Neurocomputing 74 (2011) 3072–3078

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

A fast recursive algorithm based on fuzzy 2-partition entropy approach for threshold selection

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ARTICLE INFO

Article history:
Received 13 August 2010
Received in revised form
28 January 2011
Accepted 17 April 2011
Communicated by M. Sato-Ilic
Available online 23 May 2011

Keywords:
Image segmentation
Fuzzy c-partition
Thresholding
Recursive algorithm
Entropy

ABSTRACT

The fuzzy c-partition entropy approach for threshold selection is an effective approach for image segmentation. The approach models the image with a fuzzy c-partition, which is obtained using parameterized membership functions. The ideal threshold is determined by searching an optimal parameter combination of the membership functions such that the entropy of the fuzzy c-partition is maximized. It involves large computation when the number of parameters needed to determine the membership function increases. In this paper, a recursive algorithm is proposed for fuzzy 2-partition entropy method, where the membership function is selected as *S*-function and *Z*-function with three parameters. The proposed recursive algorithm eliminates many repeated computations, thereby reducing the computation complexity significantly. The proposed method is tested using several real images, and its processing time is compared with those of basic exhaustive algorithm, genetic algorithm (GA), particle swarm optimization (PSO), ant colony optimization (ACO) and simulated annealing (SA). Experimental results show that the proposed method is more effective than basic exhaustive search algorithm, GA, PSO, ACO and SA.

1. Introduction

Image segmentation is an important step in a middle-level image processing. It can be used in various image-processing applications such as object tracking [1], character recognition [2], document analysis [3], visual inspection of defects [4], etc. Image thresholding is a popular tool for image segmentation. It is essentially a classification problem, pixels whose gray value is greater than the given threshold value are classified to one class, otherwise, classified into another class. In the past years, many automatic threshold selection methods have been proposed by optimizing a certain criterion function such as between-class variance [5], information entropy [6–9], classification error [10] and so on. It is believed that there exist fuzziness and uncertainty in image processing. Owing to this fact, fuzzy set theory was adopted by several authors for image segmentation purpose. In [11], Cheng et al. proposed a new automatic threshold selection method based on the concept of fuzzy c-partition. In Cheng's method, two fuzzy sets for object and background were defined using parameterized membership function. The two fuzzy sets form a fuzzy 2-partition of the image. The selection of optimal threshold was converted to find the best parameters of the membership function such that the entropy of fuzzy c-partition is maximized. In [12,13], Cheng extended the own method to two-dimension case. Zhao et al. [14] proposed another threshold selection method based on probability partition and fuzzy 3-partition. On the basis of Zhao's method, Tao adopted another definition of fuzzy entropy [15,16], which is similar to Kapur's definition of entropy [17].

Though the fuzzy c-partition entropy approach can result in better segmentation than existing methods, however, the computation burden of these methods is very large because the search space increases when the number of parameters of the membership function increases. For example, if the membership function has three parameters and the image have 256 gray level, the search space is close to $256^3 = 16\,777\,216$. Such procedure is characterized by high computation cost. To alleviate computation difficulty, many researchers resort to intelligent optimization methods such as genetic algorithm [15], simulated annealing [11], particle swarm optimization [18] and ant colony optimization [16]. Nevertheless, it can not guarantee that the global optimization can be obtained using intelligent optimization methods. In other words, optimal segmentation results can not be guaranteed. One effective strategy to reduce the computation burden is to eliminate the repeated computations in search process. Based on this idea, Benabdelkader and Boulemden [19]

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doi:10.1016/j.neucom.2011.04.010

proposed a recursive algorithm for 2-level thresholding based on fuzzy 2-partition, where the membership function is a trapezium function with two parameters.

In this paper, we propose a fast recursive algorithm for thresholding method based on fuzzy c-partition and probability partition. The membership functions are *S*-function and *Z*-function with three parameters, which are utilized by several authors [14–16]. The proposed recursive algorithm eliminates much repeated computations, thus the computation complexity reduces dramatically. The rest of the papers is organized as follows. Section 2 presents an overview of fuzzy 2-partition entropy approach. In Section 3, the proposed recursive algorithm are described in details. In Section 4, we evaluate the performance of the proposed recursive algorithm using some images. Finally, conclusions are presented in Section 5.

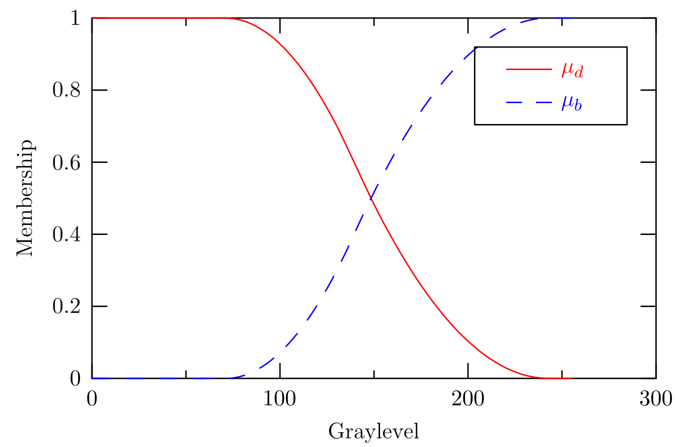

Fig. 1. The plot of membership function with (a, b, c)=(71, 139, 243).

