Neurocomputing 186 (2016) 127–138

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Synchronization of complex dynamical networks with uncertain inner coupling and successive delays based on passivity theory

B. Kaviarasan [a], R. Sakthivel [a,b,*], Yongdo Lim [b]

[a] Department of Mathematics, Sri Ramakrishna Institute of Technology, Coimbatore 641010, India
[b] Department of Mathematics, Sungkyunkwan University, Suwon 440-746, South Korea

ARTICLE INFO

Article history:
Received 31 August 2015
Received in revised form
12 December 2015
Accepted 19 December 2015
Communicated by Hongyi Li
Available online 6 January 2016

Keywords:
Complex dynamical networks (CDNs)
Synchronization
Uncertain inner coupling matrix
Additive time-varying delays

ABSTRACT

This paper is discussed with the problem of passivity based synchronization for a class of complex dynamical networks (CDNs) consisting uncertain inner coupling matrix together with successive time-varying delays via a state feedback delayed controller. Due to occurrence of uncertainties in coupling strengths, the considered CDNs take account of an uncertain inner coupling strength which is more general than the previously existing inner coupling strengths. Specifically, the uncertainties encountering in coupling terms are characterized with the aid of interval matrix approach. Also, by introducing a simple linear transformation, the corresponding error system is formulated. Then, based on the information about control delay term, two cases are considered namely, differentiable and non-differentiable. More precisely, by constructing an appropriate Lyapunov–Krasovskii functional (LKF) containing triple integral terms in respect of Kronecker product, for both the cases, some sufficient criteria are established in terms of linear matrix inequalities (LMIs) to guarantee the robust synchronization of the addressed CDNs based on passivity property. And the established criteria optimistically reduce the $\mathcal{L}_2$ gain level from the disturbance to the output vector. Subsequently, the desired state feedback gain matrix is designed in terms of the solution to a convex optimization problem. Finally, a numerical example is presented to demonstrate the effectiveness of the proposed theoretical results.

1. Introduction

In nature, most of the dynamical systems can be represented in the form of various complex dynamical networks (CDNs). Generally, a CDN consists of a very large number of interconnected nodes in which each node represents some prescribed contents. To specify, some well-known examples of CDNs are disease transmission networks, social networks, cell metabolisms, food webs, electricity distribution networks [1,2]. Moreover, CDNs are classified into different categories such as scale free networks [3], random networks [4] and small world networks [5]. It should be pointed out that since the dynamics of CDNs are coupled with topological evolution, the qualitative analysis on their dynamical behaviors is more challenging and interesting issue compared to single dynamic systems. Because of this fact, research communities have eagerly concentrated on the study of CDNs and have reported some delightful results on CDNs in the literature, see for example [6–8].

On the other hand, some researchers have paid their great attention to investigate the various dynamical behaviors of CDNs such as self-organization, synchronization, spatio-temporal chaos, auto-waves, and spiral waves, see [9–11]. Among them synchronization has become the fascinating dynamical behavior of CDNs and has been widely discussed in the literature [12–14]. As is well-known, synchronization is the process between two or more dynamical systems to attain a common behavior by tuning some prescribed properties of their motion [15]. In order to do this, the node in CDNs needs to exchange its own information with its neighbors. Meanwhile, during the process of synchronization, the connections among the nodes are very critical. Therefore, the synchronization analysis mainly concentrates on nodes connections and topological structure of the CDNs. In this relation, diverse kinds of synchronization have been proposed by the control communities including exponential synchronization [16], projective lag synchronization [17] and cluster synchronization [18]. After that, few of the researchers have employed various control methods such as pinning control [19], sampled-data control [20] and adaptive control [21] to achieve synchronization in CDNs.

It is obvious that time delay naturally exists in all kinds of dynamical systems. Moreover, in practice, it is also a main reason to affect the stability and the performances of dynamical systems.

* Corresponding author at: Department of Mathematics, Sungkyunkwan University, Suwon 440-746, South Korea.
E-mail address: krsakthivel@yahoo.com (R. Sakthivel).

http://dx.doi.org/10.1016/j.neucom.2015.12.071
0925-2312/

Therefore, it is necessary and important to include the concept of time delay while discussing about the dynamical networks. Recently, much progress has been achieved in the study of CDNs with time or time-varying delays, for instance refer [22–24]. Following these seminal works, researchers have begun to investigate the successive or additive delay components for control systems which are more general than of single component; see for example [25–27] and the references therein. Also, it has been showed that the additive delays gave less conservative results over the single component [28]. Therefore, we should also pay great attention to additive time-varying delays which give more important in both theoretical and practical purposes. Besides, in the existing literature, it is noticed that most of the works on time-varying delayed systems have been studied under the assumptions that the range of time-varying delay is from 0 to a certain upper bound and its derivative exists which is also bounded. But in practice, the time-varying delay may belong to any interval which means that the lower bound needs not to be zero. Moreover, it may not necessarily be differentiable and this phenomenon permits the delay function to be a fast time-varying function. Took these facts into account, some of the researchers have reported few fruitful results in the previous literature, see for instance [29–31]. Hence worth, it is of great significance to investigate the synchronization of CDNs with non-differentiable interval time-varying delay.

