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Exponential stability of complex-valued neural networks with mixed delays ☆

Xiaohui Xu [a,*], Jiye Zhang [b], Jizhong Shi [c]

[a] School of Transportation and Automotive Engineering, Xihua University, Chengdu 610039, PR China
[b] State Key Laboratory of Traction Power, Southwest Jiaotong University, Chengdu 610031, PR China
[c] College of Engineering, Zhejiang Normal University, Jinhua 321004, PR China

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ABSTRACT

This paper investigates the problem of the dynamic behaviors of a class of complex-valued neural networks with mixed time delays. Some sufficient conditions for assuring the existence, uniqueness and exponential stability of the equilibrium point of the system are derived using the vector Lyapunov function method, homeomorphism mapping lemma and the matrix theory. The obtained results not only are convenient to check, but also generalize the previously published corresponding results. A numerical example is used to show the effectiveness of the obtained results.

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1. Introduction

Since the last decades there have been some important researches on the dynamic behaviors of neural networks due to their widespread applications in the associative memory, signal processing, pattern recognition, etc. [1,2]. Although numerous works [3–17] have been published in recent years on the stability analysis of the equilibrium points of various neural networks, little attention has been paid to the study on the dynamic behaviors of complex-valued neural networks. In fact, complex-valued neural networks (CVNN for short) make it possible to solve some problems which cannot be solved with their real-valued counterparts. For example [18–22], the XOR problem and the detection of symmetry problem cannot be solved with a single real-valued neuron, but they can be solved with a single complex-valued neuron with the orthogonal decision boundaries, which reveals the potent computational power of complex-valued neurons. Besides, CVNN has more different and more complicated properties than the real-valued ones. Therefore it is necessary to study the dynamic behaviors of the systems deeply.

In the past decades, there have been some researches on the dynamic behavior analysis of the equilibrium point of various CVNN. In [23], some sufficient conditions for judging the existence, uniqueness and global exponential stability of the equilibrium point of a class of discrete CVNN were obtained. However, time delays were not considered in the model studied in [23], which may lead to the instability of the system. In [24] a class of CVNN with constant delays was studied, and some sufficient conditions were obtained for assuring the stability of the equilibrium point of CVNN with two classes of activation functions. Usually, constant fixed time delays in the models of delayed feedback systems serve as a good approximation in simple circuits having a small number of cells. Though consider that the time delays arise frequently in practical applications, it is difficult to measure them precisely. In most situations the delays are variable and unbounded. So it is necessary to study CVNN with mixed time delays.

There have been various approaches for analyzing the dynamic behaviors of CVNN, such as the scalar Lyapunov function method [24], the LMI method [25], the energy function method [26,27], and the synthesis method [28], etc. However, to the best of our knowledge, the vector Lyapunov function method has not been used for studying the dynamic behaviors of CVNN so far. The vector Lyapunov function method has been used to study the dynamic behaviors of various real-valued neural networks, see [3,6,12,29]. It is easy to find that the obtained stability conditions in [3,6,12,29] are explicit and easy to be verified. Moreover, it is easier to construct vector Lyapunov function than to construct a scalar Lyapunov function.

Based on the above analysis, the dynamic behaviors of a class of CVNN with time-varying delays and unbounded delays will be

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* Corresponding author. Tel.: +86 2886466040.
E-mail address: xhxu@163.com (X. Xu).

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studied by using the vector Lyapunov function method in this paper. Some sufficient conditions for judging the existence, uniqueness and exponential stability of the equilibrium point of the system are established.

2. Preliminaries

To make reading easier the following notations will be used. Let C denote complex number set. Let $z=x+yi$ be the complex number, here i denotes the imaginary unit, i.e. $i=\sqrt{-1}$. For complex number vector $z\in C^n$, let $|z|=(|z_1|,|z_2|,\dots,|z_n|)^T$ be the module of the vector z, and $||z||=\sqrt{\sum_{k=1}^n |z_k|^2}$ be the norm of the vector z, here $(.)^T$ denotes the transpose of vector.

