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New delay-interval-dependent stability criteria for static neural networks with time-varying delays

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ABSTRACT

This paper introduces an effective approach to study the stability of static neural networks with interval time-varying delay using delay partitioning approach and tighter integral inequality lemma. By decomposing the delay interval into multiple equidistant subintervals and multiple nonuniform subintervals, some suitable Lyapunov–Krasovskii functionals are constructed on these intervals. A set of novel sufficient conditions are obtained to guarantee the stability analysis issue for the considered system. These conditions are expressed in the framework of linear matrix inequalities, which heavily depend on the lower and upper bounds of the time-varying delay. It is shown, by comparing with existing approaches, that the delay-partitioning approach can largely reduce the conservatism of the stability results. Finally, three examples are given to show the effectiveness of the theoretical results.

1. Introduction

Various classes of neural networks have been active research topics in the past few year, due to its practical importance and successful applications in many areas such as aerospace, data mining, signal filtering, parallel computing, robotic and telecommunications, see e.g. [1,2]. This led to significant attraction of many researchers, like mathematicians, physicists, computer scientists and biologist. The achieved applications heavily depend on the dynamic behaviors of the equilibrium point of neural networks. That is, stability is one of the main properties of neural networks, which is a crucial feature in the design of neural networks.

It is well-known that time delays are always unavoidably encountered in the implementation of neural networks due to the finite switching speed of neurons and amplifiers. So the issue of stability analysis of neural networks with time delays attracts many researchers and a large number of stability results have been reported in the literature [3–6]. The obtained results can be classified into two types: delay-dependent criteria [7–15] and delay-independent criteria [16,17]. Generally speaking, delay-dependent stability criteria are usually less conservative than delay-independent ones especially when the size of the delay is small. And, pursuing the delay-dependent stability criteria is of much theoretical and practical value.

Depending on the modeling approaches, neural networks can be modeled either as a static neural network model or as a local field neural network model [18,19]. The local neural network and static neural network can be transferred equivalently from one to the other under some assumptions, but these assumptions cannot always be satisfied in many applications [20]. That is, local field neural network models and static neural network models are not always equivalent. Thus, it is necessary and important to study them separately.

In [21], the global exponential stability criteria is obtained for static recurrent neural networks to ensure the existence and uniqueness of the equilibrium, based on the nonlinear measure. The authors in [22], investigated the problem for static neural network with constant delay using delay partitioning approach and Finsler's Lemma. Li et al. [23] employed a unified approach in stability analysis of generalized static neural networks with time-varying delays and linear fractional uncertainties by utilizing some novel transformation and discretized scheme. In [24], stability criteria is derived for both delay-independent and delay-dependent conditions using augmented Lyapunov functional and

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it realizes the decoupling of the Lyapunov function matrix and the coefficient matrix of the neural networks. The stability and dissipativity problems of static neural networks with time-varying delay were investigated in [25]. Sun et al. [26] presented the stability criteria for a class of static neural networks by constructing new augmented Lyapunov functional which fully uses the information about the lower bound of the delay and contains some new double integral and triple integral terms. Nevertheless, the results obtained in [24–26] are based on simple Lyapunov–Krasovskii functionals and are still conservative. Therefore, there is much room for further investigation. This motivates us to carry out this work.

In this paper, our research efforts are focused on developing a new approach to analyze the stability of neural networks with interval time-varying delays. In order to obtain some less conservative sufficient conditions, firstly, we decompose the delay interval $[-h_2, 0]$ into $[-h_2, -h_1]$ and $[-h_1, 0]$. Secondly we decompose the delay interval $[-h_1, 0]$ into m equidistant sub intervals. Furthermore, we choose different weighting matrices that is $[-h_1, 0] = \bigcup_{i=1}^{m}[-i\frac{h_1}{m}, -(i-1)\frac{h_1}{m}]$. Lastly, we decompose the delay interval $[-h_2, -h_1]$ into r nonuniform subintervals and we choose different weighting matrices that is $[-h_2, -h_1] = \bigcup_{j=1}^{r}[-h_1 - jq, -h_1-(j-1)q]$, with $q = \frac{h_2-h_1}{r}$. The innovation of the method includes employment of a tighter integral inequality and construction of an appropriate type of Lyapunov functional. Finally, two numerical examples are shown to illustrate the merits of the proposed methods.

Notations. Throughout this paper, $\mathcal{R}^n$ and $\mathcal{R}^{n\times m}$ denotes the n-dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. The notation $X \geq 0$ (respectively, $X > 0$), where X is symmetric matrices, means that X is positive semi definite (respectively, positive definite). The subscript T denotes the transpose of the matrix. The notation "$*$" is used as an ellipsis for terms that are induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problem formulation

Consider the following static neural networks with interval time-varying delay:

$$\dot{u}(t) = -Au(t) + g(Wu(t-d(t))+J), \tag{1}$$

where $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T$ denotes the state vector, $A = diag(a_1, a_2, \ldots, a_n)$ with $a_i > 0$, $i = 1, 2, \ldots, n$, $g(Wu(\cdot)) = [g_1(W_1 u(\cdot)), g_2(W_2 u(\cdot)), \ldots, g_n(W_n u(\cdot))]^T$ is the activation function. $W = [W_1^T, W_2^T, \ldots, W_n^T]^T$ is the delayed connection weight matrix. $J = [j_1, j_2, \ldots, j_n]^T$ is a constant input. $d(t)$ is the time-varying delay and satisfies

$$0 \leq h_1 \leq d(t) \leq h_2 \tag{2}$$

and

$$\dot{d}(t) \leq \mu, \tag{3}$$

where h_1, h_2 are known positive scalars, and μ is a constant. It is assumed that the neuron activation functions $g(\cdot)$ satisfy the following condition.

