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Global exponential stability of neural networks with discrete and distributed delays and general activation functions on time scales

Bo Zhou [a], Qiankun Song [b,*], Huiwei Wang [a]

[a] College of Information Science & Engineering, Chongqing Jiaotong University, Chongqing 400074, PR China
[b] Department of Mathematics, Chongqing Jiaotong University, Chongqing 400074, PR China

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ABSTRACT

By employing time scale calculus theory, free weighting matrix method and linear matrix inequality (LMI) approach, several delay-dependent sufficient conditions are obtained to ensure the existence, uniqueness and global exponential stability of the equilibrium point for the neural networks with both infinite distributed delays and general activation functions on time scales. Both continuous-time and discrete-time neural networks are described under the same framework by the reported method. Illustrated numerical examples are given to show the effectiveness of the theoretical analysis. It is noteworthy that the activation functions are assumed to be neither bounded nor monotone.

1. Introduction

The theory of time scales, which was first introduced and studied by Hilger [14], has a tremendous potential for applications in some mathematical models of real processes and phenomena studied in control theory [15], population dynamics [10], economics [1] and so on. In standard dynamic economic model, a consumer receives some income in a time period and decides how much to consume and save during that same period. So, all decisions are assumed to be made at evenly spaced intervals. However, when a consumer receives income at one point in time, asset holdings are adjusted at a different point in time, and consumption takes place at yet another point in time. It is hard to overestimate the advantages of such an approach over the discrete or continuous models used in economics. The time scale model would allow exploration of a variety of situations in which timing of the decisions impacts the decisions themselves [1]. This novel and fascinating type of mathematics is more general and versatile than the traditional theory of differential and difference equations as it can mathematically describe the continuous and discrete dynamical equations under the same framework, hence it is the optimal way forward for accurate and malleable mathematical modeling. The field of dynamic equations on time scales contains, links and extends the classical theory of differential and difference equations. As it is well known, both continuous and discrete systems are very important in applications, but it is troublesome to study the stability for continuous and discrete systems, respectively. Therefore, it is significant to study the systems on time scales in which the continuous-time and discrete-time systems are unified.

In the past a few decades, many important results on stability analysis have been reported for continuous and discrete neural networks with delays, see, e.g. [3,5–9,11,13,17–34] and the references therein. It is noticed that, although the signal propagation is sometimes instantaneous and can be modeled with discrete delays, it may also be distributed during a certain time period so that the distributed delays should be incorporated in system. In other words, it is often the situation that the neural network system possesses both discrete and distributed delays [27].

Recently, the theory of time scales has received much attention. The article and book on time scales, by Agarwal et al. [2] and Bohner et al. [4], summarized and organized the time scale calculous theory, respectively. In [8], Chen et al. studied the global exponential stability of delayed BAM networks on time scales, in which several stability conditions were obtained. These results were extended to some more general classes of BAM neural networks on time scales in [7,17], respectively. It is noted that in [7,8,17], the stability results are delay-independent; that is, they do not include any information on the size of delays. It is known that delay-dependent stability conditions, which employ the information on the size of delays, are generally

* Corresponding author.
E-mail addresses: qiankunsong@163.com, qiankunsong@gmail.com (Q. Song).

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less conservative than delay-independent ones especially when the size of the delay is small [16,32].

Motivated by the above discussions, the object of this paper is to study the neural networks with both infinite distributed delays and general activation functions on time scales. The activation functions are assumed to be neither bounded nor monotone. The proposed existence, uniqueness and stability conditions in this paper are expressed in terms of LMIs, which are easy to be checked by recently developed algorithms solving LMIs. Furthermore, three examples with simulation are given to show the effectiveness of the theoretical analysis.