In this paper, consider the complex-valued neural networks with time-varying delays and unbounded delays, which can be described by

$$\frac{dz_k(t)}{dt}=-d_kz_k(t)+\sum_{j=1}^n[a_{kj}f_j(z_j(t))+b_{kj}f_j(z_j(t-\tau_{kj}(t)))+p_{kj}\int_{-\infty}^t\theta_{kj}(t-s)f_j(z_j(s))ds]+J_k \tag{1}$$

where $z_k\in C$ represents the state of neuro k, $k=1,2,\dots,n$, n is the number of neuros, $A=(a_{kj})_{n\times n}\in C^{n\times n}$, $B=(b_{kj})_{n\times n}\in C^{n\times n}$ and $P=(p_{kj})_{n\times n}\in C^{n\times n}$ are the connection weight matrices, $J=(J_1,J_2,\dots,J_n)^T\in C^n$ is external constant input vector, $f(z(.))=(f_1(z_1(.)),f_2(z_2(.)),\dots,f_n(z_n(.)))^T$ represents activation function, $D=diag(d_1,d_2,\dots,d_n)\in R^{n\times n}$ with $d_k>0$ is the self-feedback connection weight matrix, $\tau_{kj}(t)(k,j=1,2,\dots,n)$ are bounded functions and $\tau=\max_{1\le k,j\le n}\sup_{t\ge0}\tau_{kj}(t)$, θ_{kj}: $[0,+\infty)\to[0,+\infty)$ are piecewise continuous functions, and satisfy

$$\int_0^{+\infty}e^{\beta s}\theta_{kj}(s)ds=\mu_{kj}(\beta),\ k,j=1,2,\dots,n \tag{2}$$

here $\mu_{kj}(\beta)$ is continuous on $[0,\delta)$, and $\mu_{kj}(0)=1$, $\delta>0$.

Assume that the initial conditions of Eq. (1) are $z_k(s)=\phi_k(s)$, here $\phi_k(s)$ are bounded continuous on $(-\infty,0]$, $k=1,2,\dots,n$.

Let $z_k=x_k+iy_k$, then activation function $f_k(z_k)$ can be expressed by separating it into its real part and imaginary part as

$$f_k(z_k)=f_k^R(x_k,y_k)+if_k^I(x_k,y_k) \tag{3}$$

where $f_k^R(x_k,y_k):\ R^2\to R$, $f_k^I(x_k,y_k):\ R^2\to R$, $k=1,2,\dots,n$.

Furthermore, Eq. (1) can be rewritten by separating it into its real part and imaginary part as

$$\begin{aligned}\dot{x}_k(t)=&-d_kx_k(t)+\sum_{j=1}^n[a_{kj}^Rf_j^R(x_j(t),y_j(t))-a_{kj}^If_j^I(x_j(t),y_j(t))]\\&+\sum_{j=1}^n[b_{kj}^Rf_j^R(x_j(t-\tau_{kj}(t)),y_j(t-\tau_{kj}(t)))\\&-b_{kj}^If_j^I(x_j(t-\tau_{kj}(t)),\ y_j(t-\tau_{kj}(t)))]\\&+\sum_{j=1}^n\int_{-\infty}^t\theta_{kj}(t-s)[p_{kj}^Rf_j^R(x_j(s),\ y_j(s))\\&-p_{kj}^If_j^I(x_j(s),\ y_j(s))]ds+u_k^R\end{aligned} \tag{4}$$

$$\begin{aligned}\dot{y}_k(t)=&-d_ky_k(t)+\sum_{j=1}^n[a_{kj}^Rf_j^I(x_j(t),y_j(t))+a_{kj}^If_j^R(x_j(t),y_j(t))]\\&+\sum_{j=1}^n[b_{kj}^Rf_j^I(x_j(t-\tau_{kj}(t)),y_j(t-\tau_{kj}(t)))\\&+b_{kj}^If_j^R(x_j(t-\tau_{kj}(t)),y_j(t-\tau_{kj}(t)))]\\&+\sum_{j=1}^n\int_{-\infty}^t\theta_{kj}(t-s)[p_{kj}^Rf_j^I(x_j(s),y_j(s))+p_{kj}^If_j^R(x_j(s),y_j(s))]ds+u_k^I\end{aligned} \tag{5}$$

here $k=1,2,\dots,n$, $A^R=(a_{kj}^R)_{n\times n}$ and $A^I=(a_{kj}^I)_{n\times n}$ are, respectively, the real part and imaginary part of A, $B^R=(b_{kj}^R)_{n\times n}$ and $B^I=(b_{kj}^I)_{n\times n}$ are, respectively, the real part and imaginary part of B, $P^R=(p_{kj}^R)_{n\times n}$ and $P^I=(p_{kj}^I)_{n\times n}$ are, respectively, the real part and imaginary part of P, $u^R=(u_1^R,u_2^R,\dots,u_n^R)^T$ and $u^I=(u_1^I,u_2^I,\dots,u_n^I)^T$ are, respectively, the real part and imaginary part of u.

