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Global exponential synchronization of generalized stochastic neural networks with mixed time-varying delays and reaction-diffusion terms ☆

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ABSTRACT

This paper investigates the synchronization problem of generalized stochastic neural networks with mixed time-varying delays and reaction-diffusion terms using linear feedback control. Lyapunov stability theory combining with stochastic analysis approaches is employed to derive sufficient criteria ensuring the coupled chaotic generalized stochastic neural networks to be globally exponentially synchronized. The generalized neural networks model considered includes reaction-diffusion Hopfield neural networks, reaction-diffusion bidirectional associative memory neural networks, and reaction-diffusion cellular neural networks as its special cases. It is theoretically proven that these synchronization criteria are more effective than some existing ones. This paper also presents some illustrative examples and uses simulated results of these examples to show the feasibility and effectiveness of the proposed scheme.

1. Introduction

Since the pioneering work on Hopfield neural networks in [5], the investigation of the dynamics of neural networks has been the subject of much recent activity due to their promising potential applications such as signal processing, pattern recognition, automatic control engineering, artificial intelligence, optimization, fault diagnosis and associative memories. Such applications heavily depend on the dynamical behaviors. Therefore, the qualitative analysis of the dynamical behaviors is a necessary step for the practical design and application of neural networks. Recently, considerable attention has been made on the study of various types of neural network models such as Hopfield neural networks model, bidirectional associative memory neural networks model as well as cellular neural networks model and a large body of work has been reported in the literature (see, e.g., [8,13,18,21,22,32] and the references therein).

It is well known that there exist time delays in the information processing of neurons. Since time delays as a source of instability and bad performance always appear in many neural networks owing to the finite speed of information processing, the stability analysis for the delayed neural networks has received considerable attention. However, in these recent publications, most research on delayed neural networks has been restricted to simple cases of discrete delays. Since a neural network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths, it is desired to model them by introducing distributed delays. Therefore, both discrete and distributed delays, especially both discrete and distributed time-varying delays should be taken into account when modeling realistic neural networks [7,12,16,19,27,28,34]. At the same time, the synchronization problems of coupled chaotic neural networks with mixed time-varying delays have also been intensively investigated in the last two decades due to its potential applications in various technological fields, including chaos generators design, secure communications, chemical reactions, biological neural networks, information processing, power systems protection, etc. (see [2,7,24,25,37] and the references therein).

In addition, noise disturbance is a major source of instability and may lead to poor performances in neural networks. In real nervous systems, the synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. It has also been known that neural networks could be stabilized or destabilized by certain stochastic inputs [1,9]. On the other hand, diffusion effects cannot be avoided in the neural networks when electrons are moving in asymmetric electromagnetic fields. So we must consider that the activations vary in space as well as in time. Hence, great attention has been paid on the synchronization analysis for delayed neural networks with stochastic perturbation

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and reaction-diffusion terms, and some initial results have been obtained. For example, in [33], the synchronization scheme was discussed for a class of delayed neural networks with reaction-diffusion terms by using inequality techniques and Lyapunov method; the authors discussed the global exponential stability and synchronization of the delayed reaction-diffusion neural networks with Dirichlet boundary conditions under the impulsive control in terms of p-norm in [6]; in [14], the problem of asymptotic synchronization for a class of neural networks with reaction-diffusion terms and discrete time-varying delays was investigated; the feedback control approach has been proposed to synchronize a class of stochastic reaction-diffusion neural networks with discrete time-varying delays and Dirichlet boundary conditions by Ma et al. in [17]; Sheng and Gao [23] studied the asymptotic and exponential synchronization for a class of bidirectional associative memory neural networks with reaction-diffusion terms and discrete constant delays; in [31], the authors proposed a linear feedback controller to guarantee global exponential synchronization for a class of reaction-diffusion cellular neural networks with Dirichlet boundary conditions and discrete time-varying delays.

