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Global existence of periodic solutions in a six-neuron BAM neural network model with discrete delays ☆

Changjin Xu [a,*], Xiaofei He [b], Peiluan Li [c]

[a] Guizhou Key Laboratory of Economics System Simulation, Guizhou College of Finance and Economics, Guiyang 550004, PR China
[b] Zhangjiajie College of Jishou University, Zhangjiajie 427000, PR China
[c] Department of Mathematics and Statistics, Henan University of Science and Technology, Luoyang 471003, PR China

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ABSTRACT

In this paper, a six-neuron BAM neural network model with discrete delays is considered. Using the global Hopf bifurcation theorem for FDE due to Wu [Symmetric functional differential equations and neural networks with memory, Trans. Am. Math. Soc. 350 (1998) 4799–4838] and the Bendixson's criterion for high-dimensional ODE due to Li and Muldowney [On Bendixson' criterion, J. Differential Equations 106 (1994) 27–39], a set of sufficient conditions for the system to have multiple periodic solutions are derived when the sum of delays is sufficiently large.

1. Introduction

Since Hopfield [1] constructed a simplified neural network model and Marcus and Westervelt [2] proposed a neural network model incorporating time delay for the no instantaneous transmission of information from one neuron to another, there has been considerable activity on the dynamics properties (including stable, unstable, oscillatory and chaotic behavior) of neural networks with delays. Many excellent and interesting results have been obtained (see [15–32]). In many applications, the properties of periodic solutions are of great interest. There are a great many papers which deal with the periodic phenomena of neural networks, for example, Dong and Sun [3] investigated the global existence of periodic solution in a special neural network model with two delays:

$$\begin{cases} \dot{u}_1(t) = -u_1(t) + a_{12}f_{12}(u_2(t-\tau_2)), \\ \dot{u}_2(t) = -u_2(t) + a_{21}f_{21}(u_2(t-\tau_1)). \end{cases} \tag{1.1}$$

Wei and Li [4] investigated the global existence of periodic solution in the following tri-neuron network model with delays:

$$\begin{cases} \dot{u}_1(t) = -a_1u_1(t) + f_1(u_3(t-\tau_1)), \\ \dot{u}_2(t) = -a_2u_2(t) + f_2(u_1(t-\tau_2)), \\ \dot{u}_3(t) = -a_3u_3(t) + f_3(u_2(t-\tau_3)). \end{cases} \tag{1.2}$$

Cao and Xiao [5] studied the local stability and local Hopf bifurcation on the following simplified BAM neural network with two delays:

$$\begin{cases} \dot{x}_1(t) = -\mu_1x_1(t) + c_{21}f_1(y_1(t-\tau_2)) \\ \qquad\qquad + c_{31}f_1(y_2(t-\tau_2)) + c_{41}f_1(y_3(t-\tau_2)), \\ \dot{y}_1(t) = -\mu_2y_1(t) + c_{12}f_2(x_1(t-\tau_1)), \\ \dot{y}_2(t) = -\mu_3y_2(t) + c_{13}f_3(x_1(t-\tau_1)), \\ \dot{y}_3(t) = -\mu_4y_3(t) + c_{14}f_4(x_1(t-\tau_1)). \end{cases} \tag{1.3}$$

Yu and Cao [6] studied the local stability and local Hopf bifurcation on the following BAM neural network with delays:

$$\begin{cases} \dot{x}_1(t) = -\mu_1x_1(t) + c_{11}f_{11}(y_1(t-\tau_3)) + c_{12}f_{12}(y_2(t-\tau_3)), \\ \dot{x}_2(t) = -\mu_2x_2(t) + c_{21}f_{21}(y_1(t-\tau_4)) + c_{22}f_{22}(y_2(t-\tau_4)), \\ \dot{y}_1(t) = -\mu_3y_1(t) + d_{11}g_{11}(x_1(t-\tau_1)) + d_{12}g_{12}(x_2(t-\tau_2)), \\ \dot{y}_2(t) = -\mu_4y_2(t) + d_{21}g_{21}(x_1(t-\tau_1)) + d_{22}g_{22}(x_2(t-\tau_2)). \end{cases} \tag{1.4}$$

Sun et al. [7] studied the global Hopf bifurcation of system (1.4). Motivated by [3–7], Xu et al. [8] investigated the nontrivial periodic solutions bifurcating from local Hopf bifurcation of the

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* Corresponding author. Tel./fax: +86 732 2680745.
E-mail address: xcj403@126.com (C. Xu).

