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Finite-time H_∞ control for one-sided Lipschitz systems with auxiliary matrices

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ABSTRACT

In this paper, finite-time H_∞ control is investigated for a class of one-sided Lipschitz systems by using the Finsler's lemma. Finite-time boundedness conditions are firstly provided for the one-sided Lipschitz system. The proposed conditions are less conservative since a new Lyapunov function with an additional matrix is constructed, and auxiliary matrices are introduced to make the Lyapunov matrix separate from the system matrix. Then, finite-time H_∞ controller is designed by dynamic output feedback. The observer gain and output feedback control gain can be designed in one step optimization. Further discussions and comparisons are also presented in two aspects. At last, numerical simulations are given to verify the validity of the proposed methods.

1. Introduction

In the past few decades, researchers have paid considerable attention on Lipschitz nonlinear systems [1–5]. Recently, some researchers have transferred interest from Lipschitz nonlinear systems to one-sided Lipschitz nonlinear systems [6–11]. The reason is that Lipschitz conditions may fail to be utilized when the Lipschitz constant is large [6,9]. Therefore, one-sided Lipschitz nonlinear systems have been extensively studied by researchers.

It should be noted that the results in the aforementioned papers related to stability and performance criteria are mostly defined over an infinite time interval. In some practical applications, the main concerns are the behavior of the considered system do not exceed a given bound in a finite time interval, that is, large values of the states are not acceptable in the presence of saturations [16]. In this sense, it is reasonable to define stability for a system as its states, with some initial conditions, remains within a given bound in the fixed finite time interval. For this purpose, definition of finite time stability (FTS) is presented in [12,13]. Later, the definition of FTS is extended to definition of finite-time boundedness (FTB) by taking external disturbances into consideration [16]. By linear matrix inequality (LMI) theory, computationally appealing conditions are presented to ensure FTS and FTB for linear systems. The problem of FTB by dynamic output feedback is also considered in [14]. The proposed approach allowed to fit the FTB control problem in the general framework of the LMI approach to the multi-objective synthesis. Sub-optimal controller can be derived by a two-step optimization. There are some results on finite-time stabilization for discrete-time systems [15]. The disturbances considered in [15,16] are assumed to be constant. The results on FTB for linear systems with time-varying disturbances and for linear time-varying systems can be referred to [17,18] and [19], respectively. In order to reduce conservativeness, new sufficient conditions for FTB are derived by using the Finsler's lemma [21]. However, the synthesis of FTS and FTB controllers by the proposed conditions is intractable. More recently, definition of finite-time H_∞ control is presented for a class of linear systems [20]. A state feedback controller is designed to ensure that the closed-loop system is FTB, and at the same time to reduce the effect of the disturbance input on the output to a prescribed level. The results in [20] have been extended to one-sided Lipschitz nonlinear systems [22] and time-delayed nonlinear systems [23].

In this paper, motivated by the ideas in [21,22], we consider robust finite-time H_∞ control for one-sided Lipschitz systems based on Finsler's lemma. Firstly, sufficient conditions for FTB are presented. The proposed conditions are less conservative since a new Lyapunov function is constructed, and auxiliary matrices are introduced to make the Lyapunov matrix separate from the system matrix. Then, a

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dynamic output feedback controller is designed to ensure the closed-loop systems are FTB, and the effect of the disturbance input on the output to a prescribed level simultaneously. The observer gain and output feedback control gain can be designed in one step optimization. Further discussions and comparisons are also presented in two aspects: robust finite-time H_∞ control by state feedback, and finite-time stabilization by output feedback for a class of linear systems considered in [14]. Finally, numerical simulations are given to verify the validity of the proposed approach. The main contributions of this paper includes: a new Lyapunov function with an auxiliary matrix is constructed, and computationally tractable and less conservative sufficient conditions are presented based on the Finsler's lemma.

This paper is organized as follows. In Section 2, some definitions and robust finite-time H_∞ control are presented. The problem of finite-time H_∞ control by dynamic output feedback is studied for a class of one-sided Lipschitz nonlinear systems by using the Finsler's lemma in Section 3. Section 4 gives further discussions and comparisons in two aspects. In Section 5, numerical simulations are provided to illustrate the validity of the proposed design methods. Finally, the paper is concluded in Section 6.

