\forall\xi_b\neq 0$, which is equivalent to

$$[f_b(x_b(k))-l_b^-x_b(k)][f_b(x_b(k))-l_b^+x_b(k)]\leq 0. \tag{2}$$

In this paper, a model of coupled discrete-time neural networks with interval time-varying delays in network couplings is considered as

$$x_i(k+1)=Ax_i(k)+W_1f(x_i(k))+W_2f(x_i(k-h(k)))+\sum_{j=1}^{N}g_{ij}\Gamma x_j(k-h(k)),$$
$$i=1,2,\ldots,N, \tag{3}$$

where N is the number of couple nodes, $x_i(k)=[x_{i1}(k),\ldots,x_{in}(k)]^T\in\mathbf{R}^n$ is the state vector of the ith node, $\Gamma\in\mathbf{R}^{n\times n}$ is the constant inner-coupling matrix of nodes, which describe the individual coupling between networks, $G=[g_{ij}]_{N\times N}$ is the outer-coupling matrix representing the coupling strength and the topological structure of the networks satisfies the diffusive coupling connections:

$$g_{ij}=g_{ji}\geq 0\ \ (i\neq j),\quad g_{ii}=-\sum_{j=1,i\neq j}^{N}g_{ij}\ \ (i,j=1,2,\ldots,N).$$

For the convenience of stability analysis for the network (3), the following Kronecker product and its properties are used.

Lemma 1 (*Kronecker product, Graham [17]*). *Let $\otimes$ denotes the notation of Kronecker product. Then, the following properties of Kronecker product are easily established*:

(i) $(\alpha A)\otimes B=A\otimes(\alpha B)$,
(ii) $(A+B)\otimes C=A\otimes C+B\otimes C$,
(iii) $(A\otimes B)(C\otimes D)=(AC)\otimes(BD)$.

Let us define

$$\mathbf{x}(k)=[x_1(k),\ldots,x_N(k)]^T,\mathbf{f}(\mathbf{x}(k))=[f(x_1(k)),\ldots f(x_N(k))]^T.$$

Then, with Kronecker product in Lemma 1, the network (3) can be represented as

$$\mathbf{x}(k+1)=I_N\otimes(A\mathbf{x}(k)+W_1\mathbf{f}(\mathbf{x}(k))+W_2\mathbf{f}(\mathbf{x}(k-h(k))))+(G\otimes\Gamma)\mathbf{x}(k-h(k)). \tag{4}$$

The aim of this paper is to investigate the delay-dependent synchronization stability analysis of the network (4) with interval time-varying delays in network coupling. In order to do this, the following definition and lemmas are needed.

Definition 1 (*Liu and Chen [18]*). The network (3) is said to be asymptotically synchronized if the following condition holds:

$$\lim_{t\to\infty}\|x_i(k)-x_j(k)\|=0,\quad i,j=1,2,\ldots,N.$$

Lemma 2 (*Cao et al. [3]*). *Let $U=[u_{ij}]_{N\times N}$, $P\in\mathbf{R}^{n\times n}$, $x^T=[x_1,x_2,\ldots,x_n]^T$, and $y^T=[y_1,y_2,\ldots,y_n]^T$. If $U=U^T$ and each row*