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Tracking control of modified Lorenz nonlinear system using ZG neural dynamics with additive input or mixed inputs ☆

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ABSTRACT

The tracking-control problem of a special nonlinear system (i.e., the extension of a modified Lorenz chaotic system) with additive input or the mixture of additive and multiplicative inputs is considered in this paper. It is worth pointing out that, with the parameters fixed at some particular values, the modified Lorenz nonlinear system degrades to the modified Lorenz chaotic system. Note that, due to the existence of singularities at which the nonlinear system fails to have a well-defined relative degree, the input–output linearization method and the backstepping design technique cannot solve the tracking-control problem. By combining Zhang neural dynamics and gradient neural dynamics, a new effective controller-design method, termed Zhang-gradient (ZG) neural dynamics, is proposed for the tracking control of the modified Lorenz nonlinear system. With singularities conquered, this ZG neural dynamics is able to solve the tracking-control problem of the modified Lorenz nonlinear system via additive input or mixed inputs (i.e., the mixture of additive and multiplicative inputs). Both theoretical analyses and simulative verifications substantiate that the tracking controllers based on the ZG neural dynamics with additive input or mixed inputs not only achieve satisfactory tracking accuracy but also successfully conquer the singularities encountered during the tracking-control process. Moreover, the applications to the synchronization, stabilization and tracking control of other nonlinear systems further illustrate the effectiveness and advantages of the ZG neural dynamics.

1. Introduction

The chaos theory is considered to be one of the emerging research fields widely encountered in a variety of scientific and engineering fields. Being a system of three ordinary differential equations, the first chaotic system was developed in 1963 for atmospheric convection by Lorenz [1], which is shown as follows:

$$\begin{cases} \dot{x}(t)=a(y(t)-x(t)), \\ \dot{y}(t)=cx(t)-x(t)z(t)-y(t), \\ \dot{z}(t)=x(t)y(t)-bz(t), \end{cases} \tag{1}$$

where $x(t)$, $y(t)$ and $z(t)$ are the system states with constants $a=10$, $b=8/3$ and $c=28$. Note that, in the rest of this paper, the argument t is frequently omitted for presentation convenience, e.g., by writing $x(t)$ as x. A new chaotic system modified from Lorenz chaotic system (1) with fewer terms was presented in [2], which is simpler than those of existing seven-term or six-term ones and shown as follows:

$$\begin{cases} \dot{x}=a(y-x), \\ \dot{y}=-xz, \\ \dot{z}=xy-b, \end{cases} \tag{2}$$

where $a=1$ and $b=1$ for additive input or $b=1$ with a being a variable of mixed inputs in this paper.

Two main directions are frequently considered for the investigations of chaotic systems. The first direction includes (i) utilizing the chaotic behaviors for practical applications, such as true random number generators and encryption and secure communication [3,4]; and (ii) generating chaotic system with more chaotic behaviors, such as hyperchaotic systems [5]. The other direction is to get rid of chaotic behaviors existing in physical systems such as electrical circuits, fluid dynamics and mechanical devices, since

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the systems may perform unexpected behaviors under the influence of chaos [6]. Both the investigations of the above directions involve control, especially the tracking control of chaos, which has attracted lots of attention [7,8]. In this paper, the tracking control of modified Lorenz chaotic system (2) using additive input is investigated. In view of the fact that a chaotic system only remains chaotic with particularly fixed parameters and may become a general nonlinear system with parameters being changed, the ZG controller group using the mixture of additive and multiplicative inputs is also applied to the tracking control of a more general nonlinear system (i.e., the modified Lorenz nonlinear system). The proposed control method can be extended to many other nonlinear systems, such as the synchronization of two chaotic systems, the stabilization of complex dynamical networks and the tracking control of other chaotic systems via additive control inputs, which are also studied in this paper.

Input–output linearization (IOL) is an effective method for solving nonlinear tracking-control problems [9,10]. However, with the existence of singularities at which the nonlinear system fails to have a well-defined relative degree, the method of IOL fails to solve the corresponding tracking-control problem. With some nonlinear terms that may generate singularities neglected, a method for constructing the approximate nonlinear systems that are input–output linearizable was presented in [11], which attempted to overcome the singularity drawback. As [12] pointed out, this method does not work well when the system is away from the singularities, because of the approximation error generated by the dropped terms. In [13], a switching scheme provided an idea of switching between approximate and exact IOL, so as to avoid the singularities and enhance the tracking performance. However, this approach costs much in terms of the implementation since it requires two or more controllers for solving a single problem. The backstepping design technique, being a kind of systematic synthetic technique to controller, is a recursive procedure that combines the choice of a Lyapunov function with the design of feedback control. The aforementioned methods generally bring in the operation of division, which leads to the potential possibility of generating singularities.

