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A novel approach for estimation of optimal embedding parameters of nonlinear time series by structural learning of neural network

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Abstract

In this work a novel approach for estimation of embedding parameters for reconstruction of underlying dynamical system from the observed nonlinear time series by a feedforward neural network with structural learning is proposed. The proposed scheme of optimal estimation of embedding parameters can be viewed as a global non-uniform embedding. It has been found that the proposed method is more efficient for estimating embedding parameters for reconstruction of the attractor in the phase space than conventional uniform embedding methods. The simulation has been done with Henon series and three other real benchmark data sets. The simulation results for short term prediction of Henon Series and the bench mark time series with the estimated embedding parameters also show that the estimated parameters with proposed technique are better than the estimated parameters with the conventional method in terms of the prediction accuracy. The proposed technique seems to be an efficient candidate for prediction of future values of noisy real world time series.

Keywords: Chaos; Nonlinear time series; Strange attractor; Embedding theorem; Embedding parameters; Structural learning; Neural network

1. Introduction

Nonlinear time series are ubiquitous and a lot of them e.g., stock market or exchange rate, meteorological data or network traffic flow, are significant to our society and life. In order to understand and predict future values of nonlinear time series, it is important to analyze the time series to extract knowledge of the underlying dynamical system. Time series are generally sequences of measurements of one or more observable variables of an underlying dynamical system, whose state changes with time as a function of its current state vector. Linear dynamical systems evolve over time to an attracting set of points that are called fixed point attractors and the time series derived from such a system have a regular appearance. There are many linear modelling algorithms for analysing those time series. However, some nonlinear dynamical systems evolve to a chaotic attractor or a strange attractor. The path of the state vector through the attractor is non-periodic, exhibits highly irregular geometrical pattern and sensitive to initial condition. The generated time series shows a complex appearance and behaviour. Many real world observed time series are of such chaotic nature, the actual variables of the underlying dynamical system that contribute to state vector are unknown. We need nonlinear methods for their analysis and modelling in order to predict their future behaviour.

One standard approach for prediction of future values of such a chaotic time series involves the reconstruction of the chaotic dynamics of the phase space from the observed (measured) values of the state vector and thereby predicting the evolution of the measured variable. The *embedding theorem* of Takens [37] guarantees that the space of time delayed vectors with sufficiently large dimension (actually there is an upper bound provided for the embedding dimension) will capture the structure of the original phase space. Unfortunately embedding theorem does not provide any values for the embedding parameters i.e. delay time τ

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and embedding dimension m, a good choice is needed for correct reconstruction of the attractor. Therefore, estimation of the optimal embedding parameters for reconstruction of the nonlinear dynamics has been studied as an important problem. Abarbanel [1] suggested some heuristics for estimation of delay time and embedding dimension.

Artificial neural networks are popular and efficient models for modelling nonlinear phenomenon where various inputs are combined to predict unknown data and they have been extensively used for nonlinear time series analysis. The most popular choice of neural architecture is the multilayer feedforward neural network with back propagation (BP) learning algorithm. Though widely used, the multilayer network with BP learning suffers from some serious drawbacks. The most severe shortcoming is inability to reveal the interpretation of hidden units and connection weights in connection to the underlying distribution of the data model. Structural learning or pruning techniques are one of the approaches that aim to minimize the problems of BP learning algorithm. A variety of structural learning techniques have been proposed in several papers [10,16,27,32,33,26,6].

Although the initial researches on structural learning [32,33] aimed to improve generalization ability of neural network, later it is shown that structural learning or pruning techniques with regularization are capable of discovering rules in classification problems [10,16] and selection of essential input components or analysis of input–output relations in time series prediction problems [27,26,6]. The use of regularizer in neural network learning has also has been used in automatic relevance detection of input units to the target concept [24,29] and the magnitude of connection weights has been used in selection of inputs [40,23]. Neural network with several tuning parameters is also a good candidate for optimal model selection from various alternatives. Neural network with hierarchical Bayesian learning has also been successfully applied for estimation of embedding dimension for generated time series data in [28].

In order to accurately reconstruct the chaotic dynamics of nonlinear time series for prediction of its future values, we propose an algorithm for optimal estimation of embedding parameters from the nonlinear time series by neural network model trained by structural learning with regularizer. An initial version of the algorithm has been compared with existing theoretically motivated heuristic approaches using Lorenz data in [25]. In this paper we present the proposed algorithm in more detail and simulation results of artificial data as well as some benchmark real data sets to show its effectiveness. In the next two sections, the problem of estimation of embedding parameters for modelling and prediction of time series followed by the related work for its implementation have been discussed. In the following section the proposed scheme for optimal model selection for estimation of embedding parameters has been described. The next section represents simulation experiments and their results followed by the final section with discussion and concluding remarks.

2. Reconstruction of chaotic attractor

2.1. Time series and dynamical system

The relation between a time series and its underlying dynamical system can be expressed by the following equations:

$$\boldsymbol{u}(t+1) = \boldsymbol{F}[\boldsymbol{u}(t)] + \xi(t), \tag{1}$$

$$y(t) = \boldsymbol{g}[\boldsymbol{u}(t)] + \boldsymbol{\eta}(t), \tag{2}$$

where $\boldsymbol{u}(t)$ is the state vector of the dynamical system at time t, the function $\boldsymbol{F}$ represents the state change of dynamical system. $y(t)$ is the observed value at time t while $\boldsymbol{g}$ represents the observation function, $\boldsymbol{\xi}(t)$ and $\boldsymbol{\eta}(t)$ are dynamical noise and observational noise at time t, respectively. Eqs. (1) and (2) represent that we cannot directly obtain the state $\boldsymbol{u}(t)$ of original dynamical system.

2.2. Embedding theorem and delay coordinate embedding

Embedding theorem, which is developed by Takens [37] and expanded by Sauer et al. [3], guarantees that, with a single observed time series $y(t)$, we can obtain the following $\boldsymbol{f}$ which has one-to-one correspondence to the original dynamical system.

$$\begin{aligned} \boldsymbol{v}(t+1) &= \boldsymbol{f}(\boldsymbol{v}(t)), \\ \boldsymbol{v}(t) &= (y(t), y(t+\tau), \ldots, y(t+(m-1)\tau)), \end{aligned} \tag{3}$$

where $\boldsymbol{f}$ denotes reconstructed dynamical system, $\boldsymbol{v}$ denotes time delay coordinate vector, m is called *embedding dimension* and τ is called *delay time*. An example of embedding of Lorenz System with the effect of different values of m and τ on reconstruction of the original system is shown in Fig. 1 where (a) is original Lorenz attractor with x, y, z variables, (b) is a single time series of variable x, (c)–(f) are reconstructed attractor from variable x with various m and τ. From the example it is clear that for correct reconstruction of the attraction, a fine estimation of the parameters (m and τ) is needed.

2.3. Estimation of embedding parameters

There are variety of heuristic techniques for estimating the embedding parameters, m and τ, the details can be found in [1,3,14]. Here we briefly review the most representative ones. For the estimation of delay time τ, average mutual information (AMI) [8] at varying sample rates is computed and the first minimum is taken as the appropriate sample rate. The most commonly used techniques to estimate the embedding dimension m are false nearest neighbour (FNN) [15] and singular value analysis [2].