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Variable-time impulses in BAM neural networks with delays

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ABSTRACT

In this paper, the globally exponential stability of BAM neural networks with time delays and impulses has been studied. Different from most existing publications, the case of variable time impulses is dealt with in the present paper, i.e., impulse occurring is not at fixed instants but depends on the states of systems. By using Lyapunov function and inequality technique, some globally exponential stability criteria of BAM neural networks with time delays and variable-time impulses have been established. When the proposed results can also be applied to the case of fixed-time impulses, it provides new stability conditions for the case of fixed-time impulses. Numerical examples are also given to illustrate the effectiveness of our theoretical results.

1. Introduction

In recent years, the BAM neural networks first proposed by Kosko [1–3] have been attracting more and more attention of researchers. The original form of BAM neural networks is described by

$$\begin{cases} \frac{dx(t)}{dt} = -Ax(t)+B\tilde{f}(y(t))+S, \\ \frac{dy(t)}{dt} = -Cy(t)+D\tilde{g}(x(t))+R, \quad t \geq t_0. \end{cases} \tag{1}$$

These networks consist of two layers, namely, the X-layer and the Y-layer. $x=(x_1,x_2,\ldots,x_m)^T$ denotes the activations of the set of m neurons in X-layer, $y=(y_1,y_2,\ldots,y_n)^T$ denotes the activations of the set of n neurons in Y-layer. $S=(s_1,s_2,\ldots,s_m)^T$ and $R=(r_1,r_2,\ldots,r_n)^T$ are the external inputs. The positive definite diagonal matrices $A=\mathrm{diag}(a_1,a_2,\ldots,a_n)$ and $C=\mathrm{diag}(c_1,c_2,\ldots,c_n)$ denote the decay rates of the neurons. $B=[b_{ij}]_{m\times n}$ and $D=[d_{ji}]_{n\times m}$ are the connection weight matrices, which denote the strengths of connectivity between the X-layer and the Y-layer, respectively. $\tilde{f}(y)=(\tilde{f}_1(y_1),\tilde{f}_2(y_2),\ldots,\tilde{f}_n(y_n))^T$ and $\tilde{g}(x)=(\tilde{g}_1(x_1),\ \tilde{g}_2(x_2),\ldots,\ \tilde{g}_m(x_m))^T$ are activation functions. In application, we will pay attention to the dynamical behaviors of BAM neural networks. Therefore, it is worthy to investigate the stability of the equilibrium points [4].

In system (1), it is implicitly assumed that the neurons process inputs and produce outputs instantaneously, and such outputs are transmitted to the receiving neurons instantly. However, it is well known that time delays are unavoidable due to finite switching speeds of the amplifiers [5] and influence severely the stability of networks in some cases. Introducing time delays into BAM neural networks, the model (1) can be modified as follows:

$$\begin{cases} \frac{dx(t)}{dt} = -Ax(t)+B\tilde{f}(y(t-\sigma))+S, \\ \frac{dy(t)}{dt} = -Cy(t)+D\tilde{g}(x(t-\tau))+R, \quad t \geq t_0, \end{cases} \tag{2}$$

where $\tilde{f}(y(t-\sigma))=(\tilde{f}_1(y_1(t-\sigma_1)),\tilde{f}_2(y_2(t-\sigma_2)),\ldots,\tilde{f}_n(y_n(t-\sigma_n)))^T$, $\tilde{g}(x(t-\tau))=(\tilde{g}_1(x_1(t-\tau_1)),\tilde{g}_2(x_2(t-\tau_2)),\ldots,\tilde{g}_m(x_m(t-\tau_m)))^T$, $\tau_i(i=1,2,\ldots,m)$ and $\sigma_j(j=1,2,\ldots,n)$ are nonnegative constants representing the time lags of neural processing and delivery of signals. About the stability analysis on the equilibrium points of (2) or its generalized form, please refer to the Refs. [6–16].

