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Brief Papers

New inequality-based approach to passivity analysis of neural networks with interval time-varying delay

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ABSTRACT

This paper is concerned with the problem of passivity analysis of neural networks with an interval time-varying delay. Unlike existing results in the literature, the time-delay considered in this paper is subjected to interval time-varying without any restriction on the rate of change. Based on novel refined Jensen inequalities and by constructing an improved Lyapunov–Krasovskii functional (LKF), which fully utilizes information of the neuron activation functions, new delay-dependent conditions that ensure the passivity of the network are derived in terms of tractable linear matrix inequalities (LMIs) which can be effectively solved by various computational tools. The effectiveness and improvement over existing results of the proposed method in this paper are illustrated through numerical examples.

1. Introduction

During the last decade, we have witnessed increasing interest in studying asymptotic behavior and control of neural networks due to their potential applications in various fields such as image and signal processing, pattern recognition, associative memory, parallel computing, and solving optimization problems [1,2]. When modeling and implementing artificial neural networks and general complex dynamical networks, time-delay is often encountered in real applications due to the finite switching speed of amplifiers [3] which usually becomes a source of poor performance, oscillation, divergence and even instability [4]. Considerable attention has been devoted to address various issues of neural networks with delays recently (see, for example, [5–9]). For simple circuits with a small number of cells, the use of fixed constant delays may provide a good approximation when modeling them. However, in practical implementation, neural networks usually have a spatial nature due to the presence of an amount of parallel pathways with a variety of axon sizes and lengths. As a consequence, the time-delay in neural networks is usually time-varying and belongs to an interval the lower bound of which is not restricted to be zero. Therefore, the study of neural networks with interval time-varying delay is more relevant and important in practice which has attracted increasing interest recently [10–13].

On the other hand, known as part of a broader and a general theory of dissipativeness, passivity theory plays an important role in stability analysis and control of dynamical systems [14,15]. The main point of passivity theory is that the passive properties of the system can keep the system internally stable [16]. Specifically, the passive system utilizes the product of input and output as the energy provision and embodies the energy attenuation character. A passive system only burns energy without energy production, and thus passivity represents the property of energy consumption [17]. The problem of passivity performance analysis has also been extensively applied in many areas such as signal processing, fuzzy control, sliding mode control and networked control. Over the past few years, increasing attention has been devoted to this topic and a number of important results have been reported, for example, in [18–32] and especially in [33–36]. Particularly, in [33], by employing the Wirtinger-based integral inequality (WBI) to estimate the derivative of an augmented LKF, some improved delay-dependent passivity conditions were derived for a class of neural networks with discrete and distributed delays. In [35], a general model of coupled reaction–diffusion neural networks was considered. By designing some adaptive strategies to tune the coupling strengths among network nodes and utilizing some techniques used in estimating the storage function, sufficient conditions ensuring passivity and synchronization of the network were derived. The problem of passivity analysis of uncertain neural networks with interval time-varying delay was also studied in [36]. A novel LKF, which does not require the positiveness of all symmetric matrices, was first constructed. Then, by using Jensen's

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inequality combined with free-weighting matrix method, some less conservative delay-dependent conditions ensuring the passivity of the system were proposed.

However, it should be pointed that: (i) in most of the aforementioned contributions, the time-delay is restricted to be slowly varying (for example, the condition $\dot{\tau}(t) \le \sigma < 1$ should be met) and/or the lower bound of delay is set to be zero; (ii) the use of Jensen integral inequality and its variants usually introduces undesirable conservatism in the derived conditions; and (iii) in the tradeoff between the feasibility of the region of delays and the computational cost, the use of extra slack variables seems to be ineffective while increasing much computational burden. In general, constructing a more suitable LKF and especially reducing the enlargement in estimating the derivative of the LKF are essential and require further investigation [33]. Besides that time-delay is usually ignored in the system output in mots existing results. Indeed, due to some practical reasons such as the finite speed of the data processing through a low-rate communication channel or sensor technology, time-delay associated with the output naturally arises in a variety of engineering applications to which we must consider the impact of time-delay in the output of the systems [37–39].

Motivated by the above discussions, in this paper we further investigate the problem of passivity analysis of a class of neural networks with time-varying delay in both the system state and output. In comparison to existing literature, the novelty of the present paper is threefold. Firstly, the time-delay under consideration in state and output is subjected to interval time-varying without any restriction on the rate of change. This means that the results obtained in this paper can be applied for systems with fast time-varying delay the upper bound and lower bound of which are positive constants. Secondly, some novel refined Jensen-based inequalities [40,41], which were theoretically and numerically shown to be effective in reducing the conservatism of the stability conditions than the ones based on Jensen's inequality or Wirtinger based integral inequality, are utilized to manipulate the derivative of LKF candidates. Thirdly, an improved LKF, which fully utilizes information of the neuron activation functions and the time-delay in single, double and triple integral terms, is constructed. On the basis of these factors, new delay-dependent conditions that ensure the passivity of the system are derived in terms of tractable LMIs without using any extra free-weighting matrix. The proposed method in this paper allows one to achieve benefit in reducing both the conservatism and the computational cost of the derived conditions.

