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Stability analysis of Takagi–Sugeno stochastic fuzzy Hopfield neural networks with discrete and distributed time varying delays ☆

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ABSTRACT

In this paper, the global stability problem of Takagi–Sugeno (T–S) stochastic fuzzy Hopfield neural networks (TSSFHNNs) with discrete and distributed time varying delays is considered. A novel LMI-based stability criterion is obtained by using Lyapunov functional theory to guarantee the asymptotic stability of TSSFHNNs with discrete and distributed time varying delays. Here we choose a generalized Lyapunov functional and introduce a parameterized model transformation with free weighting matrices to it, in order to obtain stability region. In fact, these techniques lead to generalized and less conservative stability condition that guarantee the wide stability region. The proposed stability conditions are demonstrated with numerical examples. Comparison with other stability conditions in the literature shows that our conditions are the more powerful ones to guarantee the widest stability region.

1. Introduction

In recent years, the well-known Hopfield neural network has been extensively studied and successfully applied in many areas such as combinatorial optimization, signal processing and pattern recognition, see for example [7,8]. Recently, it has been realized that significant time delays as a source of instability and bad performance may occur in neural processing and signal transmission. Thus, the stability problem of Hopfield neural networks has became interesting and many sufficient conditions have been proposed to guarantee the asymptotic or exponential stability for the neural networks with various type of time delays, see for details [3,4,9,11,14,16,33–35,37].

Although discrete time delays in the delayed feedback neural networks having a small number of cells serve usually as good approximation of the prime models, a real system is usually affected by external perturbations. Therefore, it is significant and of prime importance to consider stochastic effects to the stability property of the neural networks with delays (see [23,32,38]).

Since neural networks usually have a spatial extend due to the presence of a multitude of parallel pathways with a variety of axon sizes and length, and hence there is a distribution of propagation delays over a period of time. It is worth noting that, although the signal propagation is sometimes instantaneous and can be modeled with discrete delays, it may also be distributed during a certain time period so that the distributed delays should be incorporated in the model. In other words, it is often the case that the neural network model possesses both discrete and distributed delays [20]. In recent years, it is noted that stability of Hopfield neural networks, cellular neural networks and bidirectional associative memory neural networks with distributed delays has been discussed in [15,17,19,20,24,25].

Fuzzy systems in the form of the Takagi–Sugeno (T–S) model [29] have attracted rapidly growing interest in recent years [2,30]. T–S fuzzy systems are nonlinear systems described by a set of IF–THEN rules. It has shown that the T–S model can give an effective way to represent complex nonlinear systems by some simple local linear dynamic systems with their linguistic description. Some nonlinear dynamic systems can be approximated by the overall fuzzy linear T–S models for the purpose of stability analysis [2,30]. Originally, Tanaka and his colleagues have provided a sufficient condition for the quadratic stability of the T–S fuzzy systems in the sense of Lyapunov in [31] by considering a Lyapunov function of the sub-fuzzy systems of the T–S fuzzy systems. The concept of incorporating fuzzy logic into a neural network is proposed in some papers [10,21,26–28,36].

Based on the above discussions, we shall generalize the ordinary T–S fuzzy models to express a class of stochastic Hopfield neural network with discrete and distributed time

