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# 3-Points Convex Hull Matching (3PCHM) for fast and robust point set registration

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ABSTRACT

Point set registration plays a crucial role in numerous computer vision applications. This paper proposes a novel and general approach called three-point convex hull matching (3PCHM) for registering two point sets with similarity transform. First, convex hulls are extracted from both point sets. Triangular patches on the surface of convex hulls are specified by predefining their normal vectors, thus guaranteeing that all points are located on the same side of any randomly selected triangle plane. Second, the potential similar triangle pair set is obtained by comparing the length ratio of the edges on the two extracted convex hulls. Thereafter, the transformation parameters for each pairwise triangle are calculated by minimizing the Euclidean distance between the corresponding vertex pairs. Furthermore, a k-dimensional (k-d) tree is used to accelerate the closest point search for the whole point sets. Third, outliers that may lead to significant errors are discarded by integrating the random sample consensus algorithm for global optimization. Experiments show that the proposed 3PCHM is robust even with the existence of noise and outliers and is effective in cases of part-to-part registration and part-to-whole registration.



## 1. Introduction

The development of computer graphics, computer vision, virtual reality, and augmented reality in recent years has increased the research attention on 3D model generation and manipulation techniques. In all relative techniques, registration and alignment are the most important aspects for the quantitative analysis of 3D models. A 3D model can be represented as a mesh or cloud point, and the registration technique aims to find the best matching geometric warping among models at different representations. This technology has significant applications in the fields of photogrammetry, motion tracking, camera pose recovering, and object identification.

In the past two decades, numerous methods have been developed for the registration of point sets and 3D models. The most famous method is the iterative closest point (ICP), which was proposed by Besl and Mckay [1] in 1992. In ICP, the transformation between two different point sets is optimized by minimizing Euclidian distance between every corresponding point pair of two sets. Considering the simplicity and effectiveness of ICP for point sets with relatively small variations, ICP has been widely used in different fields and referenced by numerous studies. However, the ICP algorithm is highly dependent on the initial geometry of registering point sets and can be easily trapped into a local minimum.

Numerous ICP variants have greatly increased our understanding of matching problems. The ICP algorithm generally assumes that the points to be registered have homogeneous Gaussian noise. On the basis of this hypothesis, Granger et al. [2,3] extended the ICP algorithm by using expectation-maximization (EM) principles to estimate the Gaussian weights of the matches, thus resulting in the EM–ICP algorithm. They also proposed a coarse-to-fine annealing scheme to avoid local minimum. By decimating the point sets, the computation time explosion at coarse levels can be reduced. EM–ICP provides better repeatability, superior accuracy, and higher computation efficiency than the original ICP. Instead of calculating the one-to-one corresponding relationship between each point by using the nearest neighbor criterion, Myronenko [4,5] and Jian et al. [6] assumed that each model point corresponds to a weighted sum of the scene points; thus, the point sets can be represented as Gaussian mixture models (GMMs). Point set registration can also be used to align two Gaussian mixtures by minimizing their discrepancies. Rather than providing additional prior affinity measures [7] for ICP algorithms, GMM-based algorithms [8] statistically estimate the

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discrepancy between point sets and significantly improve the computational accuracy and robustness of point set registration.

ICP variants generally employ the Euclidean distance as the distance metric. The Euclidean distance theoretically guarantees that corresponding matches can be found by tentative iterations. However, some of these algorithms are rendered ineffective by noise, occlusion, and partial point scarcity. An effective way to modify the distance metric with respect to the whole point set is to find local invariant features [9], such as surface orientation [10], curvature [11], and normal [12] or congruent shapes [13]. Some algorithms aim to establish a pairwise relationship and distinguish a limited number of invariable features from numerous points with respect to similarity transformation, thus resulting in a significant reduction in computation complexity. However, feature extraction and matching procedures are sensitive to imaging noise and resolution. Thus, higher levels of geometric descriptors such as shape contexts [14,15], spin images [16], similarity graphs [17], and log-polar height maps [18] have been proposed for point set registration. These approaches generally integrate structural or topological information into the registration scheme, thus allowing the accurate estimation of camera parameters under poor initialization and the handling of partial or missing structures. Unfortunately, these techniques are ill suited for point sets with unknown densities or sparse distributions. Furthermore, localization uncertainty for point sets may be highly anisotropic because of different imaging techniques. Therefore, the commonly assumed isotropic noise distribution for point sets can be deemed as an ill-posed theorem, thus leading to the use of SoftICP [19], SoftAssign [20], or A-ICP [21] for the estimation and refinement of tentative correspondences by the integration of anisotropic weight estimation.