To the best of the authors' knowledge, synchronization problem of generalized stochastic neural networks model with mixed time-varying delays and reaction-diffusion terms which includes reaction-diffusion Hopfield neural networks model, the reaction-diffusion bidirectional associative memory neural networks model and reaction-diffusion cellular neural networks model has not been studied in the literature and it is interesting to study this problem both in theory and in applications, so there exist open room for further improvement. This situation motivates our present investigation. This paper is concerned with global exponential synchronization problem of coupled chaotic generalized stochastic neural networks with mixed time-varying delays and reaction-diffusion terms.

The organization of this paper is as follows: in the next section, model description and preliminary results are presented; in Section 3, a simple linear feedback control scheme is proposed to ensure global exponential synchronization of generalized stochastic neural networks with mixed time-varying delays and reaction-diffusion terms; numerical simulations will be given in Section 4 to demonstrate the effectiveness and feasibility of our theoretical results. Finally, conclusions are drawn in Section 5.

Notation. Throughout this paper, $\mathbf{R}^n$ and $\mathbf{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively; the notation $C^{2,1}(\mathbf{R}^+\times\mathbf{R}^n;\mathbf{R}^+)$ denotes the family of all nonnegative functions $V(t,x(t))$ on $\mathbf{R}^+\times\mathbf{R}^n$ which are continuously twice differentiable in x and once differentiable in t; $(\Omega,\mathcal{F},\mathcal{P})$ is a complete probability space, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$; $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$.

2. Modeling and preliminary

In this paper, we consider a class of generalized neural networks with mixed time-varying delays and reaction-diffusion effects described by

$$\frac{\mathrm{d}u_i(t,x)}{\mathrm{d}t}=\sum_{k=1}^{l}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i(t,x)}{\partial x_k}\right)-c_i(u_i(t,x))+\sum_{j=1}^{n}a_{ij}f_j(u_j(t,x))+\sum_{j=1}^{n}b_{ij}g_j(u_j(t-\tau_{ij}(t),x))+\sum_{j=1}^{n}d_{ij}\int_{t-\sigma_{ij}(t)}^{t}h_j(u_j(s,x))\,\mathrm{d}s+J_i, \tag{2.1}$$

where $i=1,2,\ldots,n$, n is the number of neurons in the networks; $x=(x_1,x_2,\ldots,x_l)^T\in\Omega\subset\mathbf{R}^l$ and $\Omega=\{x=(x_1,x_2,\ldots,x_l)^T\,|\,|x_k|<m_k, k=1,2,\ldots,l\}$ is a bound compact set with smooth boundary $\partial\Omega$ and mes $\Omega>0$ in space $\mathbf{R}^l$; $u(t,x)=(u_1(t,x),u_2(t,x),\ldots,u_n(t,x))^T$ with $u_i(t,x)$ corresponds to the state of the ith neural unit at time t and in space x; J_i denotes the bias of the ith neurons; a_{ij}, b_{ij} and d_{ij} are, respectively, the connection strength, the time-varying delay connection weight, and the distributed time-varying delay connection strength of the jth neuron on the ith neuron; $f_j(t,x)$, $g_j(t,x)$ and $h_j(t,x)$ are the activation functions of the jth neurons at time t and in space x; $D_{ik}\geq 0$ corresponds to the transmission diffusion operator along the ith neuron; $0<\tau_{ij}(t)\leq\tau$ is the time-varying delay along the axon of the jth unit from the ith unit; $0<\sigma_{ij}(t)\leq\sigma$ denotes the distributed time-varying delay along the axon of the jth unit from the ith unit; $c_i(\cdot)>0$ represents the rate with which the ith neuron will reset its potential to the resting state when disconnected from the network and external inputs.

