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Exponential stability analysis of stochastic reaction-diffusion Cohen–Grossberg neural networks with mixed delays ☆

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ABSTRACT

In this paper, we study a new class of stochastic Cohen–Grossberg neural networks with reaction-diffusion and mixed delays. Without the aid of nonnegative semimartingale convergence theorem, the method of variation parameter and linear matrix inequalities technique, a set of novel sufficient conditions on the exponential stability for the considered system is obtained by utilizing a new Lyapunov–Krasovskii functional, the Poincaré inequality and stochastic analysis theory. The obtained results show that the reaction-diffusion term does contribute to the exponentially stabilization of the considered system. Therefore, our results generalize and improve some earlier publications. Moreover, two numerical examples are given to show the effectiveness of the theoretical results and demonstrate that the stability criteria existed in the earlier literature fail.

1. Introduction

In the past years, the stability problem for a class of Cohen–Grossberg neural networks has received much research attention, and many good results related to this problem have been reported, see, e.g. [1–3,5–8,11,15,16,18,19,21,24,25,27] and the references therein. The reason for causing this result is twofold. On one hand, Cohen–Grossberg neural networks have been widely applied in many areas including signal processing, image processing, pattern recognition, fault diagnosis, associative memory, combinatorial optimization, and so on. And on the other hand, this model generalizes some other models such as the Hopfield neural networks, recurrent neural networks, cellular neural networks and bi-directional associative memory neural networks. Therefore, it is important to study the stability problem for a class of Cohen–Grossberg neural networks in theories and applications.

However, a large number of results existed in the literature mainly focused on the traditional neural network models, which are described by ordinary differential equations. But in the factual operations, diffusion effects cannot be avoided in the neural network model when electrons are moving in asymmetric electromagnetic fields. To cope with this case, we must consider the state variables that are varying with the time and space variables. It has been recognized that the neural network with diffusion term should be expressed by partial differential equations. It is inspiring that the subject of neural networks with diffusion term has gained many researchers, and there have appeared lots of results on the stability analysis in the literature [4,6,8–10,12,13,16–19,23]. For instance, Cui and Lou [4] studied the global asymptotic stability for a class of bi-directional associative memory neural networks with distributed delays and reaction-diffusion terms. Li and Song [6] investigated the global exponential stability of reaction-diffusion recurrent neural networks with delays. By establishing an integro-differential inequality with impulsive initial conditions and applying *M*-matrix theory, Li and Li [8] obtained some sufficient conditions ensuring the existence, uniqueness, global exponential stability and global robust exponential stability of equilibrium point for impulsive Cohen–Grossberg neural networks with distributed delays and reaction-diffusion terms. Liang and Cao [10] studied the existence, uniqueness and global exponential stability of the equilibrium point of delayed reaction-diffusion recurrent neural networks by using the properties of diffusion operator and the general Halanay inequality. Based on the topological degree theory and linear matrix inequality technique, Pan and Zhong [16] obtained a set of sufficient conditions

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to ensure the existence, uniqueness and global exponential stability of the equilibrium point. However, all of the above-mentioned works did not consider the effects of noise perturbations. In other words, the discusses about those results were restricted to only the case of deterministic neural networks.

On the other hand, noise disturbance is a major source for causing instability and poor performances in neural networks. In fact, the synaptic transmission in real neural networks can be viewed as a noisy process introduced by random fluctuations from the release of neurotransmitters and other probabilistic causes. Therefore, noise disturbances should be taken into account when studying the stability of neural networks, and the corresponding neural networks with noise disturbances are called stochastic neural networks. With respect to stochastic Cohen–Grossberg neural networks with diffusion term, however, there are only few results in the literature on the stability. Therefore, there is enough room to develop new approaches and techniques to study the stability of stochastic Cohen–Grossberg neural networks with diffusion term.

Motivated by the above discussion, in this paper we investigate the exponential stability for a class of stochastic reaction-diffusion Cohen–Grossberg neural networks with mixed delays. By using a new Lyapunov–Krasovskii functional, the Poincaré inequality and stochastic analysis theory, a set of novel sufficient conditions is obtained to guarantee the exponential stability for this class of new Cohen–Grossberg neural networks. It should be mentioned that the approach and technique provided here are quite different from the existed approaches such as the nonnegative semimartingale convergence theorem [14], the method of variation parameter [24] and linear matrix inequalities technique [22]. It is also worth pointing out that the stability criteria obtained in [14,22,24] were independent on the reaction-diffusion term, which means that the role of diffusion terms for exponentially stabilizing reaction-diffusion Cohen–Grossberg neural networks was ignored. However, different from those given in [14,22,24], our obtained results are dependent on the reaction-diffusion term and show that the reaction-diffusion term does contribute to the exponentially stabilization for the considered system. Therefore, our results generalize and improve some earlier publications. Moreover, two numerical examples are given to show the effectiveness of the theoretical results and demonstrate that the stability criteria existed in the earlier literature fail.

The remainder of this paper is organized as follows. In Section 2, we introduce the model of a new class of stochastic Cohen–Grossberg neural networks with both reaction-diffusion and mixed delays, and present some necessary assumptions. Without the aid of nonnegative semimartingale convergence theorem, the method of variation parameter and linear matrix inequalities technique, our main results are established by using some new approaches and techniques in Section 3. In Section 4, two numerical examples are given to show the effectiveness of the obtained results. Finally, in Section 5, the paper is concluded with some general remarks.

