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Multiple resonances with time delays and enhancement by non-Gaussian noise in Newman–Watts networks of Hodgkin–Huxley neurons

Yinghang Hao, Yubing Gong*, Xiu Lin

School of Physics, Ludong University, Yantai, Shandong 264025, PR China

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ABSTRACT

In this paper, we study the effect of time delay on the spiking activity in Newman–Watts small-world networks of Hodgkin–Huxley neurons with non-Gaussian noise, and investigate how the non-Gaussian noise affects the delay-induced behaviors. It was found that, as the delay increases, the neuron spiking intermittently performs the most ordered and synchronized behavior when the delay lengths are integer multiples of the spiking periods, which shows multiple temporal resonances and spatial synchronizations, and reveals that the locking between the delay lengths and the spiking periods might be the mechanism behind the behaviors. It was also found that the delay-optimized spiking behaviors could be enhanced when non-Gaussian noise's deviation from the Gaussian noise is appropriate. These results show that time delay and non-Gaussian noise would cooperate to play more constructive and efficient roles in the information processing of neural networks.

1. Introduction

In the last decades, the constructive role of noise has been extensively studied in various nonlinear dynamics, the most prominent of which is stochastic resonance (SR) and coherence resonance (CR) [1–7]. SR is characterized by the optimization of the output signal-to-noise ratio in a nonlinear dynamical system following the addition of a weak external signal, and CR refers to the resonant response of a dynamical system to pure noise. Among these studies, many have been dedicated to the SR and CR in various neuronal systems [2–7]. It is well known that a single neuron in the vertebrate cortex connects to more than 10 000 postsynaptic neurons via synapses forming complex networks [8]. Therefore, it is necessary to employ neural networks to account for the dynamics of neural systems, and randomly adding a number of long-range shortcuts among neurons representing random connections is reasonable and feasible due to the presence of random information transmission among the neurons [9–11]. In recent decade, many SR and CR phenomena have been found in complex neuronal networks, such as array-enhanced CR in coupled FitzHugh–Nagumo neurons [12,13], CR in Watts–Strogatz networks of Hodgkin–Huxley (HH) neurons [14,15], SR in small-world networks of overdamped bistable oscillators [16], Watts–Strogatz networks of Rulkov map [17], Newman–Watts networks of HH neurons [18], and two-dimensional spatially extended neuronal networks [19–21]. In all these works, noise is always assumed to be of Gaussian behavior. However, non-Gaussian noise has been experimentally found in sensory systems like neurons of crayfish [22] and rat skin [23], as well as in calcium oscillations in hepatocytes [24]. In the last decades, two types of non-Gaussian noises, i.e., Lévy noise and a particular kind of non-Gaussian colored noise (NGN), have attracted much attention. It has been shown that Lévy stochastic processes are very common in economic and social systems [25,26] and the NGN widely exists in nonlinear dynamical systems [27–36]. The non-Gaussian character of the NGN is characterized simply by a parameter q that measures the NGN's deviation from the Gaussian noise ($q=1$) and determines the NGN's probability distribution function. Thus, such a particular form allows one to easily control the deviation from the Gaussian behavior by changing a single parameter q. Very recently, we have found the NGN-induced CR in a single HH neuron and an array of coupled HH neurons [37,38].

Firing synchronization in neuronal networks is another important dynamical phenomena, since the synchronization of coupled neurons may elucidate how the coherent spontaneously synchronized oscillations, which have been observed in the brain cortex, are established in many neural systems [39–41]. In the past years, people have found many synchronization phenomena, such as noise-induced synchronization in modified HH (MHH) neurons [42], a ring neuronal network [43], two coupled map-based neurons [44], a square lattice noisy neuronal network [45],

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* Corresponding author. Tel.: +86 535 6697550; fax: +86 535 6672870.
E-mail address: gongyubing09@hotmail.com (Y. Gong).