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varying delays. The main purpose of this paper is to study the global stability results of TSSFHNNs with discrete and distributed time varying delays in terms of LMIs. The main advantage of the LMI based approach is that the LMI stability conditions can be solved numerically using MATLAB LMI toolbox [5] which implements the state of art interior-point algorithms [1]. We also provide numerical examples to demonstrate the effectiveness of the proposed stability results.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n\times m$ real matrices. The superscript "T" denotes the transpose and the notation $X\geq Y$(respectively, $X>Y$) where X and Y are symmetric matrices, means that X–Y is positive semi -definite (respectively positive definite). $\lambda_{min}(A)$ means the smallest eigen value of the matrix A. Moreover let $(\Omega,\mathcal{F},\{\mathcal{F}\}_{t\geq 0},P)$ be a complete probability space with a filtration $\{\mathcal{F}\}_{t\geq 0}$ satisfying the usual conditions (i.e the filtration contains all P-null sets and is right continuous). $C([-\rho,0];R^n)$ denotes the family of continuous functions ϕ form $[-\rho,0]$ to R^n with norm $\|\phi\|=sup_{-\rho\leq\theta\leq 0}|\phi(\theta)|$, where $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. Denote by $L^p_{\mathcal{F}_0}([-\rho,0];R^n)$ the family of all $\mathcal{F}_0$ measurable $C([-h,0];R^n)$-valued random variables $\xi=\{\xi(\theta):-\rho\leq\theta\leq 0\}$ such that $sup_{-\rho\leq\theta\leq 0}E|\xi(\theta)|^p<\infty$ where $E\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P.

2. System description and preliminaries

Consider the following Hopfield neural networks with both discrete and distributed delays described by,

$$\dot{u}(t)=-D(u(t))+Ag(u(t))+Bg(u(t-h(t)))+C\int_{t-\tau(t)}^{t}g(u(s))\,ds+I, \quad (1)$$

where $u(t)=[u_1(t),u_2(t),\ldots,u_n(t)]^T$ is the neural state vector, the matrix $D=diag\{d_1,d_2,\ldots,d_n\}$ is a diagonal matrix and $d_i>0$, $i=1,\ldots,n$. $A\in\mathbf{R}^{n\times n},B\in\mathbf{R}^{n\times n}$ and $C\in\mathbf{R}^{n\times n}$ are the discretely delayed connection weight matrices and the distributively delayed connection weight matrix respectively. $g(u(t))=[g_1(u_1(t)),g_2(u_2(t)),\ldots,g_n(u_n(t))]^T\in\mathbf{R}^n$ denotes the neuron activation function with $g(0)=0$. $h(t)>0$ denote the discrete time varying delay. The term $\int_{t-\tau(t)}^{t}g(u(s))\,ds$ denotes the distributively delayed connection. $\tau(t)>0$ denote the distributed time varying delay. $h(t)>0$ and $\tau(t)>0$ are assumed to satisfy $0\leq h(t)\leq\overline{h},0\leq\tau(t)\leq\overline{\tau},\dot{\tau}(t)\leq\tau^*<1$ and $\dot{h}(t)\leq h^*<1$ where $\overline{h},\overline{\tau},\tau^*$ and h^* are known constants. $I=(I_1,I_2,\ldots,I_n)\in R^n$ is a constant external input vector.

It is well known that bounded activation functions always guarantee the existence of an equilibrium point for Hopfield neural network (1). For convenience, we shift the equilibrium point $u^*=(u_1^*,u_2^*,\ldots,u_n^*)^T$ to the origin by translation $x(t)=u(t)-u^*$, which yields the following system:

$$\dot{x}(t)=-Dx(t)+Af(x(t))+Bf(x(t-h(t)))+C\int_{t-\tau(t)}^{t}f(x(s))\,ds,$$

where $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T\in R^n$ is the state vector of the transformed system and $f(x(\cdot))=(f_1(x(\cdot)),\ldots,f_n(x(\cdot)))^T,f(x(\cdot))=g(x(\cdot)+u^*)-g(u^*)$.

Based on the discussions in the previous section in this paper, we generalize the ordinary T–S fuzzy models to express a complex system whose consequent parts are a set of stochastic Hopfield neural networks with discrete and distributed time varying delays. The model of Takagi–Sugeno fuzzy Hopfield neural networks with discrete and distributed time varying delays is described as follows.