Given that iterative optimization is needed to minimize the sum of squared distances between corresponding points, several strategies have been designed to improve the effectiveness of registration procedures (e.g., the Levenberg–Marquardt (LM) algorithm [22], dynamic programming [23,24], annealing scheme [25], and multi-resolution strategies [26]). The LM–ICP [22], which is capable of yielding a large basin of convergence than common techniques, is proposed by integrating the LM algorithm into the optimization kernel function. The random sample consensus (RANSAC) algorithm [27] iteratively estimates the parameters of a predefined model of observed data by removing outliers; this method is effective for refining the results of point set registration [28,29]. The four-point congruent sets (4PCS) approach is proposed by extracting all coplanar four-point set from source point sets that are approximately congruent to the given set of coplanar four-point in the target point sets under rigid transformation [13]. By integrating the RANSAC algorithm, the 4PCS is capable of calculating the global transformation by using a set of sampling coplanar points. On the contrary, recent developments in graphics hardware and software have motivated people to accelerate registration procedures by parallelization [30] and graphics processing unit (GPU) [31] implementations, which have been widely acknowledged as effective ways for fast registration.

All of the above-mentioned methods have greatly improved the technique of point set registration. However, given that the complexity of the registration problem is closely correlated with the noise distribution, density, and sparsity of two point sets, many challenges for robust point set registration still exist. We propose a novel three-point convex hull matching (3PCHM) method on the basis of previous studies. First, convex hulls are extracted from both point sets to be registered. The specification of triangle patches is then conducted by using normal vectors on the convex hulls. Thereafter, a similar triangle pair set is obtained by comparing the length ratio of each triangle on the convex hull of the two point sets; this step also assists the computation of the scaling factor for the similarity transformation. The optimization of the transformation parameters including the rotation and translation for each triangle pair is realized by minimizing the Euclidean distance between the corresponding vertex pairs. Pairs that may lead to significant errors are discarded by using the RANSAC algorithm to achieve global optimization. The main contribution of the proposed algorithm is twofold: first, considering that the invariant property of the extracted 3D convex hull is used for point set registration, the process is independent of the initial pose and the alignment of point sets. Second, the registration procedure is robust and efficient with respect to the initial transformation because of the utilization of limited number triangle pairs for computation.

## 2. Method

Suppose we have two finite sized point sets with similarity transformation to be registered. Let $P = \{p_1, p_2, \dots, p_{N_P}\}$ represent the source point set and $Q = \{q_1, q_2, \dots, q_{N_Q}\}$ represent the target point set. Both $P$ and $Q$ are assumed subsets of the vector space $\mathbb{R}^3$, $N_P$ and $N_Q$ are the number of $P$ and $Q$ respectively. The registration approach is addressed to obtain the optimal transformation between spaces. Fig. 1 shows the basic procedural flow of the 3PCHM algorithm with its key processes. The registration procedure can be divided into the following stages:

a) The first stage is the formation of the convex hull and involves the extraction of the convex hull of both 3D point sets.
b) The second stage is triangle matching, wherein the length ratios of each triangle are computed and ordered. Thereafter,

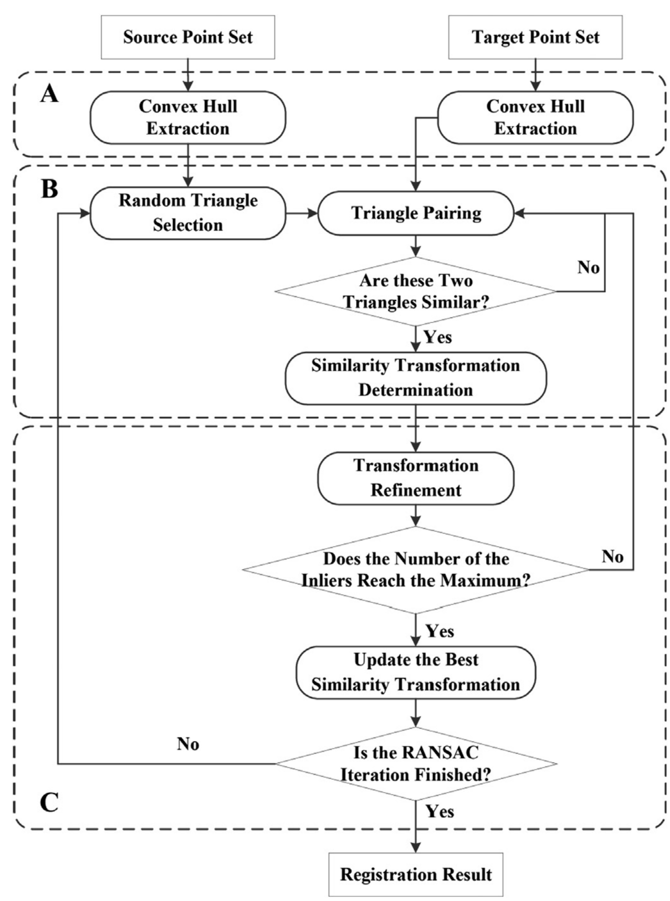


**Fig. 1.** Flow chart of the 3PCHM algorithm. (A) Formation of convex hull. (B) Triangle matching and registration. (C) Global optimization.