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following system:

$$\begin{cases}\dot{x}_1(t)=-\mu_1x_1(t)+c_{11}f_{11}(y_1(t-\tau_4))+c_{12}f_{12}(y_2(t-\tau_4))\\ \qquad +c_{13}f_{13}(y_3(t-\tau_4)),\\ \dot{x}_2(t)=-\mu_2x_2(t)+c_{21}f_{21}(y_1(t-\tau_5))+c_{22}f_{22}(y_2(t-\tau_5))\\ \qquad +c_{23}f_{23}(y_3(t-\tau_5)),\\ \dot{x}_3(t)=-\mu_3x_3(t)+c_{31}f_{31}(y_1(t-\tau_6))+c_{32}f_{32}(y_2(t-\tau_6))\\ \qquad +c_{33}f_{33}(y_3(t-\tau_6)),\\ \dot{y}_1(t)=-\mu_4y_1(t)+c_{41}f_{41}(x_1(t-\tau_1))+c_{42}f_{42}(x_2(t-\tau_2))\\ \qquad +c_{43}f_{43}(x_3(t-\tau_3)),\\ \dot{y}_2(t)=-\mu_5y_2(t)+c_{51}f_{51}(x_1(t-\tau_1))+c_{52}f_{52}(x_2(t-\tau_2))\\ \qquad +c_{53}f_{53}(x_3(t-\tau_3)),\\ \dot{y}_3(t)=-\mu_6y_3(t)+c_{61}f_{61}(x_1(t-\tau_1))+c_{62}f_{62}(x_2(t-\tau_2))\\ \qquad +c_{63}f_{63}(x_3(t-\tau_3)).\end{cases} \tag{1.5}$$

It is worth pointing out that it is important mathematical subject to investigate if these nontrivial periodic solutions of system (1.5) exist globally. In this paper, we will devote our attention to the global existence of periodic solutions to system (1.5).

In order to establish the main results for model (1.5), it is necessary to make the following assumptions:

(H1) For $i=1,2,3,4,5,6,\ j=1,2,3,$ constants $\mu_i>0$, $f_{ij}\in C^k, f_{ij}(0)=0$ $(k=1,2,3,\ldots)$, and there exists $L>0$ such that $|f_{ij}(u)|\le L$ for all $u\in R$.
(H2) $\tau_1+\tau_4=\tau_2+\tau_5=\tau_3+\tau_6=\tau$.

The purpose of this paper is to investigate the global existence of nontrivial periodic solutions for model (1.5). Recently, a great deal of papers have been devoted to this topic [3,4,7,8,10,11,28–30], One method used is the ejective fixed point argument developed by [9]. Our method for showing the existence of nontrivial periodic solutions is the S^1-equivariant degree (see [10,11]). More precisely, we shall use a global Hopf bifurcation result due to Wu [10] for functional differential equations, which was established using purely topological argument. Meanwhile, the Bendixson's criterion for ordinary differential equations will be used to rule out the existence of nontrivial periodic solution for zero delays. We would like to mention that there are few papers related to the high-dimensional neural networks system with multiple delays. To the best of our knowledge, it is the first time to deal with the global existence of nontrivial periodic solutions of system (1.5).

This paper is organized as follows. In Section 2, the preliminary results on the stability of the equilibrium and the existence of Hopf bifurcation at the equilibrium for system (1.5) in [8] are stated, the general mathematical framework: a global Hopf bifurcation theory of Wu [10], is outlined and some results which will be useful in Section 3 are derived. The global existence of nontrivial periodic solutions is discussed in Section 3. In Section 4, numerical simulations are carried out to illustrate the validity of the main results. Some main conclusions are drawn in Section 5.

2. Preliminary results

Let

$$\begin{cases}u_1(t)=x_1(t-\tau_1),\\ u_2(t)=x_2(t-\tau_2),\\ u_3(t)=x_3(t-\tau_3),\\ u_4(t)=y_1(t),\\ u_5(t)=y_2(t),\\ u_6(t)=y_3(t)\end{cases} \tag{2.1}$$

then system (1.5) takes the following equivalent form:

$$\begin{cases}\dot{u}_1(t)=-\mu_1u_1(t)+c_{11}f_{11}(u_4(t-\tau))+c_{12}f_{12}(u_5(t-\tau))\\ \qquad +c_{13}f_{13}(u_6(t-\tau)),\\ \dot{u}_2(t)=-\mu_2u_2(t)+c_{21}f_{21}(u_4(t-\tau))+c_{22}f_{22}(u_5(t-\tau))\\ \qquad +c_{23}f_{23}(u_6(t-\tau)),\\ \dot{u}_3(t)=-\mu_3u_3(t)+c_{31}f_{31}(u_4(t-\tau))+c_{32}f_{32}(u_5(t-\tau))\\ \qquad +c_{33}f_{33}(u_6(t-\tau)),\\ \dot{u}_4(t)=-\mu_4u_4(t)+c_{41}f_{41}(u_1(t))+c_{42}f_{42}(u_2(t))+c_{43}f_{43}(u_3(t)),\\ \dot{u}_5(t)=-\mu_5u_5(t)+c_{51}f_{51}(u_1(t))+c_{52}f_{52}(u_2(t))+c_{53}f_{53}(u_3(t)),\\ \dot{u}_6(t)=-\mu_6u_6(t)+c_{61}f_{61}(u_1(t))+c_{62}f_{62}(u_2(t))+c_{63}f_{63}(u_3(t)).\end{cases} \tag{2.2}$$

