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Decentralized adaptive coupling synchronization of fractional-order complex-variable dynamical networks

Quan Xu [a,b,*], Shengxian Zhuang [b], Sijia Liu [b], Jian Xiao [b]

[a] School of Technology, Xihua University, Chengdu 610039, China
[b] School of Electrical Engineering, Southwest Jiaotong University, Chengdu 610031, China

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A B S T R A C T

In this paper, we combine decentralized adaptive control with the fractional-order techniques to investigate the synchronization of fractional-order complex-variable dynamical networks. A new lemma is proposed for estimating the Caputo fractional derivatives of Hermitian quadrtic Lyapunov functions. Based on local information among neighboring nodes, an effective fractional-order decentralized adaptive strategy to tune the coupling gains among network nodes is designed. This analysis is further extended to the case where only a small fraction of coupling gains are choosen to be adjusted. By constructing suitable Lyapunov functions and utilizing the proposed lemma, two sufficient criteria are derived to guarantee the network synchronization by using the proposed adaptive laws. Finally, numerical examples are given to validate the theoretical results.

1. Introduction

It is well known that numerous natural and man-made systems can be modeled as complex dynamical networks. Examples include social networks, food webs, epidemic spreading networks, biological networks, scientific citation networks, Internet networks, World Wide Web, electric power grids, and so on [1,2]. In recent years, extensive efforts have been made to understand and study the topology and dynamics of complex networks. Particularly, as a typical collective behavior of complex networks, synchronization has received increasing attention due to its potential applications in many real scenarios [3]. So far, many systematic results on different synchronization patterns, such as complete synchronization, lag synchronization, generalized synchronization, cluster synchronization, etc., have been obtained for many kinds of complex networks (see Refs. [4–9] and relevant references therein).

However, all the above results on synchronization mainly concentrated on integer-order and real-variable dynamical networks. It has been recognized that the real objects are generally fractional and fractional calculus allows us to describe and model a real object more accurately than the classical integer-order methods. Not surprisingly, the dynamics and control problem of fractional-order systems has attracted increasing attention from various fields [10–14]. Particularly, synchronization in fractional-order complex dynamical networks [15–18] has currently become an interesting and open problem.

On the other hand, there are a lot of physical systems involving complex-variables, which can be modeled by complex-variable dynamical systems or complex-variable dynamical networks. For example, the complex-variable Lorenz system has been introduced to describe and simulate detuned laser and rotating fluids [19]. After the complex Lorenz model, many other complex-variable dynamical systems have been proposed, such as complex nonlinear oscillators [20], complex dynamos system [21], many kinds of chaotic systems with complex-variable [21–25], etc. In recent years, the synchronization and control problem of complex-variable chaotic systems [21–25] and complex-variable dynamical networks [26–30] has been extensively investigated, and some well-known results have been obtained. Among them, some new kinds of synchronization for complex-variable dynamical systems and networks, such as complex complete synchronization [24], complex projective synchronization [22,26,29,30], complex modified projective synchronization [25], etc., have been widely investigated due to their potential applications in secure communication. Further, in Ref. [31–33], the authors investigated the chaotic phenomena and synchronization in the newly proposed fractional-order complex Lorenz system, fractional-order complex Chen system, and fractional-order complex T system. However, the synchronization of fractional-order complex-variable dynamical networks has not yet been investigated. From a control perspective, the aim here is to find some appropriate controllers such that

* Corresponding author at: School of Electrical Engineering, Southwest Jiaotong University, Chengdu 610031, China
E-mail address: quanxnjd@sina.com (Q. Xu).

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the controlled fractional-order complex-variable dynamical networks are synchronized.

In nature and technology, some consensus phenomena have been reported, which are based on the nearest-neighbor interaction rules. Examples include flocking, the rendezvous problem, multi-vehicle cooperative control, and so on [34]. Motivated by this, decentralized adaptive control has been proposed for interconnected systems [35]. Recently, the decentralized adaptive strategies have been proposed to tune the coupling gains so as to guarantee synchronization in diffusively coupled complex networks, see (Refs. [34,36–39]). To our knowledge, the decentralized adaptive strategies can be also used to tune the gains of feedback controllers. However, adding one more control to network nodes is redundant and too expensive, since a diffusively coupled complex network could be synchronized by designing suitable coupling gains among the network nodes. Compared with the centralized adaptive strategies developed in Refs. [40,41], the coupling gains are adapted based on local information exchanged among neighboring nodes. It should be noted that the obtained results for integer-order real-variable dynamical systems cannot be applied directly to fractional-order complex-variable dynamical systems. Therefore, it is important and interesting to investigate the synchronization of fractional-order complex-variable dynamical networks by using the decentralized adaptive strategies.

