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Stability analysis for discrete delayed Markovian jumping neural networks with partly unknown transition probabilities

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ABSTRACT

This paper addresses the analysis problem of asymptotic stability for a class of uncertain neural networks with Markovian jumping parameters and time delays. The considered transition probabilities are assumed to be partially unknown. The parameter uncertainties are considered to be norm-bounded. A sufficient condition for the stability of the addressed neural networks is derived, which is expressed in terms of a set of linear matrix inequalities. A numerical example is given to verify the effectiveness of the developed results.

1. Introduction

The last few decades have witnessed successful applications of neural networks in diverse fields including associative memory, fault diagnosis, pattern recognition and image processing, etc. The study of neural networks has therefore gained persistent research interest from the early 80s, see [7,8,11,13,21,22] and the references therein. As a major concern, the investigations on the stability of neural networks have attracted considerable research attention due mainly to the fact that the stability of the equilibrium points ensures that the stored memory can be retrieved.

It is well known that time delays are often unavoidable in a variety of industrial and engineering systems, and the existence of time delays may lead to oscillation, instability and poor performances of systems (see, e. g., [6,12,18]). In recent years, fruitful literature has been available for neural networks with time delays. Many different types of time delays, such as constant delays, time-varying delays and distributed delays, have been taken into account and lots of delay-independent and delay-dependent results have been obtained for neural networks (see, for example [4,10,19,23]). Moreover, the time-delay neural networks often involve with parameter uncertainties that constitute another main cause for degrading the system performances or even leading to instability. Nevertheless, the corresponding stability analysis for discrete time-delay neural networks has received much less attention than their continuous-time counterpart.

On the other hand, Markovian jump systems have gained particular research attention in the past two decades. Such class of systems is a special class of stochastic hybrid systems with finite operation modes, and is more appropriate to model the dynamic systems subject to abrupt variation in their structures, such as component failures, sudden environmental disturbance and abrupt variations of the operating points of a nonlinear system [2,5,20]. In practice, a neural network may be subject to abrupt changes in its structure or network modes jumps, which are commonly governed by a Markovian chain. Recently, several significant research results on the time-delay neural networks with Markovian jumping parameters have been reported by using linear matrix inequality approach, *M*-matrix theory and topological degree theory. It is worth mentioning that most of the aforementioned results have been based on the implicit assumption that the complete knowledge of transition probabilities is available. This ideal assumption inevitably limits the application scope of the established results since, in practice, it is difficult to obtain precisely all the transition probabilities. Very recently, some initial efforts have been devoted to the study of the systems with partially unknown transition probabilities [24,25]. However, the robust stability analysis problem for time-delay neural networks with partly unknown transition probabilities has not been fully investigated in the literature, and this motivates our present work of this paper.

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In this paper, we aim to address the robust stability problem for discrete neural networks with partially unknown transition probabilities and time delays. The parameter uncertainties are assumed to be norm-bounded. By constructing a suitable Lyapunov–Krasovskii function, a sufficient stability criterion is obtained such that the considered neural networks is asymptotically stable. The proposed criterion is given by solving a set of linear matrix inequalities, which can be checked efficiently by using recently developed convex optimization algorithms. A simulation example is developed to demonstrate the validity of the proposed methods.

Notation: The notation used in the paper is fairly standard. The superscript "T" stands for matrix transposition, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all real matrices of dimension $m\times n$, $\mathbb{N}^+$ stands for the sets of positive integers, and I and 0 represent the identity matrix and zero matrix, respectively. The notation $P>0$ means that P is real symmetric and positive definite; the notation $|\cdot|$ stands for the the Euclidean vector norm. In symmetric block matrices or complex matrix expressions, we use an asterisk $*$ to represent a term that is induced by symmetry, and diag$\{\cdots\}$ stands for a block-diagonal matrix. In addition, $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|y\}$ will, respectively, represent expectation of x and expectation of x conditional on y. If A is a matrix, $\lambda_{\min}$ stands for the smallest eigenvalue of A. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulation

In this paper, we consider a discrete-time n-neuron Markovian jumping neural network described by the following dynamical equation:

$$\begin{aligned} x(k+1) &= (C(r(k))+\Delta C(r(k)))x(k)+(A(r(k)) \\ &\quad +\Delta A(r(k)))f(x(k))+((B(r(k))+\Delta B(r(k)))f(x(k-d)), \\ x(k) &= \phi(k), k\in[-d,0), \end{aligned} \tag{1}$$

where $x(k)=[x_1(k)\ x_2(k)\ \ldots\ x_n(k)]^T$ is the neural state vector; $f(x(k))=[f_1(x_1(k))\ f_2(x_2(k))\ \ldots\ f_n(x_n(k))]^T$ represents the nonlinear activation function with the initial condition $f(0)=0$; $C(r(k))=\text{diag}\{c_1(r(k)),c_2(r(k)),\ldots,c_n(r(k))\}$ describes the rate with which the each neuron will reset its potential to the resting state in isolation when disconnected from the networks and external inputs; $A(r(k))=[a_{ij}(r(k))]_{n\times n}$ and $B(r(k))=[b_{ij}(r(k))]_{n\times n}$ are, respectively, the connection weight matrix and the delayed connection weight matrix; $d\geq 0$ denotes the discrete time delay; $\phi(k)$ describes the initial condition. In addition, $\Delta A(r(k))$, $\Delta B(r(k))$ and $\Delta C(r(k))$ are time-varying parameter uncertainties that satisfy