By the hypothesis (H1), it is easy to see that (2.2) has a unique equilibrium $u_*(0,0,0,0,0,0)$. Under the hypotheses (H1) and (H2), the linear equation (2.2) at $u_*(0,0,0,0,0,0)$ takes the form:

$$\begin{cases}\dot{u}_1(t)=-\mu_1u_1(t)+\alpha_{11}u_4(t-\tau)+\alpha_{12}u_5(t-\tau)+\alpha_{13}u_6(t-\tau),\\ \dot{u}_2(t)=-\mu_2u_2(t)+\alpha_{21}u_4(t-\tau)+\alpha_{22}u_5(t-\tau)+\alpha_{23}u_6(t-\tau),\\ \dot{u}_3(t)=-\mu_3u_3(t)+\alpha_{31}u_4(t-\tau)+\alpha_{32}u_5(t-\tau)+\alpha_{33}u_6(t-\tau),\\ \dot{u}_4(t)=-\mu_4u_4(t)+\alpha_{41}u_1(t)+\alpha_{42}u_2(t)+\alpha_{43}u_3(t),\\ \dot{u}_5(t)=-\mu_5u_5(t)+\alpha_{51}u_1(t)+\alpha_{52}u_2(t)+\alpha_{53}u_3(t),\\ \dot{u}_6(t)=-\mu_6u_6(t)+\alpha_{61}u_1(t)+\alpha_{62}u_2(t)+\alpha_{63}u_3(t),\end{cases} \tag{2.3}$$

where $\alpha_{ij}=c_{ij}f'_{ij}(0)$ $(i=1,2,3,4,5,6, j=1,2,3)$. Then the associated characteristic equation of (2.3) is

$$p_1(\lambda)e^{\lambda\tau}+p_2(\lambda)+p_3(\lambda)e^{-\lambda\tau}+p_4(\lambda)e^{-2\lambda\tau}=0, \tag{2.4}$$

where p_i $(i=1,2,3,4)$ are defined by Appendix A.

For the benefit of readers, we stated the main results of Xu et al. [8].

Lemma 2.1. *If* (*H*1)–(*H*4) *hold, then for system* (1.5), *its zero solution is asymptotically stable for* $\tau\in[0,\tilde{\tau}_0)$ *and unstable when* $\tau>\tilde{\tau}_k$ $(k=0,1,2,\ldots)$, $\pm i\omega_0$ *are a pair of imaginary roots of* (2.4), *and system* (1.5) *undergoes a Hopf bifurcation at the origin when* $\tau=\tilde{\tau}_k$, *i.e., system* (1.5) *has a branch of periodic solutions bifurcating from the zero solution near* $\tau=\tilde{\tau}_k$, *where the meaning of conditions* (*H*3) *and* (*H*4) *are listed in Appendix A and* $\tilde{\tau}_k=\min\{\tau_1^{(k)},\tau_2^{(k)}\}$ $(k=0,1,2,\ldots)$, *where* $\tau_1^{(k)}$ *and* $\tau_2^{(k)}$ *are defined by* (2.18) *and* (2.21) *in* [8], *respectively, the definition of* ω_0, *one can see* [8].

To extend the local Hopf bifurcation branches described in Lemma 2.1 for large delay values, we apply a global Hopf bifurcation result of Wu [10]. For the convenience of the reader, we briefly explain in the following.

Let X be the Banach space of bounded continuous mappings $x:R\to R^n$ with the supreme norm. For $x\in X, t\in R$, define $x_t\in X$ as $x_t\in s=x(t+s)$ for $s\in R$. Consider a functional differential equation

$$x'(t)=F(x_t,\alpha,T) \tag{2.5}$$

parameterized by two real parameters $(\alpha,T)\in R\times R_+$, where $R_+=(0,+\infty)$ and $F:X\times R\times R_+\to R^n$ is completely continuous. Restrict F to the subspace of constant functions x, which is identified with R^n, we obtain a mapping $\widehat{F}=F|_{R^n\times R\times R_+}\to R^n$. Assume that

(A1) $\widehat{F}$ is C^2.
(A2) $D_x\widehat{F}(x,\alpha,T)|_{(\hat{x}_0,\alpha_0,T_0)}$ is an isomorphism at each stationary solution $(\hat{x}_0,\alpha_0,T_0)$.
Define the characteristic matrix at a stationary solution $(\hat{x}_0,\alpha_0,T_0)$ of (2.5), as

$$\Delta_{(\hat{x}_0,\alpha_0,T_0)}(\lambda)=\lambda Id-D_\varphi F(\hat{x}_0,\alpha_0,T_0)(e^{\lambda}Id), \tag{2.6}$$

where $D_\varphi F(\hat{x}_0,\alpha_0,T_0)$ is the derivative of $\widehat{F}(x,\alpha,T)$ with respect to φ at $(\hat{x}_0,\alpha_0,T_0)$ (see Assumption (A3)).
(A3) $\widehat{F}(x,\alpha,T)$ is differential with respect to φ. The characteristic matrix $\Delta_{(\hat{y}(\alpha,T),\alpha,T)}(\lambda)$ is continuous in $(\alpha,T,\lambda)\in B_{\varepsilon_0}(\alpha_0,T_0)\times C$.