Notations: The notations are quite standard. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $m\times n$ real matrices. Matrix I represents the n-dimensional identity matrix. The subscript T denotes the matrix transposition. The notation $X\geq Y$ (respectively, $X>Y$) means that $X-Y$ is positive semi-definite (respectively, positive definite). $\|\cdot\|$ is the Euclidean norm operator. $\rho_{\max}(P)$ and $\rho_{\min}(P)$ are defined as the largest and the smallest eigenvalue of matrix P, respectively. Set $(-\infty,b]_{\mathbb{T}}$ is defined as: $(-\infty,b]_{\mathbb{T}}:=\{t\in\mathbb{T},-\infty<t\leq b\}$. Sometimes, the arguments of a function or a matrix will be omitted in the analysis when no confusion can arise.

2. Problems formulation and preliminaries

In this paper, we consider the following neural networks on time scale $\mathbb{T}$. $\mathbb{T}$ is an arbitrary nonempty closed subset of $\mathbb{R}$.

$$x^{\Delta}(t)=-Cx(t)+Df^{*}(x(t))+Ef^{*}(x(t-\tau(t)))+F\int_{-\infty}^{t}K(t-s)f^{*}(x(s))\,\Delta s+J, \tag{1}$$

for $t\in\mathbb{T}$, where $x^{\Delta}(t)$ is the delta derivative of function $x(t)$, which means that

$$x^{\Delta}(t)=\begin{cases}\lim\limits_{s\to t}\dfrac{x(t)-x(s)}{t-s}, & \sigma(t)=t,\\ \dfrac{x(\sigma(t)-x(t))}{\sigma(t)-t}, & \sigma(t)>t,\end{cases}$$

with $\sigma(t):=\inf\{s\in\mathbb{T}:s>t\}$.

In system (1), $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))^T$, $x_i(t)$ is the state of the ith neuron at time t. $f^*(x(t))=(f_1^*(x_1(t)),\ f_2^*(x_2(t)),\ldots,f_n^*(x_n(t)))^T$, $f^*(x(t-\tau(t)))=(f_1^*(x_1(t-\tau_1(t))),f_2^*(x_2(t-\tau_2(t))),\ldots,f_n^*(x_n(t-\tau_n(t))))^T$, $f_i^*(\cdot)$ is the activation function of the ith neuron. $J=(J_1,J_2,\ldots J_n)^T$ is the constant input vector. $\tau_i(t)$ corresponds to the discrete delay of neural networks which satisfies if $t\in\mathbb{T}$, then $t-\tau_i(t)\in\mathbb{T}$ and $0\leq\tau_i(t)\leq\tau$ (τ is a constant). $K(t)=\mathrm{diag}(k_1(t),k_2(t),\ldots,k_n(t))$, $k_i(\cdot)$ is the delay kernel function which is assumed to satisfy the underlying Assumption (H2). $C=\mathrm{diag}(c_1,c_2,\ldots,c_n)$, where c_i describes the rate with which the ith neuron will reset its potential to the resting state in isolation when disconnected from the networks and external inputs, $D=(d_{ij})_{n\times n}$ is the connection weight matrix, $E=(e_{ij})_{n\times n}$ is the discrete delay connection weight matrix and $F=(f_{ij})_{n\times n}$ is the distributed delay connection weight matrix.

The initial conditions of system (1) are of the forms

$$x_i(s)=\phi_i(s),\quad s\in(-\infty,0]_{\mathbb{T}},$$

with $\phi_i(s)(i=1,2,\ldots,n)$ is bounded and continuous on $(-\infty,0]_{\mathbb{T}}$.

In the following, we will give some useful definitions, which can be found in [4,2,28].

Definition 1 (*Bohner and Peterson [4]*). Let $\mathbb{T}$ be a time scale, we define the graininess $\mu(t):\mathbb{T}\to\mathbb{R}^+$, by

$$\mu(t)=\sigma(t)-t.$$

In the above definition, if $\mu(t)=0$, we say that t is right-dense; while if $\mu(t)>0$, we say that t is right-scattered.

Throughout this paper, we always assume that time scale $\mathbb{T}$ has a bounded graininess $\mu(t)\leq\overline{\mu}<\infty$.

