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New conditions on synchronization of memristor-based neural networks via differential inclusions ☆

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ABSTRACT

In this paper, a general framework named Filippov-framework and some new analytic techniques are presented for dealing with the exponential synchronization of memristor-based neural network system with time-varying delay. Several new sufficient conditions are established to guarantee two different types of exponential synchronization of the coupled networks based on master–slave (drive-response) concept and discontinuous state-feedback controller. In addition, we also provide an estimation of the exponential synchronization rate which depends on the time delay and system parameters. These conditions are good improvement and extension of the existing works on synchronization control of memristive or switching networks. Finally, the validity of our method and theoretical results is demonstrated by concrete examples with numerical simulations.

1. Introduction

In the early 1970s, a new circuit element named memristor (i.e., memory resistor) was firstly predicted by Professor Chua [1]. From the theoretical point of view, memristor possesses the distinctive ability to memorize the passed quantity of electric charge. Due to its unique properties, the memristor is very different from other existing three circuit elements: resistor, inductor and capacitor. However, in the subsequent forty years, memristor did not cause much attention of researchers because it is only an ideal circuit element. In 2008, the first practical memristor device was invented by members of Hewlett-Packard (HP) Lab [2,3]. This new circuit prototype of memristor is based on nanotechnology and exhibits the feature of pinched hysteresis just as the neurons in the human brain have [4,5]. Therefore, by using memristor element, it is more realistic to design neural network model for emulating the human brain. It should be noted that memristive system has its own physical characteristics. Firstly, it possesses memory characteristic. Secondly, it possesses nanometer dimensions. In some classical electronic circuits, suppose we replace the resistors or diodes with memristors, then we can build some new memristive system or circuits. For example, in 2008, Itoh and Chua designed some new nonlinear oscillatory circuits by replacing the “Chua's diode” in the canonical Chua's oscillator with a memristor [44]. As shown in Fig. 1, a memristive circuit has been obtained by replacing the Chua's diode with a flux-controlled memristor. Especially, in the field of artificial neural networks, some memristive neural network circuits can also been builded by replacing the resistors with memristors (see, for example, [17,18,33]). Up to now, considerable efforts have been devoted to study memristor-based neural networks [6–11]. It is noted that the memristor-based neural network model is basically switching dynamical system and its switching rule depends on network's state. This class of switching dynamical neuron system is usually described by the ordinary (or functional) differential equation with discontinuous right-hand side. However, the additional difficulties will arise since the classical theory of differential equation have been shown to be invalid to deal with discontinuous systems. In this case, the given vector field is no longer continuous and the solutions in the conventional sense might not exist. Fortunately, Filippov proposed a novel method and framework in 1964 (see [12]). Actually, by constructing Filippov set-valued map named Filippov-regularization, the differential equations with discontinuous right-hand sides could be transformed into differential inclusions. According to the theory of differential inclusion, dynamical behaviors of differential equations possessing discontinuous right-hand sides can be handled under this new framework of

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Filippov solution. Recently, the authors in [13–18] investigated the dynamical behaviors of memristor-based neural network via Filippov differential inclusion theory and framework. But the main conditions in many of these literature were not correct. Moreover, there is not much research concerning more complex dynamical behaviors such as periodic oscillation, finite time stability, chaos, bifurcation and synchronization for memristor-based neural networks.

As far as we know, synchronization is a typical collective behavior in nature and it includes many different types such as complete synchronization, anti-synchronization, phase synchronization, quasi-synchronization, lag synchronization, projective synchronization, and so on. Actually, synchronization means the dynamics of nodes share the common time-spatial property. So we can understand an unknown dynamical system from the well-known dynamical systems by synchronization. Since the pioneering work of Pecora and Carroll in 1990 (see [19]), the chaotic synchronization has become a hot research topic due to its potential applications in various science and engineering fields such as information science, meteorology, biological systems and secure communications [20–23]. Especially, in the field of neural networks, the problems of synchronization have been extensively and intensively investigated because of their practical significance. There are many excellent results on different types of synchronization for neural networks described by differential equations with continuous or discontinuous right-hand sides (see, for example, [24–31]). However, the study of synchronization for neural networks possessing discontinuous property is not an easy work. It is worth noting that some complex nonlinear behaviors including chaos and periodic oscillatory usually appear even in a simple network of memristor (see [43–45]), so it is of practical importance and great necessity for us to give a detailed analytical investigation of synchronization control of memristor-based neural networks with the basic oscillator. Moreover, the synchronization control of memristive neural networks plays important roles in many potential applications such as super-dense nonvolatile computer memory and neural synapses (see [46]). In recent years, based on Filippov-framework, the interest of synchronization issues is shifting to the memristor-based neural networks which possess discontinuous switching jumps with respect to states. For example, the paper [32] concerned the problem of global exponential synchronization for a class of memristor-based Cohen–Grossberg neural networks with mixed delays by designing a novel feedback controller. In [33–35], the authors investigated the synchronization of memristor-based neural networks with time delays by using Lyapunov functional method. In [36,37], the anti-synchronization criteria were obtained for memristive neural networks, respectively. In [38], the complete periodic synchronization control was considered for memristor-based neural networks with time-varying delays. It should be pointed out that, because of abrupt changes at certain instants during the dynamical processes, memristor-based neural networks exhibit some especial state-dependent nonlinear switching or discontinuous behaviors which are different from the discontinuities of discontinuous neuron activations. Hence, the analytical technology for studying synchronization of neural networks with discontinuous activations may not be applicable to deal with the synchronization of memristive neural networks because the discontinuities of these two classes of neural networks are different. In short, there exist more difficulties and challenges in investigating the synchronization of memristor-based neural networks due to the special discontinuous switching features of memristor and the lack of effective analysis methods.

