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Brief Papers

Stability analysis of Hopfield neural networks perturbed by Poisson noises

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ABSTRACT

In this paper, the stability problem is investigated for Hopfield neural networks (HNNs) perturbed by Poisson noises. Note that Poisson process can better reflect the dynamical behaviors of jump stochastic noises which exist widely in neurons. A stability criterion for HNNs perturbed by Poisson noises is presented by employing a combination of the martingale theory and measure theory. Finally, a simulation example is given to illustrate the effectiveness of the proposed stability criteria.

1. Introduction

During last decades, HNNs have been extensively investigated due to their wide applications such as classification of patterns, signal processing, associative memories, reconstruction of moving images, and optimization problems [1–4]. As is known to all, it is very important to consider stability problems of such neural networks before these applications. Therefore, stability problems of HNNs have attracted considerable attention in [5–14] and the references therein.

In practice, the synaptic transmission in real nervous systems is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. Hence, it is significant to consider stochastic effects on the stability property of neural networks, and many results related to this topic have been reported in [15–24]. For example, [15] investigated the exponential stability problem of stochastic Hopfield neural networks perturbed by white noises. When time delays arise in such stochastic neural networks, delay-dependent stability conditions are proposed in [17–24] by Lyapunov–Krasovskii functional method, free weighting matrix technique and delay partitioning technique, respectively.

In most of published papers focusing on stability analysis of stochastic neural networks, stochastic effects were described by Wiener process. However, there exist some jump stochastic phenomena such as synaptic noises [25–27], spike trains [28,29] in neurons, and Wiener process cannot describe these stochastic phenomena effectively [25–29,36–38]. Recently, researchers have recognized that Poisson process is a natural model for such stochastic phenomena [25–31]. For example, in the leaky integrate-and-fire neuronal model of [27], Poisson processes P^+ and P^- were introduced to model excitatory and inhibitory synaptic noises resulting from synaptic inputs, respectively. Nowadays, Poisson process as a model of activity of a neuron was experimentally observed many times and on very different neuronal structures [32–35]. Based on the above reasons, when neural networks are subject to above jump stochastic noises, it is necessary to investigate stochastic neural networks perturbed by Poisson noises. However, to the best of our knowledge, the stability problem of neural networks perturbed by Poisson noises has not been investigated yet. Research in this area should be important and useful, which motivates us to carry out the present work.

In this paper, we investigate the stability problem for HNNs perturbed by Poisson noises. By utilizing the martingale theory and measure theory, a stability criteria is presented. Finally, a simulation example is given to illustrate the effectiveness of the proposed stability criteria.

Notation: In this paper, unless otherwise specified, we will employ the following notation. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ and $\mathbb{E}(\cdot)$ be the expectation operator with respect to the probability measure. If A is a vector or matrix, its transpose is denoted by A^T. If P is a square matrix, $P>0$ $(P<0)$ means that is a symmetric positive (negative) definite matrix of appropriate dimensions while $P\geq 0$ $(P\leq 0)$ is a symmetric positive (negative) semidefinite matrix. I

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stands for the identity matrix of appropriate dimensions. Let $|\cdot|$ denote the Euclidean norm of a vector and its induced norm of a matrix. Unless explicitly specified, matrices are assumed to have real entries and compatible dimensions. $L^2(\Omega)$ denotes the space of all random variables X with $\mathbb{E}|X|^2<\infty$, it is a Banach space with norm $\|X\|_2=(\mathbb{E}|X|^2)^{1/2}$. $\mathcal{L}_2[0,\infty)$ is the space of square integrable functions over $[0,\infty)$. The symbol '$*$' within a matrix represents the symmetric terms of the matrix, e.g. $\begin{pmatrix} X & Y \\ * & Z \end{pmatrix}=\begin{pmatrix} X & Y \\ Y^T & Z \end{pmatrix}$. If a function is right continuous with left limits, this function is called càdlàg function. If a function is left continuous with right limits, this function is called càglàd function. Moreover, a stochastic process is said to be càdlàg if it almost surely has sample paths which are right continuous with left limits. A stochastic process is said to be càglàd if it almost surely has sample paths which are left continuous with right limits.

2. Problem formulation and preliminaries

Consider the following HNNs perturbed by Poisson noises:

$$(\Sigma):dx(t)=[-Ax(t)+Bf(x(t))]\,dt+Cx(t-)\,d\lambda(t), \tag{1}$$

$$x(0)=\xi, \tag{2}$$

where $x(t)=[x_1(t),x_2(t),\dots,x_n(t)]^T\in\mathcal{R}^n$ is the state; $\lambda(t)$ is a one-dimension Poisson process defined on $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\ge 0},\mathbb{P})$ with parameter $\lambda>0$; $f(x(\cdot))=\left[f_1(x_1(\cdot)),f_2(x_2(\cdot)),\dots,f_n(x_n(\cdot))\right]^T$ with $f_i(x_i(\cdot))$ being the activation functions, and $x(0)$ is the initial condition.

Remark 1. As pointed out in [41,42], stochastic differential equation (1) should be interpreted as meaning the corresponding stochastic integral equation:

$$x(t)=x(0)+\int_0^t[-Ax(s)+Bf(x(s))]\,ds+\int_0^t Cx(s-)\,d\lambda(s), \tag{3}$$

where $\int_0^t Cx(s-)\,d\lambda(s)$ is the stochastic integral with respect to the Poisson process $\lambda(s)$, whose definition can be found in [40,42]. It might appear strange that one uses the left limit $Cx(t-)$ rather than $Cx(t)$ as integrand in (1). Actually, [39] has pointed out clearly that "When a Poisson process $\lambda(t)$ jumps, i.e., $\Delta\lambda(t)=1$, then $x(t)$ jumps from $x(t-)$ to $x(t)$, where the jump size is given by C. It would not make much sense if the jump size depended on the post-jump state $x(t)$. It is rather convenient to assume that the jump size is determined by the state just before the jump occurs '$-$' which is formally $x(t-)$. Thus, the jump size itself is given by $Cx(t-)$."