Notation: The notation used in this paper is fairly standard. Let $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space with vector norm $\|\cdot\|$ and the set of $n\times m$ matrices, respectively. For matrices $A,B\in\mathbb{R}^{n\times m}$, $\text{col}\{A,B\}$ and $\text{diag}\{A,B\}$ denote the block matrices $\left[\begin{smallmatrix}A\\B\end{smallmatrix}\right]$ and $\left[\begin{smallmatrix}A&0\\0&B\end{smallmatrix}\right]$, $\text{Sym}(A)$ stands for $A+A^T$. A matrix P is symmetric positive definite, write $P>0$, if $P^T=P$ and $x^TPx>0$ for all $x\in\mathbb{R}^n, x\neq 0$. Let $\mathbb{S}_n^+$ denote the set of symmetric positive definite matrices in $\mathbb{R}^{n\times n}$. We also denote by $\mathbb{D}_n^+$ the set of positive diagonal matrices, that is, a matrix $D=\text{diag}\{d_1,d_2,\dots,d_n\}\in\mathbb{D}_n^+$ if $d_i>0$ $(i=1,2,\dots,n)$.

2. Preliminaries

We consider a class of neural networks with interval time-varying delay of the form

$$\begin{cases}\dot{x}(t)=-Ax(t)+W_0g(x(t))+W_1g(x(t-\tau(t)))+u(t), & t\ge 0,\\ y(t)=C_1g(x(t))+C_2g(x(t-\tau(t)))+C_3u(t), & t\ge 0,\\ x(t)=\phi(t), & t\in[-\tau_2,0],\end{cases} \tag{1}$$

where n denotes the number of neurons in the network, $x(t)=[x_1(t),x_2(t),\dots,x_n(t)]^T\in\mathbb{R}^n$ is the neuron state vector, $y(t)\in\mathbb{R}^n$ is the output vector and $u(t)$ is the external input of the network, $g(x(t))=[g_1(x_1(t)),g_2(x_2(t)),\dots,g_n(x_n(t))]^T\in\mathbb{R}^n$ denotes the activation function, $g(x(t-\tau(t)))=[g_1(x_1(t-\tau(t))),g_2(x_2(t-\tau(t))),\dots,g_n(x_n(t-\tau(t)))]^T\in\mathbb{R}^n$, $A=\text{diag}\{a_1,a_2,\dots,a_n\}$ is a positive diagonal matrix and W_0, W_1 are interconnection weight matrices, C_1,C_2,C_3 are given real matrices, $\phi(t)\in\mathbb{R}^n$ is initial condition.

In this paper, we assume the delay $\tau(t)$ is time-varying satisfying

$$0\le\tau_1\le\tau(t)\le\tau_2 \tag{2}$$

where τ_1, τ_2 are known constants involving the lower bound and the upper bound of delay. Different from existing results in the literature, here we do not impose any condition on the rate of change of $\tau(t)$ which means that our results obtained in this paper can be applied to neural systems with fast time-varying delay.

Assume that the activation functions $g_i(x_i(t))$ are continuous, $g_i(0)=0$, and there exist constants l_i^-, l_i^+ $(i=1,2,\dots,n)$ such that

$$l_i^-\le\frac{g_i(x)-g_i(y)}{x-y}\le l_i^+,\quad \forall x,y\in\mathbb{R}, x\neq y. \tag{3}$$

Remark 1. As mentioned before, in many network models, time-delay encounters not only in the state but also in the output of the system which may be harmful to the system performance. On the other hand, in the presence of delay in the output, it is practically more complicated and challenging to analyse stability and synthesis control [37]. Therefore, it is relevant to deal with the problem of passivity analysis of neural networks with delayed output in the form of (1). In particular, when $C_2=0$ (that means no output signal is affected by time-delay) system (1) is reduced to the model with normal output which was considered in the literature, for example, [30,31,33,34,36].

Remark 2. The constants l_i^-, l_i^+ $(i=1,2,\dots,n)$ in (3) are allowed to be positive, negative or zero. As mentioned in the literature, for example, [8,13,33], condition (3) is less restrictive than the descriptions on both Lipschitz-type activation functions and sigmoid activation functions when analyzing the stability of such model of neural networks.

Let us first introduce the following definition and auxiliary lemmas which will be useful for the derivation of our results.

Definition 1. The network (1) is said to be passive if there exists a scalar $\gamma>0$ such that, under zero initial condition, the following inequality holds for all $t_f\ge 0$

$$2\int_0^{t_f}y^T(s)u(s)\,ds\ge-\gamma\int_0^{t_f}u^T(s)u(s)\,ds. \tag{4}$$

Lemma 1 (*Refined Jensen-based inequalities, Hien and Trinh [40]*). *For a given matrix $R\in\mathbb{S}_n^+$ and a function $\varphi:[a,b]\to\mathbb{R}^n$ whose derivative $\dot{\varphi}\in PC([a,b],\mathbb{R}^n)$, the following inequalities hold*

$$\int_a^b\dot{\varphi}^T(s)R\dot{\varphi}(s)\,ds\ge\frac{1}{b-a}\hat{\chi}^T\overline{R}\hat{\chi}, \tag{5}$$

$$\int_a^b\int_s^b\dot{\varphi}^T(u)R\dot{\varphi}(u)\,du\,ds\ge 2\hat{\Omega}^T\hat{R}\hat{\Omega}, \tag{6}$$

where $\overline{R}=\text{diag}\{R,3R,5R\}$, $\hat{R}=\text{diag}\{R,2R\}$, $\hat{\chi}=\left[\chi_1^T\ \chi_2^T\ \chi_3^T\right]^T$, $\hat{\Omega}=\left[\Omega_1^T\ \Omega_2^T\right]^T$, and

$$\chi_1=\varphi(b)-\varphi(a),\quad \chi_2=\varphi(b)+\varphi(a)-\frac{2}{b-a}\int_a^b\varphi(s)\,ds,$$

$$\chi_3=\varphi(b)-\varphi(a)+\frac{6}{b-a}\int_a^b\varphi(s)\,ds-\frac{12}{(b-a)^2}\int_a^b\int_s^b\varphi(u)\,du\,ds,$$