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# A delay-partitioning projection approach to stability analysis of stochastic Markovian jump neural networks with randomly occurred nonlinearities

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ABSTRACT

This paper considers the problem of mean square asymptotic stability of stochastic Markovian jump neural networks with randomly occurred nonlinearities. In terms of linear matrix inequality (LMI) approach and delay-partitioning projection technique, delay-dependent stability criteria are derived for the considered neural networks for cases with or without the information of the delay rates via new Lyapunov–Krasovskii functionals. We also establish that the conservatism of the conditions is a non-increasing function of the number of delay partitions. An example with simulation results is given to illustrate the effectiveness of the proposed approach.



## 1. Introduction

Due to the extensive applications in associative memories, pattern recognition, signal processing and the other fields [1], the study of neural networks has received a great deal of attention during the past decades. It is well known that the stability of neural networks is a prerequisite in modern control theories for these applications. However, time delays are often attributed as the major sources of instability in various engineering systems. Therefore, how to find sufficient conditions to guarantee the stability of neural networks with time delays is an important research topic [2–11]. These results can be classified into two types according to their dependence of the delay size that is, delay-dependent stability results and delay-independent ones. Delay-dependent stability criteria are generally less conservative than delay-independent ones especially for small time-delays.

Markovian jump system is an important class of stochastic models, which can be described by a set of linear systems with the transitions between models determined by a Markovian chain in a finite mode set. This kind of system has been extensively applied to model the real-world systems such as communication systems, power systems, manufacturing systems [12–15]. In real world, the systems may experience abrupt changes in their structure and parameters caused by phenomena, such as sudden environment changes and changes in the interconnections of subsystems. It is the same case that some neural networks have finite discrete modes, and the switching law among different modes is satisfied with Markovian property. In order to model such neural networks, the neural networks with Markovian jumping parameters have been introduced and many important and interesting stability criteria have been reported in the literature [6,7,16–22].

On the other hand, it should be pointed out that, however, a number of practical systems are influenced by additive randomly occurred nonlinear disturbances which are caused by environmental circumstances. The nonlinear disturbances may occur in a probabilistic way, moreover, they are randomly changeable in terms of their types and/or intensity. For example, in a networked environment, nonlinear disturbances may be subject to random abrupt changes, which may result from abrupt phenomena such as random failures and repairs of the components, environmental disturbance, etc. The stochastic nonlinearities, which are then named as randomly occurred nonlinearities (RONs), have recently attracted much attention [23–26].

Recently, by introducing free-weighting matrices [27,28], model transformation method [29], linear matrix inequality (LMI) approach

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[30,31], and adopting the concept of delay partitioning and projection [3,5], stability criteria have been established. Sufficient conditions for the robust stability of uncertain stochastic system with interval time-varying delay were derived in [27] by employing the delay partitioning approach. Delay-dependent conditions on mean square asymptotic stability of stochastic neural networks with Markovian jumping parameters are presented by using the delay partitioning method which is different from the existing ones in the literature and convex combination method in [6]. The RONs model and the sensor failure model were introduced in [24], in addition, with a novel Lyapunov–Krasovskii functional and delay partitioning technique, the reliable $H_\infty$ filtering problem was investigated for a class of uncertain discrete time-delay systems with randomly occurred nonlinearities and sensor failures. In [3], better delay-dependent stability criteria for continuous systems with multiple delay components were established for cases with or without the information of the delay rates by utilizing a delay-partitioning projection approach. Inspired by the idea of [3], we utilize the delay-partitioning projection approach in this paper. To the best of the authors' knowledge, the delay-partitioning projection approach to stability analysis of stochastic Markovian jump neural networks with randomly occurred nonlinearities has never been tackled in the previous literatures. This motivates our research.

In this paper, the problem of stability analysis of stochastic Markovian jump neural networks with time-varying delay and RONs is considered. The paper is organized as follows. Section 2 introduces model description and preliminaries. RONs are introduced to model a class of sector-like nonlinearities whose occurrence is governed by a Bernoulli distributed white sequence with a known conditional probability. In Section 3, we derive the stability results based on delay-partitioning projection approach for stochastic Markovian jump neural networks with RONs. The results include two cases, one with a specified delay rates and the other with unknown (hence arbitrary) delay rates. In addition to delay dependence, the obtained conditions are also dependent on the partitioning size, we verify that the conservatism of the conditions is a non-increasing function of the number of partitions. A numerical example is presented to illustrate the effectiveness of the obtained criteria in Section 4. And finally, conclusions are drawn in Section 5.

*Notations*: Throughout this paper, the notation is fairly standard. $R^n$ denotes the $n$-dimensional Euclidean space. $R^{n\times m}$ stands for real matrix $R$ of size $n\times m$ (simply abbreviated $R_n$ when $m=n$). $P>(<)0$ is used to define a real symmetric positive definite (negative definite) matrix. For real symmetric matrices $X$ and $Y$, the notation $X\geq Y$ (respectively, $X>Y$) means that the matrix $X-Y$ is positive semi-definite (respectively, positive definite). The symmetric terms in a symmetric matrix are denoted by $*$ and $diag\{\cdots\}$ denotes a block-diagonal matrix. The superscripts $A^T$ and $A^{-1}$ stand for the transpose and inverse of matrix $A$. $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathcal{P})$ denotes a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$, where $\Omega$ is a sample space, $\mathcal{F}$ is the $\sigma$-algebra of subset of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $E\{x\}$ stands for the expectation of the stochastic variable $x$.