2. Model description and problem formulation

In this paper we consider a new class of stochastic reaction-diffusion Cohen–Grossberg neural networks, which is described by the following integro-differential equation:

$$\begin{aligned} dy_i(t,x) = \Bigg\{ & \sum_{k=1}^{m} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial y_i(t,x)}{\partial x_k}\right) - \alpha_i(y_i(t,x)) \\ & \times \Bigg[\beta_i(y_i(t,x)) - \sum_{j=1}^{n} a_{ij} f_j(y_j(t,x)) \\ & - \sum_{j=1}^{n} b_{ij} g_j(y_j(t-\tau_1),x) - \sum_{j=1}^{n} c_{ij}\int_{t-\tau_2}^{t} h_j(y_j(s,x))\,ds\Bigg]\Bigg\}dt \\ & + \sum_{j=1}^{n}\sigma_{ij}(t,y_j(t,x),y_j(t-\tau_1,x),y_j(t-\tau_2,x))\,dw_j(t), \\ & t \ge 0,\ x\in S,\ i=1,2,\ldots,n, \end{aligned} \tag{1}$$

where $y_i(t,x)$ is the state variable of the ith neuron at time t and in space variable x, $\alpha_i(y_i(t,x))$ represents an amplification function of the ith unit at time t and in space variable x, and $\beta_i(y_i(t,x))$ is the behaved function of the ith unit at time t and in space variable x. The constants a_{ij}, b_{ij} and c_{ij} are the connection weight strengthes of the jth unit on the ith unit at time t. $f_j(y_j(t,x)), g_j(y_j(t,x))$ and $h_j(y_j(t,x))$ are the neuron activation functions of the jth unit at time t and in space variable x. The smooth function $D_{ik} = D_{ik}(t,x,y) \ge 0$ is a diffusion operator, S is a compact set with a smooth boundary ∂S of class $\mathcal{C}^2$ and measure mes $S>0$ in $\mathbb{R}^m$. The noise perturbation $\sigma_{ij}:[0,+\infty)\times\mathbb{R}\times\mathbb{R}\times\mathbb{R}\times S\to\mathbb{R}$ is a Borel measurable function, and $w_j(t), j=1,2,\ldots,n$ are scalar standard Brownian motions defined on a complete probability space $(\Omega,\mathcal{F},P)$ with a natural filtration $\{\mathcal{F}_t\}_{t\ge 0}$. The constants τ_1 and τ_2 are time delays.

Let $y=(y_1,y_2,\ldots,y_n)^T$ and $L_2(S)$ is the space of scalar value Lebesgue measurable functions on S, which is a Banach space for the L_2-norm

$$\|u\|_2 = \left(\int_S |u(x)|^2\,dx\right)^{1/2}, \quad u\in L_2(S),$$

then the norm $\|y\|$ is defined as

$$\|y\| = \left(\sum_{i=1}^{n}\|u_i\|_2^2\right)^{1/2}.$$

Take $\tau=\max\{\tau_1,\tau_2\}$, and $C([-\tau,0];\mathbb{R}^n)$ denotes the family of continuous function ϕ from $[-\tau,0]$ to $\mathbb{R}^n$ with the uniform norm $\|\phi\|=\sup_{-\tau\le s\le 0}|\phi(s)|$. Denote by $L^2_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$ the family of all $\mathcal{F}_t$ measurable, $C([-\tau,0];\mathbb{R}^n)$-valued stochastic variables $\phi=\{\phi(s,x):-\tau\le s\le 0\}$ such that $\int_{-\tau}^{0}\mathbf{E}|\phi(s)|^2\,ds<\infty$, where $\mathbf{E}[\cdot]$ stands for the correspondent expectation operator with respect to the given probability measure P. Then, the initial and the Neumann boundary condition of system (1) is given as follows:

$$\frac{\partial y_i}{\partial v} := \left(\frac{\partial y_i}{\partial x_1},\frac{\partial y_i}{\partial x_2},\ldots,\frac{\partial y_i}{\partial x_m}\right)^T = 0, \quad t\ge 0,\ x\in\partial S,\ i=1,2,\ldots,n, \tag{2}$$

$$y_i(t,x)=\phi_i(t,x), \quad -\tau\le t\le 0,\ x\in S,\ i=1,2,\ldots,n, \tag{3}$$

where $\phi\in L^2_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$.

Throughout this paper, we make the following assumptions.

Assumption 1. There exist positive constants α_i^0,α_i^1 $(i=1,2,\ldots,n)$ such that

$$0<\alpha_i^0\le\alpha_i(y_i(t,x))\le\alpha_i^1$$

for all $x\in\mathbb{R}$, $i=1,2,\ldots,n$.

Assumption 2. There exist positive constants γ_i $(i=1,2,\ldots,n)$ such that

$$y_i(t,x)\beta_i(y_i(t,x))\ge\gamma_i y_i^2(t,x)$$

for all $x\in\mathbb{R}$, $i=1,2,\ldots,n$.

Remark 1. The function $\beta_i(y_i(t,x))$ in [14,24] is required to be differentiable and its derivative is required to be over zero. However, the function $\beta_i(y_i(t,x))$ in Assumption 2 is not necessarily differentiable. For example, if taking $\beta_i(y_i(t,x))=2|y_i(t,x)|$ $(i=1,2,\ldots,n)$, then Assumption 2 is satisfied, but the conditions in [14,24] do not hold.