On the basis of the aforementioned discussion, referring to previous works [13–16,33–38], this paper considers a class of memristor-based neural networks with discontinuous switching jumps and time-varying delay which are described by the following differential equations:

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -d_i x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t)) f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(x_i(t)) g_j(x_j(t-\tau(t))) + J_i, \quad t \ge 0, \ i = 1, 2, \ldots, n, \tag{1}$$

where $x_i(t)$ denotes the voltage of the capacitor $\mathbf{C}_i$; J_i is the external input to the ith neuron; the time-varying delay $\tau(t)$ is a nonnegative continuous function satisfying $0 \le \tau(t) \le \tau$ ($\tau \ge 0$ is a constant); $f_j(x_j(t))$ and $g_j(x_j(t-\tau(t)))$ denote feedback functions without and with time-varying delay between the jth-dimension of the memristor and $x_i(t)$, respectively; $d_i > 0$ denotes the self-inhibition with which the ith neuron unit will reset its potential to the resting state in isolations when disconnected from the network; $a_{ij}(x_i(t))$ and $b_{ij}(x_i(t))$ denote memristor-based connection weights, and

$$a_{ij}(x_i(t)) = \frac{\mathbf{W}_{ij}}{\mathbf{C}_i} \times \mathrm{SGN}_{ij}, b_{ij}(x_i(t)) = \frac{\mathbf{M}_{ij}}{\mathbf{C}_i} \times \mathrm{SGN}_{ij},$$

$$\mathrm{SGN}_{ij} = \begin{cases} 1, & \text{if } i \ne j, \\ -1, & \text{if } i = j, \end{cases}$$

in which $\mathbf{W}_{ij}$ and $\mathbf{M}_{ij}$ represent the memductances of memristors $\mathbf{R}_{ij}$ and $\mathbf{F}_{ij}$, respectively. $\mathbf{R}_{ij}$ denotes the memristor between the feedback function $f_j(x_j(t))$ and $x_i(t)$. $\mathbf{F}_{ij}$ represents the memristor between the time-varying delayed feedback function $g_j(x_j(t-\tau(t)))$ and $x_i(t)$. According to the feature of the memristor and the current-voltage characteristic, the memristor-based connection weights $a_{ij}(x_i(t))$ and $b_{ij}(x_i(t))$ of the neural network satisfying the following conditions:

$$a_{ij}(x_i(t)) = \begin{cases} \hat{a}_{ij}, & \text{if } |x_i(t)| \le \Upsilon_i, \\ \check{a}_{ij}, & \text{if } |x_i(t)| > \Upsilon_i, \end{cases}$$

$$b_{ij}(x_i(t)) = \begin{cases} \hat{b}_{ij}, & \text{if } |x_i(t)| \le \Upsilon_i, \\ \check{b}_{ij}, & \text{if} |x_i(t)| > \Upsilon_i, \end{cases}$$

for $i, j = 1, 2, \ldots, n$, where the switching jumps $\Upsilon_i > 0$, $\hat{a}_{ij}$, $\check{a}_{ij}$, $\hat{b}_{ij}$ and $\check{b}_{ij}$ are constants.

In order to achieve our main results, the feedback functions f_j and g_j are assumed to satisfy the following basic conditions:

(C1) For any two different $x, y \in \mathbb{R}$, there exist positive constants L_j, Q_j ($j = 1, 2, \ldots, n$) such that

$$|f_j(x) - f_j(y)| \le L_j |x - y|, \quad |g_j(x) - g_j(y)| \le Q_j |x - y|.$$