It should be noted that all the aforementioned works about CDNs have been concentrated only on known/fixed coupling strengths. Nevertheless, in some real cases, uncertainties may encounter in the coupling strength terms due to few environmental disturbances. In the beginning stage, by employing some coupling adjustment strategies, few authors attempted to address the issues of existence of nonlinear coupling terms during the synchronization process in CDNs. After that, very recently, the concepts of uncertain inner coupling strengths and incomplete measurements of coupling terms have been introduced. More precisely, the uncertainties occurred in the coupling terms are characterized by using interval matrix method. Based on these real scenarios, very less amount of works only has been reported in the existing literature [32–34]. However, the issues and the impacts of uncertain coupling terms in CDNs during synchronization process have not been fully analyzed in the literature.

On the other hand, passivity is a familiar concept in control theory that interconnects the system inputs and outputs to the energy storage function. One of the main advantages of this concept is that it has the ability to maintain the system stability internally. Due to this peculiar property, passivity theory has been successfully employed in analysis and synthesis of nonlinear control systems [35,36]. Since a dynamical network is composed of a number of nodes that are nonlinear systems, it is natural and reasonable to expect passivity to be more helpful during network analysis and design. The key idea behind the passivity property for achieving synchronization is that it enlarges the possibility of synchronizability even if all the provided outer coupling topology cannot bring synchronization. Concerning these facts, there has been grown considerable researches in passivity analysis and passive control for a variety of dynamical systems [37–40] during the past three decades. Very recently, the problem of passivity based synchronization for CDNs has been addressed [41,42]. From the above discussions, it is clear that so far in the existing literature, very little attention has been paid on the problem of passivity based synchronization for CDNs. Moreover, to the best of authors' knowledge, very few researchers have discussed the synchronization of CDNs with uncertain inner coupling matrices. Especially, the passivity based synchronization of CDNs with uncertain inner coupling matrices has not yet been investigated which stimulates us to do the present study.

Motivated by these observations, this paper is devoted to investigate the passivity based synchronization for a class of CDNs with uncertain inner coupling strength and two additive delay components. And the main contributions of this paper can be summarized as follows:

- Inspired by the works [33,34], an uncertain inner coupling term is considered to a family of CDNs with two additive delays and external disturbance after that the passivity performance is studied for the CDNs.
- By utilizing the full information of delays' bounds, a new set of Lyapunov–Krasovskii functional is constructed and then the passivity based synchronization criteria are derived for the proposed CDNs according to the information of control delay term.
- Based on the proposed criteria, two control algorithms for the state feedback delayed controller are developed and they significantly minimize the $\mathcal{L}_2$ gain level $\gamma > 0$.

Finally, the proposed theoretical results are validated through by a numerical example and its simulations.

The rest of this paper is arranged as follows: Section 2 presents the problem formulation and some necessary preliminaries which are needed for the proof of the main results. Synchronization criteria for the undertaken CDNs are derived and the corresponding control gain is obtained in Section 3. Numerical simulations are provided to verify the theoretical results in Section 4 which is followed by the conclusion in Section 5.

Notations: The following notations are used throughout this paper. $\mathbb{R}^n$ and $\mathbb{R}^{p\times q}$, respectively, denote the n dimensional Euclidean space and the space of all $p\times q$ matrices. The notation $\mathcal{A}>0$ (respectively, $\mathcal{A}<0$), for $\mathcal{A}\in\mathbb{R}^{n\times n}$ means that the matrix $\mathcal{A}$ is real symmetric positive definite (respectively, negative definite). The superscript T stands for matrix transposition. I denotes the identity matrix with compatible dimension. The Kronecker product of matrices $\mathcal{A}\in\mathbb{R}^{l\times n}$ and $\mathcal{B}\in\mathbb{R}^{p\times q}$ is a matrix in $\mathbb{R}^{lp\times nq}$ and denoted as $(\mathcal{A}\otimes\mathcal{B})$. $\mathcal{L}_2[0,\infty)$ represents the space of square integrable vector functions over $[0,\infty)$. Moreover, the asterisk $*$ in a matrix is used to denote term that is induced by symmetry. Matrices, if they are not explicitly specified, are assumed to have compatible dimensions.

2. Problem formulation and preliminaries

In this paper, we consider a class of complex dynamical networks (CDNs) consisting of N identical nodes with uncertain inner coupling and additive time-varying delays. Then the corresponding ith node of CDNs is governed by the following differential equations:

$$\left.\begin{aligned}&\dot{x}_i(t)=f(x_i(t))+\sum_{j=1}^{N}g_{ij}Ax_j(t)+\sum_{j=1}^{N}g_{ij}Bx_j(t-\delta_1(t)-\delta_2(t))+u_i(t)+w_i(t),\\&z_i(t)=Cx_i(t),\quad i=1,2,\ldots,N,\end{aligned}\right\}\tag{1}$$

where $x_i(t)\in\mathbb{R}^n$ is the state vector of ith node; $f(x_i(t))$ represents a vector-valued nonlinear function; $u_i(t)\in\mathbb{R}^m$ is the control input of ith node; $w_i(t)$ is the network external disturbance which belongs to $\mathcal{L}_2[0,\infty)$; $z_i(t)\in\mathbb{R}^p$ is the output of ith node; $A=diag\{a_1,a_2,\ldots,a_n\}>0$ and $B=diag\{b_1,b_2,\ldots,b_n\}>0$ denote the inner coupling matrices of the network; C is a known constant matrix with appropriate dimension; $\delta_1(t)$ and $\delta_2(t)$ are the additive delay components. Let $G=[g_{ij}]_{N\times N}$ be the outer-coupling matrix representing the topological structure of the network, in which g_{ij} is defined as follows: if there is a connection between node i and node j $(j\neq i)$, then $g_{ij}>0$; otherwise $g_{ij}=0$ and the diagonal