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Behavior of morphological associative memories with true-color image patterns

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ABSTRACT

Morphological associative memories (MAMs) are a special type of associative memory which exhibit optimal absolute storage capacity and one-step convergence. This associative model substitutes the additions and multiplications used by other models by computing maximums and minimums. This type of associative model has been applied to different pattern recognition problems including face localization and gray scale image restoration. Despite of his power, MAMs have not been applied in problems that involve true-color patterns. In this paper it is described how a MAM can be applied in problems involving true-color patterns. Furthermore, a complete study of the behavior of this associative model in the restoration of true-color images is performed using a benchmark of 14 400 images altered by different type of noises.

1. Introduction

The concept of associative memory (AM) emerges from psychological theories of human and animals learning. These memories store information by learning correlations among different stimuli. When a stimulus is presented as a memory cue, the other is retrieval as a consequence; this means that the two stimuli have become associated each other in the memory.

An AM can be seen as a particular type of neural network designed to recall output patterns in terms of input patterns that can appear altered by some kind of noise. Several associative models have been described in the last years (refer for example [1–11]). Most of these AMs have several constraints that limit their applicability in complex problems. Among these constraints we could mention their capacity of storage (limited), the type of patterns (only binary, bipolar, integer or real patterns), robustness to noise (additive, subtractive, mixed, Gaussian noise, etc.).

A first attempt in formulating useful morphological neural networks was proposed by Davidson et al. [12]. Since then, only a few papers involving morphological neural networks have appeared. Refer for example to [13,14]. In 1998, Ritter et al. [8] proposed the concept of morphological associative memory (MAM) and the concept of morphological auto-associative memory (MAAM). Basically, the authors substituted the outer product by max and min operations. One year later, the authors introduced their morphological bidirectional associative memories [15]. Their properties, compared with Hopfield Associative model are completely different. For example, they exhibit optimal absolute storage capacity and one-step convergence in the auto-associative case.

This type of associative model has been applied to the reconstruction of gray scale images [9,16–20]. Despite of his power, it has not been applied to problems involving true-color patterns; neither a deep study of this associative model under true-color image patterns has been reported.

In this paper it is described how a MAM can be applied in problems involving true-color patterns. Furthermore, a complete study of the behavior of this associative model in the restoration of true-color images is performed. For this a benchmark of 14 400 images altered by different type of noises is used. In addition, the potential of the described model is tested in two real scenarios: image categorization and image restoration.

2. Basics on morphological associative memories

The basic computations occurring in the morphological network proposed by Ritter et al. are based on the algebraic lattice structure $(R, \wedge, \vee, +)$ where the symbols $\wedge$ and $\vee$ denote the binary operations of minimum and maximum, respectively.

Let $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{y} \in \mathbb{R}^m$ an input and output pattern, respectively. An association between input pattern $\mathbf{x}$ and output pattern $\mathbf{y}$ is denoted as $(\mathbf{x}^\xi, \mathbf{y}^\xi)$, where ξ is the corresponding association.

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Associative memory $\mathbf{W}$ is represented by a matrix whose components w_{ij} can be seen as the synapses of the neural network. If $\mathbf{x}^{\xi} = \mathbf{y}^{\xi} \forall \xi = 1, \ldots, p$ then $\mathbf{W}$ is auto-associative, otherwise it is hetero-associative. A distorted version of a pattern $\mathbf{x}$ to be recalled will be denoted as $\tilde{\mathbf{x}}$. If when feeding AM $\mathbf{W}$ with a distorted version of $\mathbf{x}^{\xi}$, output pattern $\mathbf{y}^{k}$ is exactly restored, we say that recalling is robust.