Recently, a model-free optimal control strategy [14] and fuzzy-neural-network based control methods [15,16] have been proposed, in addition to control methods aiming at particular fractional order chaotic systems [17–20], which all show that there exist more novel and effective control methods to be explored for nonlinear systems. Zhang neural dynamics has been developed to solve time-varying nonlinear equation [21], time-varying matrix pseudoinversion [22,23], time-varying complex Sylvester equation [24], and so on [25]. On the other hand, gradient neural dynamics is intrinsically feasible and efficient to solve time-invariant problems and has been generalized to solve time-varying problems, such as time-varying matrix inversion [26], time-varying quadratic minimization and equality-constrained quadratic programming [27], and time-varying nonlinear equation [28]. The previous studies generally exploited such Zhang neural dynamics and gradient neural dynamics individually and comparatively, and other researchers rarely consider combining them for the problem solving or discover the superiority of their combination.

In this paper, by creatively combining Zhang neural dynamics and gradient neural dynamics, a novel effective controller-design method termed Zhang-gradient (ZG) neural dynamics is proposed. Together with theoretical analyses and results, the combined ZG neural dynamics is proposed to solve the tracking-control problem of modified Lorenz chaotic system (2) in a division-free manner, which can conquer the singularity. Simulations are conducted to further substantiate the feasibility and efficacy of the novel controllers. It is worth pointing out that the IOL method, the backstepping design technique as well as the ZG neural dynamics will be compared in detail in Section 2 of the paper to clarify the differences and advantages of the new method proposed.

The rest of this paper is organized into four sections. The ZG neural dynamics is proposed to design the singularity-conquering controller for the tracking-control problem of the modified Lorenz nonlinear system with additive control input in Section 2. In addition, theoretical analyses of the controller as well as the corresponding simulation verifications are also presented in Section 2. Section 3 presents the design, theoretical analyses and the simulation verifications of the controllers with mixed control inputs. In Section 4, as an extension, the studies on the control of other nonlinear systems (i.e., the synchronization of two chaotic systems, the stabilization of complex dynamical networks and the tracking control of other chaotic systems) are illustrated. Section 5 concludes this paper with final remarks. Before ending this introductory section, it is worthwhile to point out the main contributions of this paper as follows:

(1) This paper mainly focuses on solving the tracking-control problem of a special nonlinear system (i.e., the extension of a modified Lorenz chaotic system) with singularities conquered rather than conventionally investigated nonsingular situations.
(2) Zhang neural dynamics and gradient neural dynamics are combined to construct a new controller-design method (termed ZG neural dynamics), which have been exploited individually and comparatively in previous studies but rarely combined for the stabilization control of complex dynamical networks.
(3) In addition to the conventionally investigated tracking-control problems of the special nonlinear system with additive control input, this paper also handles the ones with mixed control inputs (i.e., the mixture of additive and multiplicative inputs).
(4) The tracking-error bounds and convergence rates of the presented controllers for the tracking control of the special nonlinear system are proved theoretically in this paper.
(5) Simulation verifications illustrate that the tracking controllers based on the ZG neural dynamics not only achieve satisfactory tracking accuracy but also successfully solve the singularity problem encountered during the tracking-control process.
(6) In addition to the tracking control of the modified Lorenz nonlinear system, the ZG neural dynamics is applied to the synchronization, stabilization and tracking control of other nonlinear systems, which further illustrates the effectiveness and advantages of the ZG neural dynamics.

2. Control via additive input

In this section, the tracking-control problem of modified Lorenz chaotic system (2) equipped with a single additive control input u is firstly presented as an illustrative example. A controller based on the IOL method is presented to point out the singularity problem encountered in the tracking control. Afterwards, the detailed procedures for designing the controller to solve the singularity problem in tracking control as well as the corresponding theoretical analyses are provided.

Consider the following modified Lorenz chaotic system equipped with a single additive control input u:

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = -xz + u, \\ \dot{z} = xy - b. \end{cases} \tag{3}$$

Besides, $\vartheta = z$ denotes the output of system (3). The objective of tracking control is to design a controller such that ϑ tracks the