Besides delay effects, impulsive effects also likely exist in neural networks for switching phenomenon, frequency change or other sudden noise. In impulsive neural networks, the states of the networks will be changed instantaneous at impulse instants. The impulsive version of system (2) with fixed-time impulses can be written as follows:

$$\begin{cases} \frac{dx(t)}{dt} = -Ax(t)+B\tilde{f}(y(t-\sigma))+S, & t \neq t_k, \\ \Delta x(t_k)=\tilde{I}_k(x(t_k^-)), & k=1,2,\ldots, \\ \frac{dy(t)}{dt} = -Cy(t)+D\tilde{g}(x(t-\tau))+R, & t \neq t_k, \\ \Delta y(t_k)=\tilde{J}_k(y(t_k^-)), & k=1,2,\ldots, \end{cases} \tag{3}$$

where $x(t_k^-)=\lim_{t\to t_k-0}x(t)$ and $y(t_k^-)=\lim_{t\to t_k-0}y(t)$ are the left limits of $x(t)$ and $y(t)$ at time t_k, respectively. The impulse instants $\{t_k\}$ satisfy $t_0<t_1<t_2<\ldots$ and $\lim_{k\to\infty}t_k=+\infty$. More details about impulsive control theory and stability criteria of impulsive BAM neural networks can be referred in [17–27].

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In the real world, the impulses of many systems do not occur at fixed time, such as saving rates control systems, population control systems, some circuit control systems and so on. In these impulsive systems, the time at which impulses occur is relevant intimately to the current state. We called these systems as state-dependent impulsive differential systems or differential systems with variable-time impulses. Yang [17], Fu et al. [28], Wang and Fu [29] and Bainov and Stamova [35] have obtained much significant theories about impulsive differential systems with variable-time impulses. However, most existing publications focus on the BAM neural networks with fixed-time impulses or the delay-free case with variable-time impulses. To the best of our knowledge, few (if any) results have been reported, on both time delays and variable-time impulses.

Based on the above discussions, in this paper we consider BAM neural networks with time delays and variable-time impulses and study their stability. Some stability criteria are established using the Lyapunov function and the inequality technique. These criteria can also easily generalize the case with fixed-time impulses.

The rest of this paper is organized as follows. In Section 2, we introduce some definitions and lemmas. In Section 3, we consider the stability of impulsive BAM neural networks and establish some stability criteria. In Section 4, some numeric examples are given to illustrate the effectiveness of our theoretical results. Finally, some conclusions are drawn in Section 5.

2. Preliminaries

In this paper, let us consider BAM neural networks with time delays and variable-time impulses of the form

$$\begin{cases} \frac{du(t)}{dt} = -Au(t)+B\tilde{f}(v(t-\sigma))+S, \\ \frac{dv(t)}{dt} = -Cv(t)+D\tilde{g}(u(t-\tau))+R, & t \neq \tilde{t}_k(u,v), \\ \Delta u(t) = \tilde{I}_k(u(t^-)), \\ \Delta v(t) = \tilde{J}_k(v(t^-)), & t = \tilde{t}_k(u,v), \end{cases} \tag{4}$$

where $A,C,B,D,S,R,\tilde{I}_k,\tilde{f},\tilde{g}$ and $\tilde{J}_k$ are consistent with (1) and (2), $\tilde{t}_k \in C[R^{m+n},R_+]$ with $C[S_1,S_2]=\{\omega:S_1 \to S_2 | \omega$ is continuous on $S_1\}$, $\tilde{t}_k(x) < \tilde{t}_{k+1}(x)$, $\lim_{k\to\infty}\tilde{t}_k(x) = +\infty$ for any $x \in R^{m+n}$.

