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Synchronization control of switched linearly coupled neural networks with delay

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ABSTRACT

In this paper, synchronization control of switched linearly coupled delayed neural networks is investigated by using the Lyapunov functional method, synchronization manifold and linear matrix inequality (LMI) approach. A sufficient condition is derived to ensure the global synchronization of switched linearly coupled complex neural networks, which are controlled by some designed controllers. A globally convergent algorithm involving convex optimization is also presented to construct such controllers effectively. In many cases, it is desirable to control the whole network by changing the connections of some nodes in the complex network, and this paper provides an applicable approach. It is even applicable to the case when the derivative of the time-varying delay takes arbitrary. Finally, some simulations are constructed to justify the theoretical analysis.

1. Introduction

Synchronization and stability control of dynamical systems is an important topic in nonlinear system control [1–10,44–46] in the past decades. Recently, arrays of coupled systems have attracted much attention of researchers in different research fields. The study of synchronization of coupled neural networks is an important step for both understanding brain science and designing coupled neural networks for practical use.

Networks of coupled connection have been widely investigated [11–26] since Wang and Chen introduced an array of N linearly coupled connected complex network model [27,28]. Consider a complex dynamical network consisting of N identical linearly and diffusively coupled nodes, with each node being an n-dimensional dynamical system in [27,28] as follows:

$$\dot{x}_i(t)=f(x_i(t))+c\sum_{j=1,j\neq i}^{N}G_{ij}\Gamma(x_j(t)-x_i(t)),\quad i=1,2,\ldots,N, \tag{1}$$

where $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^T\in R^n$ $(i=1,2,\ldots,N)$ is the state vector representing the state variables of node i, $f: R^n\longrightarrow R^n$ is continuously differentiable, the constant c is the coupling strength, $\Gamma=\mathrm{diag}(\gamma_1,\gamma_2,\ldots,\gamma_n)\in R^{n\times n}$ is a constant 0–1 matrix linking the coupled variables with $\gamma_i=1$ for a specific i and $\gamma_j=0$ $(j\neq i)$, that is, there is only one 1 in the diagonal of matrix Γ and all the other components of Γ are zeros, $G=(G_{ij})_{N\times N}$ is the coupling configuration matrix representing the topological structure of the network, in which G_{ij} is defined as follows: if there is a connection between node i and node j $(j\neq i)$, then the coupling strength $G_{ij}=G_{ji}>0$; otherwise, $G_{ij}=G_{ji}=0$ $(j\neq i)$, and the diagonal elements of matrix G are defined by

$$G_{ii}=-\sum_{j=1,j\neq i}^{N}G_{ij}. \tag{2}$$

Then, in this case, the complex network (1) reduces to the model

$$\dot{x}_i(t)=f(x_i(t))+c\sum_{j=1}^{N}G_{ij}\Gamma x_j(t),\quad i=1,2,\ldots,N. \tag{3}$$

Hereafter, suppose that the network (3) is connected in the sense that there are no isolate clusters. Thus, the coupling configuration G is an irreducible matrix.

In the following, a brief introduction of recent works about synchronization of linearly coupled complex networks are given based on the model (1)–(3).

For the case that the coupling matrix G is irreducible, symmetric, and all the off-diagonal elements of G are nonegative and satisfies (2), local synchronization analysis via linearlization technique was studied in [16–20], where the eigenvalues and

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Jacobian matrix are given in the criteria of ensuring the local synchronization of the complex network. Wu and Chua [29,30] investigated the synchronization in an array of linearly coupled dynamical systems, and then a lot of works [11–13,15,21] were devoted to investigating the global asymptotical synchronization of the complex network by using the synchronization manifold and Lyapunov method. In [29], the authors defined a distance between the collective states and the synchronization manifold, based on which the a methodology was proposed to discuss global synchronization of the coupled systems.

In [11,12,15], the following linearly coupled neural networks satisfying (2) was studied:

$$\dot{x}_i(t) = -Cx_i(t) + Af(x_i(t)) + Bf(x_i(t-\tau)) + I(t) + \sum_{j=1}^{N} G_{ij}\Gamma x_j(t), \quad i = 1,2,\ldots,N, \tag{4}$$

where $C = \mathrm{diag}(c_1, c_2, \ldots, c_n) \in R^{n\times n}$ is a diagonal matrix with positive diagonal entries $c_i > 0$, $i = 1,2,\ldots,n$, $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are weight and delayed weight matrices, respectively. $I(t) = (I_1(t), I_2(t), \ldots, I_n(t))^T \in R^n$ is an external input vector, $\Gamma = \mathrm{diag}(\gamma_1, \gamma_2, \ldots, \gamma_n) \in R^{n\times n}$. $f(x_i(t)) = (f_1(x_{i1}(t)), f_2(x_{i2}(t)), \ldots, f_n(x_{in}(t)))^T \in R^n$ corresponds to the activation functions of neurons. Equivalently, system (4) can be written as

$$\begin{aligned}\dot{x}_{ik}(t) = {} & -c_k x_{ik}(t) + \sum_{l=1}^{n} a_{kl} f_l(x_{il}(t)) + \sum_{l=1}^{n} b_{kl} f_l(x_{il}(t-\tau)) \\ & + I_k(t) + \sum_{j=1}^{N} G_{ij}\gamma_k x_{jk}(t), \quad i = 1,2,\ldots,N, \ k = 1,2,\ldots,n. \end{aligned} \tag{5}$$

With the rapid development of intelligent control, hybrid systems have been widely investigated. It is found that many physical and biological models are governed by more than one dynamical system and these systems are changed depending on time. Switched systems [31–34], a special case in hybrid systems, are regarded as nonlinear systems, which are composed of a family subsystems and a rule that orchestrates the switching between the subsystems. Recently, switched systems have numerous applications in communication systems [6,35,36], control of mechanical systems, automotive industry, aircraft and air traffic control, electric power systems [37] and many other fields. In [31–34], the stability of switching system was investigated, which is a combination of discrete and continuous dynamical systems.