Definition 2 (*Bohner and Peterson [4]*). A function $f:\mathbb{T}\to\mathbb{R}$ is called rd-continuous provided it is continuous at right-dense points on $\mathbb{T}$ and its left sided limits exist at left-dense points on $\mathbb{T}$. The set of rd-continuous functions $f:\mathbb{T}\to\mathbb{R}$ is denoted by

$$C_{rd}=C_{rd}(\mathbb{T})=C_{rd}(\mathbb{T},\mathbb{R}).$$

Definition 3 (*Bohner and Peterson [4]*). A function $f:\mathbb{T}\to\mathbb{R}$ is called regressive, if $1+\mu(t)f(t)\neq 0$, for all $t\in\mathbb{T}$. Let f and g be two regressive functions on $\mathbb{T}$, we define the operators $\oplus$ and $\ominus$ by

$$f(t)\oplus g(t)=f(t)+g(t)+\mu(t)f(t)g(t),\quad f(t)\ominus g(t)=f(t)\oplus(\ominus g(t)),$$

$$\ominus g(t)=\frac{g(t)}{1+\mu(t)g(t)}.$$

Definition 4 (*Bohner and Peterson [4]*). Function f is right-dense continuous, if $F^{\Delta}(t)=f(t)$, we define the delta integral by

$$\int_a^t f(s)\,\Delta s=F(t)-F(a).$$

It is easy to check that the following formula holds:

$$\int_t^{\sigma(t)} f(s)\,\Delta s=\mu(t)f(t).$$

Definition 5 (*Agarwal et al. [2]*). If p is a regressive function, then the generalized exponential function $e_{p(t)}(t,s)$ is defined by

$$e_{p(t)}(t,s)=\exp\left\{\int_s^t \xi_{\mu(\tau)}(p(\tau))\,\Delta\tau\right\},$$

with the function $\xi_h(z)$ defined by

$$\xi_h(z)=\begin{cases}\frac{\log(1+hz)}{h} & \text{if } h\neq 0,\\ z & \text{if } h=0.\end{cases}$$

Definition 6. System (1) is globally exponentially stable on time scale $\mathbb{T}$, if there exit positive constants ε and $M(\varepsilon)$ such that the solution $X(t)=(x_1(t),\ldots,x_n(t))$ of system (1) satisfies

$$\|x(t)\|\leq M(\varepsilon)e_{\ominus\varepsilon}(t,0)\max\{\|\phi(s)\|_{\mathbb{T}},\|\phi^{\Delta}(s)\|_{\mathbb{T}}\},$$

with $\|\phi(s)\|_{\mathbb{T}}=\sup_{s\in(-\infty,0]_{\mathbb{T}}}\|\phi(s)\|$ and $\|\phi^{\Delta}(s)\|_{\mathbb{T}}=\sup_{s\in(-\infty,0]_{\mathbb{T}}}\|\phi^{\Delta}(s)\|$.

Definition 7 (*Song [28]*). A map H: $\mathbb{R}^n\to\mathbb{R}^n$ is a homeomorphism of $\mathbb{R}^n$ onto itself, if $H\in C^0$, H is one-to-one, H is onto and the inverse map $H^{-1}\in C^0$.

To prove our results, the following lemmas are necessary.

Lemma 1 (*Chen and Chen [7]*). *If $p,q\in\mathbb{R}$ then*

$$e_p(t,s)>0,$$

$$e_p(t,t)=1,$$

$$e_p(\sigma(t),s)=(1+\mu(t)p)e_p(t,s),$$

$$e_p(t,r)e_p(r,s)=e_p(t,s),$$

$$e_p(t,s)=\frac{1}{e_p(s,t)}=e_{\ominus p}(t,s).$$

Lemma 2 (*Agarwal et al. [2]*). *If f and g are two delta differentiable functions on time scale $\mathbb{T}$, then the following formula holds*

$$(fg)^{\Delta}(t)=f^{\Delta}(t)g(t)+f(\sigma(t))g^{\Delta}(t)=g^{\Delta}(t)f(t)+g(\sigma(t))f^{\Delta}(t).$$