Plant Rule k**:**

IF $\{\theta_1(t)\quad isM_{k1}\}$ and …and $\{\theta_r(t)\quad isM_{kr}\}$

THEN

$$dx(t)=\left[-D_kx(t)+A_kf(x(t))+B_kf(x(t-h(t)))+C_k\int_{t-\tau(t)}^{t}f(x(s))\,ds\right]dt$$
$$+\sigma_k(t,x(t),x(t-h(t)),x(t-\tau(t)))dw(t),$$

where $\theta_i(t)$ and $(i=1,2,\ldots,r)$ are known variables. $M_{kl}(k\in\{1,2,\ldots,m\},l\in\{1,2,\ldots,r\})$ is the fuzzy set and m is the number of model rules. The initial condition associated with this model is $x_0=\xi\in L^2_{\mathcal{F}_0}([-\rho,0];R^n),\rho=max\{\overline{h},\overline{\tau}\}$. Also $\omega(t)$ is a Wiener process (Brownian motion) on $(\Omega,\mathcal{F},\{\mathcal{F}\}_{t\geq 0},P)$ which satisfies $E\{\omega(t)\}=0,E\{\omega(t)\}^2=t$.

By inferring from the fuzzy models, the final output of TSSFHNNs is obtained by

$$dx(t)=\sum_{k=1}^{m}\eta_k(\theta(t))\{[-D_kx(t)+A_kf(x(t))+B_kf(x(t-h(t)))$$
$$+C_k\int_{t-\tau(t)}^{t}f(x(s))\,ds]\,dt+\sigma_k(t,x(t),x(t-h(t)),x(t-\tau(t)))dw(t)\}. \quad (2)$$

The weight and averaged weight of each fuzzy rule are denoted by

$$\omega_k(\theta(t))=\prod_{l=1}^{t}M_{kl}(\theta(t))\quad\text{and}\quad\eta_k(\theta(t))=\frac{\omega_k(\theta(t))}{\sum_{k=1}^{m}\omega_k(\theta(t))},$$

respectively. The term $M_{kl}(\theta(t))$ is the grade membership of $\theta_l(t)$ in M_{kl}.

We assume that

$$\omega_k(\theta(t))\geq 0,\quad\text{and}\quad\sum_{k=1}^{m}\eta_k(\theta(t))=1,\quad\text{for all } t\geq 0.$$

Throughout this paper, we make the following assumption:

(A1) There exist positive numbers l_j such that

$$0\leq\frac{f_j(x)-f_j(y)}{x_j-y_j}\leq l_j,\ j=1,2,\ldots,n.$$

for all $x_j,y_j\in R,x_j\neq y_j$ and denote $L=diag\{l_1,l_2,\ldots,l_n\}$.

(A2) There exist matrices $P_1\geq 0,P_2\geq 0,P_3\geq 0$ such that

$$trace[\sigma_k^T(t,x(t),x(t-h(t)),x(t-\tau(t)))P\sigma_k(t,x(t),x(t-h(t)),x(t-\tau(t))]$$
$$\leq x^T(t)P_1x(t)+x^T(t-h(t))P_2x(t-h(t))+x^T(t-\tau(t))P_3x(t-\tau(t))).$$

Remark 2.1. Under the assumptions (A1) and (A2), it is easy to check that functions f and σ satisfy the linear growth condition [12]. Therefore, for any initial data $\xi\in L^2_{\mathcal{F}}([-\rho,0];R^n)$ the system (2) has unique solution (or equilibrium point) denoted by $x(t;\xi)$ or $x(t)$.

Defining the following state variables for the TSSFHNNs,

$$y(t)=\sum_{k=1}^{m}\eta_k(\theta(t))[-D_kx(t)+A_kf(x(t))+B_kf(x(t-h(t)))$$
$$+C_k\int_{t-\tau(t)}^{t}f(x(s))\,ds], \quad (3)$$

$$g(t)=\sum_{k=1}^{m}\eta_k(\theta(t))[\sigma_k(t,x(t),x(t-h(t)),x(t-\tau(t)))]. \quad (4)$$

By using (3) and (4) the TSSFHNNs can be represented as

$$dx(t)=y(t)\,dt+g(t)\,d\omega(t).$$

Now we give the following definition of exponential stability for neural networks (2).