The main contribution of this paper is of threefolds. Firstly, we studied global synchronization of coupled systems with time-varying delay by using LMI and distance function from collective states to the synchronization manifold [26]. A delay-dependent condition is given to ensure the synchronization of coupled systems in this paper based on free-weighting matrix approach and cone complementarity linearization algorithm [47–49]. It is noted that the derivative of time delay can take any value. Secondly, the feedback matrix of the network is designed to adjust the configuration matrix, i.e., the connections among the nodes. Thirdly, it is very difficult to design the feedback matrix due to the complexity of the systems. So, a globally convergent algorithm involving convex optimization is presented.

The rest of the paper is organized as follows: In Section 2, preliminaries are given. In Section 3, the main results are derived. A sufficient condition is given to ensure the synchronization of switched coupled networks and a globally convergent algorithm involving convex optimization is also presented to design such controllers effectively. In Section 4, numerical simulations are constructed to justify the theoretical analysis in this paper. Finally, the conclusion is drawn.

2. Preliminaries

A set of coupled complex neural networks is considered as the individual subsystems of the switched system and the switched coupled neural network is described as follows:

$$\dot{x}_i(t) = -C_\alpha x_i(t) + A_\alpha f(x_i(t)) + B_\alpha f(x_i(t-\tau)) + I_\alpha(t) + \sum_{j=1}^{N} G_{\alpha ij} D x_j(t), \quad i = 1,2,\ldots,N, \tag{6}$$

where D is inner coupling matrix and α is a switching signal which takes its value in the finite set $\mathcal{I} = \{1,2,\ldots,\overline{N}\}$. This means that the matrices $(C_\alpha, A_\alpha, B_\alpha, I_\alpha, G_\alpha)$ are allowed to take values, at particular time, in a finite set $\{(C_1, A_1, B_1, I_1, G_1), (C_2, A_2, B_2, I_2, G_2), \ldots, (C_{\overline{N}}, A_{\overline{N}}, B_{\overline{N}}, I_{\overline{N}}, G_{\overline{N}})\}$. Throughout this paper, we assume that the switching rule α is not known priori and its instantaneous value is available in real time.

Since in most cases the time delay is not a constant, in this paper, the coupled neural network with time-varying delay is studied. Consider the state-feedback control law

$$u_{\alpha i}(t) = \sum_{j=1}^{N} K_{\alpha ij} D x_j(t), \quad i = 1,2,\ldots,N, \tag{7}$$

where

$$K_{\alpha ii} = -\sum_{j=1, j\neq i}^{N} K_{\alpha ij}. \tag{8}$$

It is useful to design a memoryless state-feedback controller $u_{\alpha i}(t)$ so that the coupled system (6) is globally synchronized. In this paper, a linearly feedback controller (7) is added to the coupled dynamical system (6)

$$\begin{aligned}\dot{x}_i(t) = {} & -C_\alpha x_i(t) + A_\alpha f(x_i(t)) + B_\alpha f(x_i(t-\tau(t))) + I_\alpha(t) \\ & + \sum_{j=1}^{N} (G_{\alpha ij} + K_{\alpha ij}) D x_j(t), \quad i = 1,2,\ldots,N. \end{aligned} \tag{9}$$

It is easy to see that one can control the synchronization of coupled neural network by adjusting the configuration coupling matrix, that is, the network topology can be changed to achieve synchronization. The architecture for such switched coupled neural networks is shown in Fig. 1. Next, we focus on global asymptotical synchronization of coupled feedback system (9).

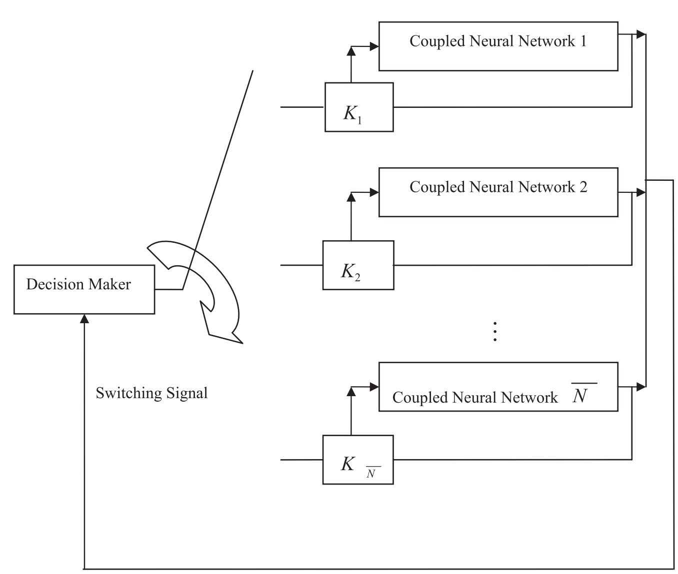

Fig. 1. Architecture of the switched coupled neural networks.