## 2. Model description and preliminaries

Consider the following delayed Markovian jump neural networks with RONs and noise perturbations:

$$\begin{aligned}\dot{x}(t)&=-A(r(t))x(t)+B(r(t))x(t-\tau(t))+\xi(t)Ef(x(t))\\&\quad+[C(r(t))x(t)+D(r(t))x(t-\tau(t))]W(t),\\x(t)&=\phi(t),\quad \forall t\in[-\tau,0],\end{aligned}\tag{1}$$

where $x(t)\in R^n$ is the state vector. $W(t)$ is a scalar zero mean Gaussian white noise process. $\phi(t)$ is a real-valued initial condition. $\{r(t)\}$ is a right-continuous Markov chain on the probability space $(\Omega,\mathcal{F},\mathcal{P})$ taking values in a finite state space $S=\{1,2,\ldots,N\}$ with generator $Q=(q_{ij})_{N\times N}$ given by

$$P\{r(t+\Delta t)=j|r(t)=i\}=\begin{cases}q_{ij}\Delta t+o(\Delta t) & \text{if } i\neq j\\ 1+q_{ii}\Delta t+o(\Delta t) & \text{if } i=j,\end{cases}\tag{2}$$

where $\Delta t>0$ and $\lim_{\Delta t\to 0}o(\Delta t)/\Delta t=0$, $q_{ij}\geq 0$ is the transition rate from $i$ to $j$ if $i\neq j$ while $q_{ii}=-\Sigma_{i\neq j}q_{ij}$. $A(r(t))=diag\{a_{i1},a_{i2},\ldots,a_{in}\}$ is a positive diagonal matrix. $B(r(t))$, $C(r(t))$, $D(r(t))$ and $E$ are known matrices. To facilitate development, in the sequel, each possible value of $r(t)$ is denoted by $i, i\in S$ and we denote

$$A(r(t))=A_i,\quad B(r(t))=B_i,\quad C(r(t))=C_i,\quad D(r(t))=D_i.\tag{3}$$

$\tau(t)$ is the time-varying delay satisfying $0\leq\tau(t)\leq\tau$. In [32], the central point of variation of the delay was introduced to study the stability for time-delay systems, which is called the DCP method. As an extension of the method, in order to derive some less restrictive stability criteria, we partition $\tau(t)$ into several components, i.e., $\tau(t)=\sum_{i=1}^m\tau_i(t)$, where $m$ is a positive integer. In this paper, we will deal with the following two cases of the time-varying delay components $\tau_i(t)$:

*Case* 1: $\tau_i(t)$ is an almost everywhere differentiable function satisfying

$$0<\tau_i(t)\leq\overline{\tau}_i<\infty,\quad \dot{\tau}_i(t)\leq\tau_i<\infty,\quad \forall t>0.\tag{4}$$

*Case* 2: $\tau_i(t)$ is a measurable (e.g. piecewise-continuous) function satisfying

$$0<\tau_i(t)\leq\overline{\tau}_i<\infty,\quad \forall t>0\tag{5}$$

where $\overline{\tau}_i$ and $\tau_i$ are constants. For convenience, we define $\overline{\tau}=\min\{\overline{\tau}_1,\overline{\tau}_2,\ldots,\overline{\tau}_m\}$, $\alpha_k(t)=\sum_{i=1}^k\tau_i(t)$ and $\overline{\alpha}_k=\sum_{i=1}^k\overline{\tau}_i$ with $\alpha_0(t)=0$, $\overline{\alpha}_0=0$ in the boundary expression of the summation. That is, $\tau_i(t)$ and $\overline{\tau}_i$ indicate a partition of the lumped time-varying delay $\tau(t)$ and $\tau$, respectively.

**Remark 1.** It should be pointed out that such delay-partitioning projection approach is very rational. The reasons are twofold. (1) The properties of $\tau_1(t)$ and $\tau_2(t)$ (let the partitioning number $m=2$ for presentation simplicity) may be sharply different in many practical situations. Thus, it is not reasonable to combine them together. (2) When $\tau(t)$ reaches its upper bound, we do not necessarily have both $\tau_1(t)$ and $\tau_2(t)$ reach their maxima at the same time. In other words, if we use an upper bound to bound the delay $\tau(t)$ we have to use the sum of the maxima of $\tau_1(t)$ and $\tau_2(t)$, however, $\tau(t)$ does not achieve this maximum value usually. Therefore, by adopting the delay-partitioning projection approach, less conservative conditions can be proposed.

Finally, $f(x(t))$ is a continuous nonlinear function. The stochastic variable $\xi(t)\in R$ accounts for the phenomena of RONs, which is a Bernoulli distributed white sequence taking values of 1 and 0 with

$$\begin{aligned}&Prob\{\xi(t)=1\}=E\{\xi(t)\}=\overline{\xi},\\&Prob\{\xi(t)=0\}=1-E\{\xi(t)\}=1-\overline{\xi},\end{aligned}\tag{6}$$

here $\overline{\xi}\in[0,1]$ is a constant, which reflects the occurrence probability of the event of the nonlinear function $f(x(t))$.

**Remark 2.** According to the given hypothesis, we have

$$E\{\xi(t)-\overline{\xi}\}=0,\quad E\{(\xi(t)-\overline{\xi})^2\}=\overline{\xi}(1-\overline{\xi}).\tag{7}$$

On the other hand, as emphasized in [33], $\xi(t)$ is a Markovian process and follows an unknown but exponential distribution of switchings.

**Assumption 1** (*Liu et al. [24]*). For $\forall x\in R^n$, the nonlinear function $f(x)$ satisfies the following sector-bounded condition:

$$[f(x)-K_1x]^T[f(x)-K_2x]\leq 0,\tag{8}$$