Assumption 1 (*Liu et al. [34]*). The neuron activation functions $g_i(\cdot)(i = 1, \ldots, n)$ are continuous, bounded and satisfy

$$b_i \leq \frac{g_i(\alpha_1)-g_i(\alpha_2)}{\alpha_1-\alpha_2} \leq l_i, \quad \forall\ \alpha_1, \alpha_2 \in \mathcal{R},\ \alpha_1 \neq \alpha_2,\ i = 1, 2, \ldots, n, \tag{4}$$

where b_i, l_i are known real constants.

The equilibrium point of system (1) whose existence is guaranteed by (4) and uniqueness has been reported in [27] is denoted by $u^* = [u_1^*, u_2^*, \ldots, u_n^*]$. Let us define $x(\cdot) = u(\cdot) - u^*$, then system (1) can be transformed into the following form:

$$\dot{x}(t) = -Ax(t) + f(Wx(t-d(t))), \tag{5}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$ is the state vector of the transformed system, $f(Wx(\cdot)) = [f_1(W_1x(\cdot)), f_2(W_2x(\cdot)), \ldots, f_n(W_n\ x(\cdot))]^T$ with $f(Wx(\cdot)) = g(Wx(\cdot)+u^*+J) - g(Wu^* +\ \ J)$. Functions $f_i(\cdot)$, $i = 1, 2, \ldots, n$, satisfy the following condition:

$$b_i \leq \frac{f_i(\alpha_1)-f_i(\alpha_2)}{\alpha_1-\alpha_2} \leq l_i, \quad f_i(0) = 0, \quad \forall \alpha_1, \alpha_2 \in \mathcal{R},\ \alpha_1 \neq \alpha_2,\ i = 1, 2, \ldots, n. \tag{6}$$

The objective of this paper is to investigate delay-dependent stability conditions for system (5). Before deriving our main results, we state the following lemmas.

Lemma 2.1 (*Han [28]*). *For any constant matrix $R \in \mathcal{R}^{n\times n}$, scalars $h > 0$, and vector function $\dot{x} : [-h, 0] \to \mathcal{R}^n$ such that the integration is well defined, then the following inequality holds:*

$$-h\int_{t-h}^{t} \dot{x}^T(s)R\dot{x}(s)\, ds \leq \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} -R & R \\ * & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}. \tag{7}$$

Lemma 2.2. *For any constant matrix $R \in \mathcal{R}^{n\times n}$, $R = R^T > 0$, scalars $b_1 \leq \tau(t) \leq b_2$ and vector function $\dot{x} : [-b_2, -b_1] \to \mathcal{R}^n$ such that the integration is well defined, then the following inequality holds:*

$$-(b_2-b_1)\int_{t-b_2}^{t-b_1} \dot{x}^T(s)R\dot{x}(s)\, ds \leq \xi^T(t)\hat{\Omega}\xi(t),$$

where

$$\xi(t) = \begin{bmatrix} x(t-b_1) \\ x(t-\tau(t)) \\ x(t-b_2) \end{bmatrix}, \quad \hat{\Omega} = \Omega + \frac{\tau(t)-b_1}{b_2-b_1}\Omega_1 + \frac{b_2-\tau(t)}{b_2-b_1}\Omega_2,$$

$$\Omega = \begin{bmatrix} -R & R & 0 \\ * & -2R & R \\ * & * & -R \end{bmatrix}, \quad \Omega_1 = \begin{bmatrix} 0 & 0 & 0 \\ * & -R & R \\ * & * & -R \end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix} -R & R & 0 \\ * & -R & 0 \\ * & * & 0 \end{bmatrix}.$$

Proof. When $b_1 \leq \tau(t) \leq b_2$, from Leibniz–Newton formula and using Jensen's inequality, we have

$$\begin{aligned} -(b_2-b_1)\int_{t-b_2}^{t-b_1} \dot{x}^T(s)R\dot{x}(s)\, ds &= -(b_2-b_1)\left[\int_{t-\tau(t)}^{t-b_1} \dot{x}^T(s)R\dot{x}(s)\, ds \right. \\ &\quad \left. + \int_{t-b_2}^{t-\tau(t)} \dot{x}^T(s)R\dot{x}(s)\, ds\right] = -\left[(b_2-\tau(t))+(\tau(t)-b_1)\right] \\ &\quad \left[\int_{t-\tau(t)}^{t-b_1} \dot{x}^T(s)R\dot{x}(s)\, ds + \int_{t-b_2}^{t-\tau(t)} \dot{x}^T(s)R\dot{x}(s)\, ds\right] = -(b_2-\tau(t)) \\ &\quad \int_{t-b_2}^{t-\tau(t)} \dot{x}^T(s)R\dot{x}(s)\, ds - (\tau(t)-b_1)\int_{t-\tau(t)}^{t-b_1} \dot{x}^T(s)R\dot{x}(s)\, ds - (b_2-\tau(t)) \\ &\quad \int_{t-\tau(t)}^{t-b_1} \dot{x}^T(s)R\dot{x}(s)\, ds - (\tau(t)-b_1)\int_{t-b_2}^{t-\tau(t)} \dot{x}^T(s)R\dot{x}(s)\, ds. \end{aligned} \tag{8}$$

The first two right-side terms of (8) are dealt using Lemma 2.1 as follows:

$$-(b_2-\tau(t))\int_{t-b_2}^{t-\tau(t)} \dot{x}^T(s)R\dot{x}(s)\, ds \leq \begin{bmatrix} x(t-\tau(t)) \\ x(t-b_2) \end{bmatrix}^T \begin{bmatrix} -R & R \\ * & -R \end{bmatrix} \begin{bmatrix} x(t-\tau(t)) \\ x(t-b_2) \end{bmatrix}, \tag{9}$$