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Robust persistent activity in neural fields with asymmetric connectivity

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Abstract

Modeling studies have shown that recurrent interactions within neural networks are capable of self-sustaining non-uniform activity profiles. These patterns are thought to be the neural basis of working memory. However, the lack of robustness challenge this view as already small deviations from the assumed interaction symmetry destroy the attractor state. Here we analyze attractor states of a neural field model composed of bistable neurons. We show the existence of self-stabilized patterns that robustly represent the cue position in the presence of a substantial asymmetry in the connection profile. Using approximation techniques we derive an explicit expression for a threshold value describing the transition to a traveling activity wave.

Keywords: Neural field; Bistability; Working memory; Spatial orientation

1. Introduction

Many models of stimulus-selective persistent activity in the brain are based on recurrent networks with 'continuous attractors'. The connectivity in these models supports the existence of self-stabilized activity profiles to represent any value along a continuous physical dimension such as direction or position [1,9,4]. A transient external input acts as a switch between a uniform rest state and one of the stable active states encoding a particular position or direction. Typically, the connections are organized in a Mexican-hat pattern with strong excitation between cells with similar preferred feature flanked by a strong "surround" inhibition. However, the continuity of the attractor states requires a perfect spatial symmetry in the connection profile (for review see [3]). Already small deviations cause a drift in the spatial position of the activity pattern. To serve as a biologically plausible model for short term memory, the network must be sufficiently robust to imprecise components. On the other hand, it has been hypothesized in the context of head direction cells in rats that an asymmetry induced drift mechanism may be actively controlled by the biological system for an updating of spatial representations during self-motion. Vestibular or proprioceptive input, for instance, may have a modulatory effect on the synapses of the head direction cell network [9,7].

The main result of the present study is that the proposed model shows the required robustness but likewise allows implementing the dynamic updating mechanism. Concretely, our analytical and numerical results reveal: (1) the existence of a threshold for the asymmetry in the weight distribution below which static activity profiles exist, (2) that these activity patterns depend to some extent on the external input, and (3) that for larger asymmetries a transition to a traveling wave occurs.

2. Model description

We study a one-dimensional field model composed of bistable neurons with a non-symmetric, homogeneous connectivity of lateral inhibition type. As a concrete example we consider a field with periodic boundary conditions representing the circular space of heading direction. The time evolution of the network is governed by the equation:

$$\tau \frac{\partial u(x,t)}{\partial t} = f(u(x,t)) + \int_0^{360} w(x-y)u(y,t)\,\mathrm{d}y + h, \tag{1}$$

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where $\tau > 0$, $h < 0$ are constants defining the time scale and the resting level of the dynamics, respectively. The cubic like shaped non-linearity f describes the bistable behavior of each neuron with a stable resting and a stable excited state. To simplify the analysis we chose a piecewise linear function given by (Fig. 1):

$$f(u) = \begin{cases} \frac{a}{2}u & \Leftarrow u \leqslant \frac{k}{2}, \\ -\frac{a}{2}\frac{k}{1-k}\left(u - \frac{1}{2}\right) & \Leftarrow \frac{k}{2} < u \leqslant 1 - \frac{k}{2}, \\ \frac{a}{2}(u-1) & \Leftarrow u > 1 - \frac{k}{2}, \end{cases} \quad (2)$$

where $0 \leqslant k \leqslant 1$ and $a < 0$.

The synaptic weight distribution function w is chosen as a rectangular profile $w(y) = b_1(H(y+d_2) - H(y-d_1)) - b2$, where H is the Heaviside step function ($H(x) = 1$ if $x > 0$ and is 0 otherwise) and $b_1 > 0$, $b_2 > 0$ are constants. The spatial range of the excitatory interactions to the "right" and to the "left" are described by $d_1 \geqslant 0$ and $d_2 \geqslant 0$, respectively, while the inhibition extends over the whole field. We assume $d_2 \leqslant d_1$ and set $A = b_1(d_1 + d_2)$ and $w_0 = A - b_2 360$. The *level of asymmetry* of the connection profile w is defined as the difference $d_1 - d_2$ relative to the total excitatory range $d_1 + d_2$, that is $ASY = (d_1 - d_2)/(d_1 + d_2)$, hence $0 \leqslant ASY \leqslant 1$. It is important to stress that the specific choice of the interaction profile w and the non-linearity f is motivated to allow for a more rigorous analysis of the steady state solutions. Numerical simulations show that smooth functions for f and w could have been chosen as well without qualitatively changing the results presented here.