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and scale-free networks of Morris–Lecar neurons [46]; burst-enhanced synchronization in an array of noisy coupled MHH neurons [47], and synchronization in a large ensemble of MHH neurons with gap junctions [48] and in small-world neuronal networks [49–53].

In neuronal systems, time delay is inherent because of both the finite speed at which action potentials propagate across neuron axons and time lapses occurring in both dendritic and synaptic processing [54]. The effects of time delay on the firing dynamics of neuronal systems have attracted much attention in recent years. It has been shown that time delay can facilitate and improve neuronal synchronization [55–57], induce various spatio-temporal patterns [58], enhance the coherence of spiral waves in noisy HH neuronal networks [59]. Very recently, Wang et al. have found interesting phenomena of delay-induced synchronization transitions in small-world and scale-free neuronal networks [60,61], as well as multiple stochastic resonances in scale-free neuronal networks [62].

The goal of this paper is to study the effect of time delay and non-Gaussian noise in the spiking activity of neuronal networks. Using Newman–Watts networks of electrically coupled HH neurons with NGN, we study how the spiking behavior changes with varying time delay and how the NGN affects the delay-induced behaviors. It is found that the neurons intermittently exhibit the most ordered and synchronized spiking behaviors when the delay lengths are integer multiples of the spiking periods. When the NGN is appropriate, the delay-optimized spiking behaviors can be enhanced.

2. Model and equations

According to the HH neuron model, the dynamics of the membrane potential $V(t)$ can be described by

$$C\frac{dV}{dt} = -g_{\mathrm{Na}}m^3h(V-V_{\mathrm{Na}})-g_{\mathrm{K}}n^4(V-V_{\mathrm{K}})-g_{\mathrm{L}}(V-V_L)+I(t)+\eta(t), \quad (1a)$$

where the constants $g_{\mathrm{Na}}=120\,\mathrm{mS/cm^2}$, $g_{\mathrm{K}}=36\,\mathrm{mS/cm^2}$, and $g_L=0.3\,\mathrm{mS/cm^2}$ are the maximal conductance of sodium, potassium, and leakage conductance, respectively. $C=1\,\mu\mathrm{F/cm^2}$ is the membrane capacitance; $V_{\mathrm{Na}}=50$ mV, $V_{\mathrm{K}}=-77$ mV, and $V_L=-54.4$ mV are the reversal potentials of sodium, potassium, and leakage currents, respectively. m^3h and n^4 are the mean portions of the open potassium and sodium ion channels within the membrane patch, respectively. We employ a sub-threshold periodic stimulus $I=6.0+\sin(0.3t)$ [63,64]. The stochastic gating variables m, h, and n obey the following Langevin equations:

$$\dot{m} = \alpha_m(V)(1-m)-\beta_m(V)m, \quad (1b)$$

$$\dot{h} = \alpha_h(V)(1-h)-\beta_h(V)h, \quad (1c)$$

$$\dot{n} = \alpha_n(V)(1-n)-\beta_n(V)n, \quad (1d)$$

with voltage-dependent opening–closing transition rates given by

$$\alpha_m(V) = \frac{0.1(V+10)}{1-\exp[-(V+40)/10]}, \quad (2a)$$

$$\beta_m(V) = 4\exp[-(V+65)/18], \quad (2b)$$

$$\alpha_h(V) = 0.07\exp[-(V+65)/20], \quad (2c)$$

$$\beta_h(V) = \frac{1}{1+\exp[-(V+35)/10]}, \quad (2d)$$

$$\alpha_n(V) = \frac{0.01(V+55)}{1-\exp[-(V+55)/10]}, \quad (2e)$$

$$\beta_n(V) = 0.125\exp[-(V+65)/80]. \quad (2f)$$

The noise term $\eta(t)$ is assumed to have a non-Gaussian distribution [35] with