Let $z^\#=(z_1^\#,z_2^\#,\dots,z_n^\#)^T$ be the equilibrium point of Eq. (1), here $z_k^\#=x_k^\#+iy_k^\#$, $k=1,2,\dots,n$.

Definition 1. The equilibrium point $z^\#$ of Eq. (1) is exponentially stable, if there exist constants $\Gamma>0$ and $\lambda>0$ such that for every $J\in C^n$ and $t\ge0$ the inequality $||z(t)-z^\#||\le\sup_{s\in(-\infty,0]}||\phi(s)-z^\#||\Gamma e^{-\lambda t}$ holds.

Assumption 1. It is assumed that $f_k(.)$ with the form of Eq. (3) satisfy the following conditions:

(i) The partial derivatives $f_k(.)$ with respect to x_k and y_k: $\partial f_k^R/\partial x_k$, $\partial f_k^R/\partial y_k$, $\partial f_k^I/\partial x_k$ and $\partial f_k^I/\partial y_k$ exist and are continuous;

(ii) The partial derivatives $\partial f_k^R/\partial x_k$, $\partial f_k^R/\partial y_k$, $\partial f_k^I/\partial x_k$ and $\partial f_k^I/\partial y_k$ are bounded, i.e. there exist positive constants l_k^{RR}, l_k^{RI}, l_k^{IR} and l_k^{II} such that $|\partial f_k^R/\partial x_k|\le l_k^{RR}$, $|\partial f_k^R/\partial y_k|\le l_k^{RI}$, $|\partial f_k^I/\partial x_k|\le l_k^{IR}$, $|\partial f_k^I/\partial y_k|\le l_k^{II}$, then it follows from the mean value theorem of multivariable functions that for any $x_k,\ x'_k,\ y_k,\ y'_k\in R$, we have

$$\begin{aligned}&|f_k^R(x_k,y_k)-f_k^R(x_k',y_k')|\le l_k^{RR}|x_k-x_k'|+l_k^{RI}|y_k-y_k'|\\&|f_k^I(x_k,y_k)-f_k^I(x_k',y_k')|\le l_k^{IR}|x_k-x_k'|+l_k^{II}|y_k-y_k'|\end{aligned} \tag{6}$$

Let $L^{RR}=diag(l_1^{RR},l_2^{RR},\dots,l_n^{RR})$, $L^{RI}=diag(l_1^{RI},l_2^{RI},\dots,l_n^{RI})$, $L^{IR}=diag(l_1^{IR},l_2^{IR},\dots,l_n^{IR})$, and $L^{II}=diag(l_1^{II},l_2^{II},\dots,l_n^{II})$.

Lemma 1 [3]. *Let $A=(a_{ij})_{n\times n}\in R^{n\times n}$ be a matrix with $a_{kj}\le0$, $(k\ne j)$. The following statements are equivalent:*

(i) $A=(a_{ij})_{n\times n}$ is a M matrix;
(ii) The real parts of all eigenvalues of A are positive;
(iii) There exists a positive vector $\xi\in R^n$ such that $A\xi>0$.

Lemma 2 [3]. *If $H(\alpha)$ is a continuous function on R^n, and satisfies the following conditions:*

(i) $H(\alpha)$ is injective on R^n;
(ii) $\lim_{||\alpha||\to\infty}||H(\alpha)||\to\infty$,then $H(\alpha)$ is a homeomorphism of R^n into itself.

3. Main results

For simplification, let

$$x=[x_1,x_2,\dots,x_n]^T,\ y=[y_1,y_2,\dots,y_n]^T,\ \alpha=[x^T,y^T]^T,\ \omega=[(u^R)^T,(u^I)^T]^T,$$

$$\tilde{f}^R(\alpha)=((f^R(x,y))^T,(f^R(x,y))^T)^T,\ \tilde{f}^I(\alpha)=((f^I(x,y))^T,(f^I(x,y))^T)^T.$$

Define a map associated with Eqs. (4) and (5) as follows

$$H(\alpha)=-\tilde{D}\alpha+Q_1\tilde{f}^R(\alpha)+Q_2\tilde{f}^I(\alpha)+\omega, \tag{7}$$

where

$$\tilde{D}=\begin{bmatrix}D&0\\0&D\end{bmatrix},\ Q_1=\begin{bmatrix}A^R+B^R+P^R&0\\0&A^I+B^I+P^I\end{bmatrix},$$

$$Q_2=\begin{bmatrix}-A^I-B^I-P^I&0\\0&A^R+B^R+P^R\end{bmatrix}.$$

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