2. Definitions and problem formulation

In this section, we introduce some useful definitions and lemma and state our main aim of the paper.

Definition 1 (*Song and He [22]*). Consider the following nonlinear system

$$\begin{cases} \dot{x}(t) = Ax(t) + Gw(t) + f(x(t)), \ x(t_0) = x_0, \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where the matrices $A \in \mathcal{R}^{n\times n}$, $G \in \mathcal{R}^{n\times l}$, and $C \in \mathcal{R}^{p\times n}$, the state $x(t) \in \mathcal{R}^n$, the output $y(t) \in \mathcal{R}^p$, $f(x(t)) : \mathcal{R}^n \to \mathcal{R}^n$ is a real nonlinear vector field, the exogenous disturbance $w(t) \in \mathcal{W} \subset \mathcal{R}^l$, and the disturbance set $\mathcal{W}$ is given as

$$\mathcal{W} = \left\{ w(t) : \int_0^T w^T(t) w(t)\, dt < d^2, \ d > 0 \right\}. \tag{2}$$

The system (1) is FTB with respect to $(c_1, c_2, \mathcal{W}, T, R)$, if

$x_0^T R x_0 \le c_1^2 \Rightarrow x^T(t) R x(t) \le c_2^2, \forall t \in [0, T], w(t) \in \mathcal{W},$

where $c_2 > c_1 > 0$, $T > 0$ and $0 < R \in \mathcal{R}^{n\times n}$.

Definition 2. The system (1) satisfies the following H_∞ performance index, if the following inequality

$$\int_0^T y^T(t) y(t)\, dt \le \gamma^2 \int_0^T w^T(t) w(t)\, dt, \tag{3}$$

holds under the zero initial condition, where γ is a positive real number.

Now, consider the following system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Gw(t) + f(x(t)), \\ y(t) = Cx(t), \ x(t_0) = x_0, \end{cases} \tag{4}$$

where the matrices $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$, $G \in \mathcal{R}^{n\times l}$, and $C \in \mathcal{R}^{p\times n}$, and the state $x(t) \in \mathcal{R}^n$, the input $u(t) \in \mathcal{R}^m$, the output $y(t) \in \mathcal{R}^p$, the exogenous disturbance $w(t) \in \mathcal{W} \subset \mathcal{R}^l$. We make the following assumptions, the matrix B has full column rank, and the nonlinear function $f(x(t))$ satisfies $f(0) = 0$ and the following conditions

(a) one-sided Lipschitz condition

$$< f(x_1) - f(x_2), x_1 - x_2 > \ \le \rho_1 (x_1 - x_2)^T (x_1 - x_2), \tag{5}$$

where ρ_1 is a real number;

(b) quadratic inner-boundedness in the region $\mathcal{D}$

$$(f(x_1) - f(x_2))^T (f(x_1) - f(x_2)) \le \rho_2 (x_1 - x_2)^T (x_1 - x_2) + \rho_3 < f(x_1) - f(x_2), x_1 - x_2 >, \tag{6}$$

for any x_1, $x_2 \in \mathcal{D}$, where ρ_2 and ρ_3 are real numbers.

In this paper, our aim is to design an output feedback controller in the form of

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + LC(\hat{x}(t) - x(t)) + f(\hat{x}(t)), \ \hat{x}(0) = 0, \\ u(t) = K\hat{x}(t), \end{cases} \tag{7}$$

such that the closed-loop (4)–(7) is FTB, and at the same time the performance index (3) is satisfied. Then, the dynamic output feedback controller (7) is called robust output feedback finite-time H_∞ controller of the one-sided Lipschitz nonlinear system.

Remark 1. The matrix B has full column rank, which can be taken nonstrict at all. Since if $\mathrm{rank}(B) = r < m$, then, there exist invertible matrices S and T such that $SBT = \left[\begin{smallmatrix} I_r & 0 \\ 0 & 0 \end{smallmatrix}\right]$, where I_r denotes r-order unite matrix. Let $y(t) = Sx(t)$, then,

$\dot{y}(t) = SAS^{-1} y(t) + SBTT^{-1} u(t) + Sw(t) + Sf(S^{-1} y(t)).$