The boundary condition of system (2.1) is

$$u_i(t,x)\big|_{\partial\Omega}=0,\quad (t,x)\in[-\overline{\tau},+\infty)\times\partial\Omega,\quad i=1,2,\ldots,n. \tag{2.2}$$

The initial value of system (2.1) is

$$u_i(s,x)=u_i^0(s,x),\quad (s,x)\in[-\overline{\tau},0]\times\Omega,\quad i=1,2,\ldots,n, \tag{2.3}$$

where $\overline{\tau}=\max\{\tau,\sigma\}$; $u_i^0(s,x)$ $(i=1,2,\ldots,n)$ is bounded and continuous on $[-\overline{\tau},0]\times\Omega$.

Remark 1. It can be easily seen that the model (2.1) is a generalized neural networks model which includes some well-known neural networks as its special cases.

(1) Let $c_i(u_i(t,x))=c_iu_i(t,x)$, $c_i>0$, $B=(b_{ij})_{n\times n}=0$, $D=(d_{ij})_{n\times n}=0$, the model (2.1) becomes the continuous-time Hopfield neural networks with reaction-diffusion terms studied in [35,40].

(2) Let $c_i(u_i(t,x))=c_iu_i(t,x)$, $c_i>0$, $A=(a_{ij})_{n\times n}=\left(\begin{smallmatrix}0 & A_1\\ A_2 & 0\end{smallmatrix}\right)$, $B=(b_{ij})_{n\times n}=\left(\begin{smallmatrix}0 & B_1\\ B_2 & 0\end{smallmatrix}\right)$, $D=(d_{ij})_{n\times n}=\left(\begin{smallmatrix}0 & D_1\\ D_2 & 0\end{smallmatrix}\right)$, and n be an even number, the model (2.1) turns into a bidirectional associative memory neural networks model with reaction-diffusion terms. Several stability conditions and synchronization schemes have been given in [3,23,26,29].

(3) Let $c_i(u_i(t,x))=c_iu_i(t,x)$, $c_i>0$, the model (2.1) is reduced into a delayed cellular neural networks model with reaction-diffusion terms which has been studied in [31,39,41].

In order to observe the global exponential synchronization behavior of system (2.1), the response (slaver) system with stochastic perturbation is designed as

$$\begin{aligned}\mathrm{d}v_i(t,x)=&\left[\sum_{k=1}^{l}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial v_i(t,x)}{\partial x_k}\right)-c_i(v_i(t,x))+\sum_{j=1}^{n}a_{ij}f_j(v_j(t,x))\right.\\&+\sum_{j=1}^{n}b_{ij}g_j(v_j(t-\tau_{ij}(t),x))\\&\left.+\sum_{j=1}^{n}d_{ij}\int_{t-\sigma_{ij}(t)}^{t}h_j(v_j(s,x))\,\mathrm{d}s+J_i+w_i(t,x)\right]\mathrm{d}t\\&+\sum_{j=1}^{n}\rho_{ij}(e_j(t,x),e_j(t-\tau_{ij}(t),x),e_j(t-\sigma_{ij}(t),x))\,\mathrm{d}\omega_j(t),\end{aligned} \tag{2.4}$$

where $v(t,x)=(v_1(t,x),v_2(t,x),\ldots,v_n(t,x))^T$ is an n-dimensional state vector of the generalized neural networks; $e(t,x)=(e_1(t,x),e_2(t,x),\ldots,e_n(t,x))^T=v(t,x)-u(t,x)$ denotes the error signal; $w(t,x)=(w_1(t,x),w_2(t,x),\ldots,w_n(t,x))^T$ is a control input to be designed; $\rho=(\rho_{ij})_{n\times n}$ is the diffusion coefficient matrix (or noise intensity matrix) and the stochastic disturbance $\omega(t)=[\omega_1(t),\omega_2(t),\ldots,\omega_n(t)]^T\in\mathbf{R}^n$ is a Brownian motion defined on $(\Omega,\mathcal{F},\mathcal{P})$, and

$$\mathbb{E}\{\mathrm{d}\omega(t)\}=0,\quad \mathbb{E}\{\mathrm{d}\omega^2(t)\}=\mathrm{d}t.$$