$$[\Delta A(r(k))\ \ \Delta B(r(k))\ \ \Delta C(r(k))]=M(r(k))F(r(k),k)[N_1(r(k))\ \ N_2(r(k))\ \ N_3(r(k))], \tag{2}$$

where $M(r(k)),N_1(r(k)),N_2(r(k)),N_3(r(k))$ are real constant matrices of appropriate dimensions, and $F(r(k),k)$ is an unknown time-varying matrix function satisfying

$$F^T(r(k),k)F(r(k),k)\leq I,\ \forall k\in\mathbb{N}^+. \tag{3}$$

The Markov chain $r(k)$ $(k\geq 0)$ takes values in a finite state space $S=\{1,2,\ldots,s\}$ with transition probability matrix $\hat{\Psi}=[\lambda_{ij}]$ given by

$$\text{Prob}\{r(k+1)=j|r(k)=i\}=\lambda_{ij},\quad \forall i,j\in S,$$

where $\lambda_{ij}\geq 0$ $(i,j\in S)$ is the transition probability from i to j and $\sum_{j=1}^{s}\lambda_{ij}=1$, $\forall i\in S$.

In this paper, we assume that some elements in the transition probability matrix $\hat{\Psi}$ are unknown, for example, the transition probability matrix $\hat{\Psi}$ may be

$$\hat{\Psi}=\begin{bmatrix}\lambda_{11} & ? & ? \\ ? & \lambda_{22} & ? \\ \lambda_{31} & \lambda_{32} & \lambda_{33}\end{bmatrix},$$

where "?" represents the unknown entries. For notation clarity, for any $i\in S$, we denote that

$$S_{\mathcal{K}}^i := \{j : \lambda_{ij} \text{ is known}\},\quad S_{\mathcal{UK}}^i := \{j : \lambda_{ij} \text{ is unknown}\}. \tag{4}$$

Also, we denote $\lambda_{\mathcal{K}}^i := \sum_{j\in S_{\mathcal{K}}^i}\lambda_{ij}$ throughout the paper.

The set S contains s modes of equation (1) and, for $r(k)=i$, the system matrices of the ith mode are denoted by $A_i+\Delta A_i$, $B_i+\Delta B_i$ and $C_i+\Delta C_i$.

Throughout the paper, we make the following assumption.

Assumption 1 (*Wang et al. [15–17]*). For the activation function $f(\cdot)$, there exist constants σ_i^- and σ_i^+ $(i=1,2,\ldots,n)$ such that

$$\sigma_i^- \leq \frac{f_i(\alpha_1)-f_i(\alpha_2)}{\alpha_1-\alpha_2}\leq \sigma_i^+. \tag{5}$$

Remark 1. The assumption based on the activation functions $f(\cdot)$ considered here has been first introduced in [15]. The activation functions described in (1) are more general than the usual sigmoid functions and the recently commonly used Lipschitz conditions, where the constants σ_i^- and σ_i^+ are allowed to be positive, negative or zero. Therefore, the activation functions could be nonmonotonic. Such an assumption will induce the less conservative results.

Before proceeding further, we introduce the following definition.

Definition 1. Neural network (1) is said to be asymptotically stable in the mean square if, for any solution $x(k)$ of (1), the following holds:

$$\lim_{k\to\infty}\mathbb{E}\{|x(k)|^2\}=0.$$

The main purpose of this paper is to deal with the asymptotic stability problem for the neural network (1).

3. Main results

Before stating our main results, we introduce the following lemmas:

Lemma 1 (*Boyd et al. [1]*). *(Schur Complement) Given constant matrices $\mathcal{S}_1,\mathcal{S}_2,\mathcal{S}_3$ where $\mathcal{S}_1=\mathcal{S}_1^{\mathrm{T}}$ and $0<\mathcal{S}_2=\mathcal{S}_2^{\mathrm{T}}$, then $\mathcal{S}_1+\mathcal{S}_3^{\mathrm{T}}\mathcal{S}_2^{-1}\mathcal{S}_3<0$ if and only if*

$$\begin{bmatrix}\mathcal{S}_1 & \mathcal{S}_3^{\mathrm{T}} \\ \mathcal{S}_3 & -\mathcal{S}_2\end{bmatrix}<0 \quad \text{or} \quad \begin{bmatrix}-\mathcal{S}_2 & \mathcal{S}_3 \\ \mathcal{S}_3^{\mathrm{T}} & \mathcal{S}_1\end{bmatrix}<0. \tag{6}$$

Lemma 2. *(S-procedure) Let $L=L^{\mathrm{T}}$ and H and E be real matrices of appropriate dimensions with F satisfying $FF^T\leq I$. Then $L+HFE+E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}}<0$, if and only if there exists a positive scalar $\varepsilon>0$ such that $L+\varepsilon^{-1}HH^{\mathrm{T}}+\varepsilon E^{\mathrm{T}}E<0$ or, equivalently,*

$$\begin{bmatrix}L & H & \varepsilon E^{\mathrm{T}} \\ H^{\mathrm{T}} & -\varepsilon I & 0 \\ \varepsilon E & 0 & -\varepsilon I\end{bmatrix}<0. \tag{7}$$

Lemma 3 (*Liu et al. [9]*). *Suppose that $\mathcal{B}=\text{diag}\{\beta_1,\beta_2,\ldots,\beta_n\}$ is a positive-semidefinite diagonal matrix. Let $x=[x_1,x_2,\ldots,x_n]^T\in\mathbb{R}^n$, and $\mathcal{H}(x)=[h_1(x_1),h_2(x_2),\ldots,h_n(x_n)]^T$ be a continuous nonlinear*