Remark 2. There is another explanation why the integrand should be $Cx(t-)$. According to the definition of stochastic integral with respect to Poisson process [40,42], the integrand of $\int_0^t Cx(\cdot)\,d\lambda(s)$ must be a predictable stochastic process. From [42], the solution $x(t)$ of the stochastic differential equation driven by Poisson process is always $\mathcal{F}(t)$ adapted and càdlàg, which cannot guarantee that $x(t)$ is a predictable process. Then, $Cx(t)$ does not satisfy the condition to be the integrand. Note that the left limit of an $\mathcal{F}(t)$ adapted and càdlàg process is a predictable process [40]. So $Cx(t-)$ is a predictable process. Therefore, we always use $Cx(t-)$ instead of $Cx(t)$ as integrand in (1) [40,41].

The following assumption is made on the neuron activation function.

Assumption 1. Each neuron activation function $f_i(\cdot), i=1,2,\dots,n$, in (1), satisfies the following condition:

$$\underline{k}_i\le\frac{f_i(\alpha)-f_i(\beta)}{\alpha-\beta}\le\overline{k}_i,\quad i=1,2,\dots,n, \tag{4}$$

where $f_i(0)=0,\alpha,\beta\in R,\alpha\ne\beta$ and $\underline{k}_i,\overline{k}_i$ are known real scalars.

To investigate the HNN (1), we will need the Itô formula for the stochastic equation (1) as follows [39,42]:

$$\begin{aligned}V(\mathcal{T},x(\mathcal{T}))&=V(0,x(0))+\int_0^{\mathcal{T}}[V_t(t,x(t))+V_x(t,x(t))(-Ax(t)\\&\quad+Bf(x(t)))]\,dt+\int_0^{\mathcal{T}}[V(t,x(t-)+Cx(t-))\\&\quad-V(t,x(t-))]\,d\lambda(t),\end{aligned} \tag{5}$$

where $\mathcal{T}>0$ is any positive constant; $V(t,x(t))$ is any non-negative function on $\mathcal{R}_+\times\mathcal{R}^n$ and is continuously twice differentiable in x and once differentiable in t.

Before stating the main results, we introduce the following useful definitions and propositions.

Definition 1. For every $x(0)=\xi$, the equilibrium point of the stochastic Hopfield neural network in (1) is said to be asymptotically stable in the mean square, if $\lim_{t\to\infty}\mathbb{E}(|x(t)|^2)=0$.

Definition 2 (*Dellacherie and Meyer [43]*). Let X_t be a positive or bounded measurable process on $(\Omega,\{\mathcal{F}_t\}_{t\ge 0},\mathbb{P})$. The predictable projection of X_t is the unique predictable process ${}^{\mathrm{p}}X_t$ which satisfies: for any predictable stopping time τ

$$\mathbb{E}(X_\tau\mathbf{1}_{\{\tau<\infty\}}|\mathcal{F}_{\tau-})={}^{\mathrm{p}}X_\tau\mathbf{1}_{\{\tau<\infty\}}\quad \text{a.s.}, \tag{6}$$

where $\mathbf{1}_{\{\tau<\infty\}}$ is the characteristic function on $\{\omega\in\Omega:\tau(\omega)<\infty\}$.

Definition 3 (*Dellacherie and Meyer [43]*). Let A_t be an integrable raw increasing process. We call the dual predictable projection of A_t is the predictable increasing process $A_t^{(p)}$ defined by

$$\mathbb{E}\left(\int_{[0,\infty[}X_s\,dA_s^{(p)}\right)=\mathbb{E}\left(\int_{[0,\infty[}{}^{\mathrm{p}}X_s\,dA_s\right) \tag{7}$$

for any bounded measurable X_t.

Proposition 1 (*Klebaner [40]*). *Let $f(t)$ be a càdlàg function on $[a,b]$. Then $f(t)$ has no more than countably many discontinuities on $[a,b]$.*

Proposition 2 (*Dellacherie and Meyer [43]*). *Let $\lambda(t)$ be a Poisson process with parameter $\lambda>0$. Then the dual predictable process of $\lambda(t)$ is λt, i.e., $\lambda(t)^{(p)}=\lambda t$.*

Proposition 3 (*Klebaner [40]*). *If $f(t)$ is an $\mathcal{F}_t$-adapted and càdlàg process, $f(t-)$ is predictable.*

Proposition 4 (*Dellacherie and Meyer [43]*). *If X_t is a measurable stochastic process and Y_t is a bounded predictable stochastic process, then*

$${}^{\mathrm{p}}(Y_tX_t)=Y_t{}^{\mathrm{p}}X_t. \tag{8}$$

3. Main results

The criterion of asymptotical stability in mean square for the neural network (1) is given in the following theorem.

Theorem 1. *The neural network (1) is asymptotically stable in the mean square if there exist a matrix $P>0$ and a diagonal matrix $H=\mathrm{diag}(h_1,h_2,\dots,h_n)>0$ such that the following linear matrix inequality (LMI) holds:*

$$\Omega=\begin{pmatrix}\Xi_1 & \Xi_2\\ * & -H\end{pmatrix}<0, \tag{9}$$