3. Results

Because of the bistability of the neurons, a localized activity pattern triggered by a transient input appears to be discontinuous with a jump in the activation level occurring at positions represented by neurons x_1 and x_2 at the left and the right side, respectively (compare Fig. 2). We apply an approximation technique similar to [8] to derive explicit formulas for three parameters which describe the activity profile: the amplitude r, the width l, and the resting state u_b. As depicted in Fig. 2A, we use a piecewise linear function to approximate the neuronal pattern below and above the jump discontinuity. This allows us to directly evaluate the equilibrium solution of Eq. (1) for a neuron x_r at resting level, a fully excited neuron x_t and the two "transition

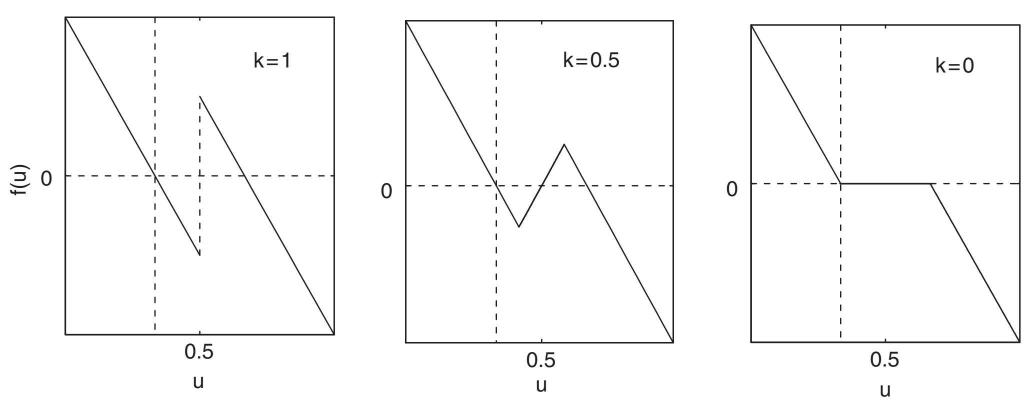

Fig. 1. Sketch of the non-linear function $f(u)$ for different values of k.

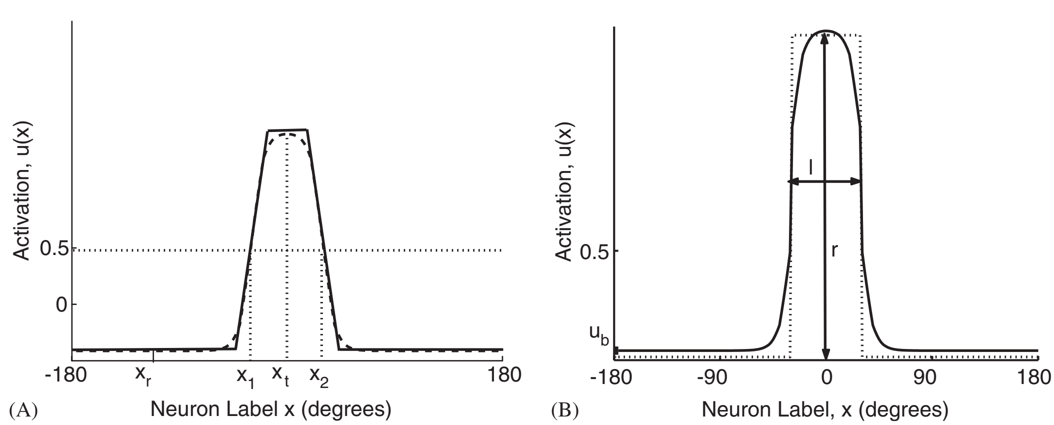

Fig. 2. (A) Piecewise linear approximation (solid line) for the activity profile below and above the jump discontinuity (dashed line). (B) Plot of a self-sustained activity profile in response to a transient input centered at $x = 0$ (solid line). The dashed line indicates the values for l, r and u_b obtained analytically by solving system 3.