2. Thresholding method based on fuzzy 2-partition entropy

Let I be a function which describes an image having 256 gray levels ranging from 0 to 255, $I(x, y)$ is a value of this function in pixel with coordinates (x, y). The number of pixels with gray level k is denoted as f_k, then, the probability of the occurrence of the gray level k in the image is

$$p(k)=f_k/N, \tag{1}$$

where N is the total of the pixels.

For segmentation purpose, the image is modeled by two fuzzy sets, i.e., *dark* and *bright*. The membership functions for these two fuzzy sets are defined as

$$\mu_d(k)=Z(k;a,b,c)=\begin{cases}1, & k\le a,\\ 1-\dfrac{(k-a)^2}{(c-a)(b-a)}, & a<k\le b,\\ \dfrac{(k-c)^2}{(c-a)(c-b)}, & b<k\le c,\\ 0, & k>c,\end{cases} \tag{2}$$

$$\mu_b(k)=S(k;a,b,c)=\begin{cases}0, & k\le a,\\ \dfrac{(k-a)^2}{(c-a)(b-a)}, & a<k\le b,\\ 1-\dfrac{(k-c)^2}{(c-a)(c-b)}, & b<k\le c,\\ 1, & k>c,\end{cases} \tag{3}$$

where $0<a\le b\le c$, k is the independent variable, a, b, and c are parameters determining the shape of the above two membership functions as shown in Fig. 1.

Obviously, $\mu_d(k)=1-\mu_b(k)$. Hence, $\mu_d(k)$ and $\mu_b(k)$ is fuzzy 2-partition of the image [11]. The probabilities of the two fuzzy events *dark* and *bright* are defined as

$$P_d=\sum_{k=0}^{255}\mu_d(k)p(k), \tag{4}$$

$$P_b=\sum_{k=0}^{255}\mu_b(k)p(k). \tag{5}$$

The fuzzy entropy of the fuzzy 2-partition is given by

$$H(a,b,c)=-P_d\log(P_d)-P_b\log(P_b). \tag{6}$$

Since different combination of (a, b, c) corresponds to different fuzzy 2-partition, so, the fuzzy entropy varies along with three parameters a, b, c. We can find an optimal combination of (a, b, c) such that the fuzzy entropy $H(a, b, c)$ has maximum value, i.e.,

$$(a^*,b^*,c^*)=\operatorname{argmax}H(a,b,c),\quad k=0,1,\ldots,255. \tag{7}$$

Then, the most appropriate threshold T can be computed as

$$\mu_d(T)=\mu_b(T)=0.5. \tag{8}$$

As is shown in Fig. 1, threshold T is the point of intersection of μ_d and μ_b. According to Eqs. (2) and (3), the solution is given as

$$T=\begin{cases}a^*+\sqrt{(c^*-a^*)(b^*-a^*)/2}, & (a^*+c^*)/2\le b^*\le c^*,\\ c^*-\sqrt{(c^*-a^*)(c^*-b^*)/2}, & a^*\le b^*\le(a^*+c^*)/2.\end{cases} \tag{9}$$

However, finding the optimal combination of (a, b, c) is not an easy work. Since a, b and c may take values from $\{0,1,2,\ldots,255\}$, the search space of parameters combination (a, b, c) is close to 256^3. Such procedure is characterized by high computation cost. In the following section, we propose a recursive algorithm to find the optimal combination of (a, b, c).

3. The proposed recursive algorithm

In the basic fuzzy 2-partition entropy approach, for each pair of (a, b, c), $\mu_d(k)$ and $\mu_b(k)$, P_b and P_d are calculated from (0,1,2) to (253,254,255). It takes too much repetitive calculation to obtain these quantities.

According to Eqs. (4) and (5), the probability of the fuzzy set *dark* can be written as

$$P_d=1-P_b, \tag{10}$$

and the entropy of the fuzzy 2-partition can be described as

$$H(a,b,c)=-\log(1-P_b)+P_b\log\left(\frac{1-P_b}{P_b}\right). \tag{11}$$

Therefore, only the probability of the fuzzy set *bright* will be computed to calculate the entire entropy of the fuzzy 2-partition.

Substituting Eq. (3) into Eq. (5), the probability of the fuzzy set *bright* is calculated as

$$P_b=\frac{1}{(c-a)(b-a)}\sum_{k=a+1}^{b}(k-a)^2p(k)-\frac{1}{(c-a)(c-b)}\sum_{k=b+1}^{c}(k-c)^2p(k)-\sum_{k=0}^{b}p(k)+1. \tag{12}$$

Eq. (12) includes four parts and the first three parts involves a sum operation. In the following, we show that the first three parts of Eq. (12) can be calculated in a recursive manner.