In this paper, it is always assumed that the existence of the solutions to (4) and the existence of equilibrium point are guaranteed by appropriate activation functions and jump operators. The readers are referred to [30,36] for more details. Let (u^*,v^*) be an equilibrium point of system (4). As usual, we shift it to the origin by the transformation $x(t)=u(t)-u^*$ and $y(t)=v(t)-v^*$. Then system (4) can be rewritten as

$$\begin{cases} \dot{x}(t) = -Ax(t)+Bf(y(t-\sigma)), \\ \dot{y}(t) = -Cy(t)+Dg(x(t-\tau)), & t \neq t_k(x,y), \\ \Delta x(t) = I_k(x(t)), \\ \Delta y(t) = J_k(y(t)), & t = t_k(x,y), \end{cases} \tag{5}$$

where $f(y(t-\sigma)) = \tilde{f}(y(t-\sigma)+v^*)-\tilde{f}(v^*)$, $g(x(t-\tau)) = \tilde{g}(x(t-\tau)+u^*)-\tilde{g}(u^*)$, $I_k(x(t)) = \tilde{I}_k(x(t)+u^*)$, $J_k(y(t)) = \tilde{J}_k(y(t)+v^*)$, $t_k(x,y) = \tilde{t}_k(x+u^*, y+v^*)$.

The initial values associated with (4) are assumed to be given by

$$\begin{cases} u(t_0+s) = \varphi_u(s), \\ v(t_0+s) = \varphi_v(s), & s \in [-\overline{\tau},0], \end{cases} \tag{6}$$

where $\varphi_u(s)$ and $\varphi_v(s)$ are real-valued continuous and bound functions defined on $[-\overline{\tau},0]$ with $\overline{\tau} = \max\{\tau_1,\tau_2,\ldots,\tau_m,\sigma_1,\sigma_2,\ldots,\sigma_n\}$. Set $\varphi(s) = [[\varphi_u(s)]^T,[\varphi_v(s)]^T]^T, s \in [-\overline{\tau},0]$. The norm of $\varphi(s)$ is defined by $\|\varphi(s)\|_{\overline{\tau}} = \sup_{-\overline{\tau}\le s \le 0}\|\varphi(s)\|_2$.

As usual, we assume that

(H₁). *There exist positive constants* $l_j^f,l_i^g (i=1,2,\ldots,m,\ j=1,2,\ldots,n)$ *such that* $|\tilde{f}_j(x_1)-\tilde{f}_j(x_2)| \le l_j^f|x_1-x_2|$, $|\tilde{g}_i(x_1)-\tilde{g}_i(x_2)| \le l_i^g|x_1-x_2|$ *for any* $x_1,x_2 \in R$. *We denote* $L_1 = diag(l_1^f,l_2^f,\ldots,l_n^f)$, $L_2 = diag(l_1^g,l_2^g,\ldots,l_m^g)$, $l_{\max}^f = \max\{l_1^f,l_2^f,\ldots,l_n^f\}$, $l_{\max}^g = \max\{l_1^g,l_2^g,\ldots,l_m^g\}$.

(H₂). $\tilde{I}_{ik}(u_i(t)) = \gamma_{ik}(u_i(t)-u_i^*)$ and $\tilde{J}_{jk}(v_j(t)) = \overline{\gamma}_{jk}(v_j(t)-v_j^*)$.

Next, we introduce some notations and cite some basic definitions and lemmas.

$G_k = \{(t,x) \in R_+ \times R^{m+n} : \tilde{t}_{k-1}(x) \le t < \tilde{t}_k(x)\}$;
$S_k = \{(t,u,v) \in R_+ \times R^m \times R^n : t = \tilde{t}_k(u,v)\}$;

$V_0 = \{V : R_+ \times R^m \times R^n \to R_+$ is continuous in G_k and locally Lipschitzian in (u,v), $V(t,0,0)=0$ for all $t \in R_+$, and the limit $\lim_{\substack{(t,x,y)\to(t_k,u,v)\\(t,x,y)\in G_k}} V(t,x,y) = V(t_k^-,u,v)$ exists$\}$; $N_k = \{t \in R_+$: there exists $x \in R^{m+n}$ such that $(t,x) \in S_k\}$