$$\frac{d\eta(t)}{dt} = -\frac{1}{r}\frac{d}{d\eta}V_q(\eta)+\frac{\sqrt{2D}}{r}\xi(t), \quad (3a)$$

where

$$V_q(\eta) = \frac{D}{r(q-1)}\ln\left[1+\frac{r}{D}(q-1)\frac{\eta^2}{2}\right], \quad (3b)$$

and $\xi(t)$ is a Gaussian white noise with vanishing mean and autocorrelation function $\langle\xi(t)\xi(t')\rangle=\delta(t-t')$, D and r are parameters related to the noise intensity and the correlation time, respectively, q stands for the deviation from Gaussian behavior ($q=1$). Note that the change of deviation depends on the value of q. For $q>1$, the deviation increases as q increases, but for $q<1$, the deviation increases as q decreases.

Theoretical analysis [35] shows that the stationary probability distribution has the form

$$P_q^{st}(\eta) = \frac{1}{Z_q}\left[1+\frac{r}{D}(q-1)\frac{\eta^2}{2}\right]^{-1/(q-1)}, \quad (4)$$

where Z_q is a normalization constant. This distribution can be normalized only for $q<3$. The first moment is always equal to zero, and the second moment is given by $\langle\eta_q^2\rangle=2D/r(5-3q)$, which is finite only for $q<5/3$. For $q>1$, the distribution has a long tail, while for $q<1$, the distribution has a cut-off and is only defined for $|\eta|<\sqrt{2D/r(1-q)}$. Clearly, when $q\to1$, the limit of $\eta(t)$ is Gaussian colored noise. The effective noise intensity $D_q=[2(2-q)/(5-3q)]^2D$ and the effective correlation time $r_q=2(2-q)r/(5-3q)$ diverge near $q=5/3$. Thus, our calculations will be restrained by $q<5/3$.

The spiking dynamics of the coupled HH neuron networks can be written as

$$C\frac{dV_i}{dt} = -g_{\mathrm{Na}}m_i^3h_i(V_i-V_{\mathrm{Na}})-g_{\mathrm{K}}n_i^4(V_i-V_{\mathrm{K}})-g_L(V_i-V_L) + I(t)+\eta_i(t)+\sum_j\varepsilon_{ij}(V_j(t-\tau)-V_i), \quad (5a)$$

$$\frac{dx_i}{dt} = \alpha_{x_i}(V_i)(1-x_i)-\beta_{x_i}(V_i)x_i, \quad (5b)$$

where $x=m$, h, n, τ is the time delay (in unit of ms). In the coupling term $\sum_j\varepsilon_{ij}[V_j(t-\tau)-V_i]$, V_i is the membrane potential of the ith neuron at time t, $V_j(t-\tau)$ is the membrane potential of jth neuron at earlier time $t-\tau$, $1\le(i,j)\le N$, N is the number of neurons, and the summation takes over all neurons; ε_{ij} is a coupling constant between the two neurons i and j, which is determined by the coupling pattern of the system and is identical for any two neurons, i.e., $\varepsilon_{ij}=\varepsilon$. $\varepsilon=0.1$ if neurons i and j are connected; $\varepsilon=0$ otherwise.

The neuronal network [47,48] used here starts with a regular ring comprising $N=60$ identical HH neurons, each neuron connecting with two nearest neighbors. Links are randomly added between non-nearest vertices. In the limit case that all neurons coupled to each other, the network contains $N(N-1)/2$ edges. Using M to denote the number of added shortcuts, the fraction of the shortcuts is given by $p=M/[N(N-1)/2]$, which is used to characterize the randomness of the network topology.

Coefficient of variation (CV) λ and standard deviation σ are used to quantitatively characterize the spiking temporal coherence and spatial synchronization, respectively. The CV for the spikes of a single neuron on the network is defined as

$$\lambda_i = \langle T\rangle/\sqrt{\langle T^2\rangle-\langle T\rangle^2}, \quad (6)$$