As is known to all, Lyapunov direct method is a standard tool to derive the synchronization criteria for integer-order complex networks. Nevertheless, the fractional Lyapunov direct method proposed in Ref. [42] is not as popular as integer-order Lyapunov direct method. It is difficult to calculate the fractional derivative of a composite Lyapunov function, because fractional derivatives of non-integer orders cannot satisfy the Leibniz rule for integer-order derivative [43], and the fractional Leibniz rule includes higher order integer and fractional derivatives of the states of the system [43]. Quite recently, Aguila-Camacho et al. [44] and Duarte-Mermoud et al. [45] introduce two lemmas for estimating the Caputo fractional derivative of a quadratic function. Thus, one can derive the synchronization conditions for fractional-order real-variable dynamical networks by using quadratic Lyapunov functions like the classic Lyapunov direct method. However, general quadratic Lyapunov functions are not valid for complex-variable dynamical systems, and Hermitian quadratic form must be used instead. Therefore, in this paper, we present a new lemma for estimating the Caputo fractional derivative of a Hermitian quadratic Lyapunov function. Then, we combine decentralized adaptive control on coupling gains with fractional-order inequality techniques to synchronize the fractional-order complex-variable dynamical networks with diffusive coupling.

The remaining of this paper is organized as follows. In Section 2, some necessary preliminaries and the model of fractional-order complex-variable networks are given. The main results of this paper are given in Section 3. In Section 4, numerical examples are provided to validate the theoretical results. Finally, some conclusions are presented in Section 5.

Notions: The standard mathematical notations will be used throughout this paper. Let $\mathbb{R}=(-\infty,+\infty)$, $\mathbb{R}^n(\mathbb{C}^n)$ be the n-dimensional Euclidean space(unitary space) and $\mathbb{R}^{m\times n}(\mathbb{C}^{m\times n})$ be the space of $m\times n$ real(complex) matrices. $x^T(B^T)$ denotes the transpose of vector x (matrix B). For any $x\in\mathbb{C}(or x\in\mathbb{C}^n)$, $\overline{x}$ denotes the conjugate of x. x^H denotes conjugate transpose of x, $\|x\|=\sqrt{x^Hx}$ denotes the norm of x. A square matrix $A\in\mathbb{C}^{n\times n}$ is Hermitian if $A=A^H$. $\lambda_{\min}(\cdot)(\lambda_{\max}(\cdot))$ denotes the minimum (maximum) eigenvalue of the corresponding matrix. Let $x=x^r+jx^i$, where $j=\sqrt{-1}$, x^r and x^i are the real and imaginary parts respectively.

2. Preliminaries and model

2.1. Fractional calculus and properties [10]

Definition 1. The uniform formula of a fractional integral with $0<\alpha<1$ is given by

$$I_t^\alpha f(t)=\frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}\frac{f(\tau)}{(t-\tau)^{1-\alpha}}d\tau, \tag{1}$$

where $t\ge t_0, f(t)$ is an arbitrary integrable function, I_t^α is the fractional integral operator, $\Gamma(\bullet)$ is the gamma function.

Definition 2. The Caputo fractional derivative with fractional-order $0<\alpha<1$can be expressed as

$$D_t^\alpha f(t)=\frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}\frac{1}{(t-\tau)^\alpha}\frac{df(\tau)}{d\tau}d\tau, \tag{2}$$

where $t\ge t_0$, D_t^α is the Caputo fractional derivative operator. Throughout this paper, we consider the Caputo definition in our fractional-order network model, since the initial conditions for the fractional-order differential equations with the Caputo derivatives take the same form as for the integer-order ones, which have clear physical meanings [10]. In the following, unless otherwise stated, we consider $\alpha\in(0,1)$.

Let us pay attention to the following properties of the fractional derivatives, which are most commonly used in applications.

Property 1.

$$D_t^\alpha(ax(t)+by(t))=aD_t^\alpha x(t)+bD_t^\alpha y(t). \tag{3}$$

Property 2.

$$D_t^\alpha f(t)=I^{1-\alpha}\dot{f}(t) \tag{4}$$

Property 3.

$$I_t^\alpha D_t^\alpha f(t)=f(t)-f(t_0),\ \forall t\ge t_0 \tag{5}$$

Property 4. *The Caputo fractional derivative of a constant function is always zero.*

Definition 3. A continuous function $\gamma:[0,t]\to[0,+\infty]$ is said to belong to class-$\mathcal{K}$ if it is strictly increasing and $\gamma(0)=0$.

Theorem 1. *(Fractional Lyapunov direct method [42]). Let $x=0$ be an equilibrium point for the fractional-order nonlinear system $D_t^\alpha x(t)=f(t,x)$. Let $V(t,x)$ be a Lyapunov function and $\gamma_i(i=1,2,3)$ be class-$\mathcal{K}$ functions such that*

$$\gamma_1(\|x\|)\le V(t,x)\le\gamma_2(\|x\|), \tag{6}$$

$$D_t^\beta V(t,x)\le-\gamma_3(\|x\|), \tag{7}$$

where $\beta\in(0,1)$. Then, the equilibrium point $x=0$ is asymptotically stable.

A new property for Caputo derivative can be stated in Lemma 1, which can facilitate estimating the fractional derivative of a common quadratic Lyapunov function.

Lemma 1. *[45]. Let $x(t)\in\mathbb{R}^n$ be a vector of derivable functions. Then, the following inequality holds*

$$D_t^\alpha\left(x^T(t)\boldsymbol{P}x(t)\right)\le 2x^T(t)\boldsymbol{P}D_t^\alpha x(t), \tag{8}$$

where $\alpha\in(0,1), t\ge t_0$ and $\boldsymbol{P}\in\mathbb{R}^{n\times n}$ is a constant, symmetric and positive definite matrix.