$d_k = d(N_{k-1},N_k) = \inf_{\substack{\overline{t}\in N_k\\ t\in N_{k-1}}} |\overline{t}-t|$; $d_k^* = \sup_{\substack{\overline{t}\in N_k\\ t\in N_{k-1}}} |\overline{t}-t|$;

$$z(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix},\ \tilde{z}(t) = \begin{pmatrix} u(t) \\ v(t) \end{pmatrix},\ Z_k = \begin{pmatrix} \tilde{I}_k(u) \\ \tilde{J}_k(v) \end{pmatrix};$$

$$F(t) = \begin{bmatrix} -Au(t)+B\tilde{f}(v(t-\sigma))+S \\ O_{n\times 1} \end{bmatrix} + \begin{bmatrix} O_{m\times 1} \\ -Cv(t)+D\tilde{g}(u(t-\tau))+R \end{bmatrix};$$

$\lambda_{max}(\cdot)$ and $\lambda_{min}(\cdot)$ denote the maximum and the minimum eigenvalues of the corresponding matrix, respectively.

Definition 1. The equilibrium point (u^*,v^*) of system (4) with (6) is said to be globally exponentially stable if there exist constants $\varepsilon > 0$ and $M > 0$ such that $\|\tilde{z}(t)-\varphi^*\| \le M|\|\varphi-\varphi^*\|_\tau e^{-\varepsilon(t-t_0)}$ where $\varphi^* = \begin{pmatrix} u^* \\ v^* \end{pmatrix}$, for any $t \ge t_0$.

The solutions of system (4) may hit the same switching surface S_k finite or infinite number of times causing "beating phenomenon" or "pulse phenomenon" [17]. Using the similar way in [31], we have the following lemma, which can guarantee the beating phenomenon does not exist. The readers refer to [30–31] for more details.

Lemma 1. *If*

1. *for any* $k=1,2,\ldots$, $\tilde{t}_k(u,v)$ *is bounded with respect to u and v;*
2. *for any* (t_0,u_0,v_0), *there exists a solution of differential system without impulse corresponding to system (4) in* $[t_0,+\infty)$;
3. $\partial\tilde{t}(\tilde{z})/\partial\tilde{z}F(t) < 1$;
4. $\partial\tilde{t}_k(\tilde{z}+sZ_k(\tilde{z}))/\partial\tilde{z}Z_k(\tilde{z}) \le 0,\ 0 \le s \le 1,\ \tilde{t}_{k+1}(\tilde{z}+Z_k) > \tilde{t}_k(\tilde{z})$.

Then there exists a solution of system (4) in $[t_0,+\infty)$, *and it hits each switching surface* S_k *exactly once in turn.*

Proof. Let $\tilde{z}(t) = \tilde{z}(t,t_0,u_0,v_0)$ be any solution of system (4) such that $0 \le t_0 < \tilde{t}_1(\tilde{u}_0,\tilde{v}_0)$. Since $\tilde{t}_1(\tilde{z})$ is bound and continuous, there is a unique $t_1 > t_0$ such that $t_1 = \tilde{t}_1(\tilde{z}(t_1))$ and $t_1 = \tilde{t}_1(\tilde{z}(t_1))$, for $t < t_1$. Hence, $\tilde{z}(t)$ hits the surface S_1 at $t=t_1$.

Now, setting $\tilde{z}_1 = \tilde{z}(t_1)$, $\tilde{z}_1^+ = \tilde{z}_1+Z_1(\tilde{z}_1)$, we can obtain from (4)

$$t_1 = \tilde{t}_1(\tilde{z}_1) \ge \tilde{t}_1(\tilde{z}_1+Z_1(\tilde{z}_1)) = \tilde{t}_1(\tilde{z}_1^+).$$

On the other hand, from (4) we have

$$\tilde{t}_2(\tilde{z}_1^+) = \tilde{t}_2(\tilde{z}_1+Z_1(\tilde{z}_1)) > \tilde{t}_1(\tilde{z}_1) = t_1,$$