Suppose we are given a couple of $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{y} \in \mathbb{R}^m$. A MAM that allows to recalling pattern $\mathbf{y}$ given input pattern $\mathbf{x}$ is given by:

$$\mathbf{W} = \mathbf{y} \boxed{\vee} (-\mathbf{x})^t = \begin{pmatrix} y_1 - x_1 & \cdots & y_1 - x_n \\ \vdots & \ddots & \vdots \\ y_m - x_1 & \cdots & y_m - x_n \end{pmatrix} \tag{1}$$

since $\mathbf{W}$ satisfies the equation $\mathbf{W} \boxed{\vee} \mathbf{x} = \mathbf{y}$ as can be verified by the simple computation

$$\mathbf{W} \boxed{\vee} \mathbf{x} = \begin{pmatrix} \bigvee_{i=1}^{n} (y_1 - x_i + x_i) \\ \vdots \\ \bigvee_{i=1}^{n} (y_m - x_i + x_i) \end{pmatrix} = \mathbf{y} \tag{2}$$

$\mathbf{W}$ is called the max product of $\mathbf{y}$ and $\mathbf{x}$. We can also denote the min product of $\mathbf{y}$ and $\mathbf{x}$ using operator $\boxed{\wedge}$.

For a given set of pattern associations $\{(\mathbf{x}^{\xi}, \mathbf{y}^{\xi}) : \xi = 1, \ldots, k\}$ we define a couple of pattern matrices $(\mathbf{X}, \mathbf{Y})$, where $\mathbf{X} = (\mathbf{x}^1, \ldots, \mathbf{x}^k)$ and $\mathbf{Y} = (\mathbf{y}^1, \ldots, \mathbf{y}^k)$. With each pair of matrices $(\mathbf{X}, \mathbf{Y})$, two natural morphological $m \times n$ memories $\mathbf{W_{XY}}$ and $\mathbf{M_{XY}}$ are defined by:

$$\mathbf{W_{XY}} = \bigwedge_{\xi=1}^{k} [\mathbf{y}^{\xi} \boxed{\wedge} (-\mathbf{x}^{\xi})] \text{and} \mathbf{M_{XY}} = \bigvee_{\xi=1}^{k} [\mathbf{y}^{\xi} \boxed{\vee} (-\mathbf{x}^{\xi})]. \tag{3}$$

From this definition it follows that

$$\mathbf{y}^{\xi} \boxed{\wedge} (-\mathbf{x}^{\xi})^t = \mathbf{y}^{\xi} \boxed{\vee} (-\mathbf{x}^{\xi})^t \tag{4}$$

which implies that

$$\mathbf{W_{XY}} \le \mathbf{y}^{\xi} \boxed{\wedge} (-\mathbf{x}^{\xi})^t = \mathbf{y}^{\xi} \boxed{\vee} (-\mathbf{x}^{\xi})^t \le \mathbf{M_{XY}}, \forall \xi = 1, \ldots, k \tag{5}$$

In terms of Eqs. (2) and (3), this last set of inequalities implies that

$$\mathbf{W_{XY}} \boxed{\vee} \mathbf{x}^{\xi} \le [\mathbf{y}^{\xi} \boxed{\wedge} (-\mathbf{x}^{\xi})^t] \boxed{\vee} \mathbf{x}^{\xi} = \mathbf{y}^{\xi} = [\mathbf{y}^{\xi} \boxed{\vee} (-\mathbf{x}^{\xi})^t] \boxed{\wedge} \mathbf{x}^{\xi} \le \mathbf{M_{XY}}, \forall \xi = 1, \ldots, k \tag{6}$$

or equivalently, that

$$\mathbf{W_{XY}} \boxed{\vee} \mathbf{X} \le \mathbf{Y} \le \mathbf{M_{XY}} \boxed{\wedge} \mathbf{X} \tag{7}$$

The complete set of theorems which guarantee perfect recall and their corresponding proofs are given in [8]. Something important to mention is that this MAM is robust either to additive noise or to subtractive noise, not both (mixed noise). While MAM $\mathbf{W_{XY}}$ is robust to subtractive noise, MAM $\mathbf{M_{XY}}$ is robust to additive noise. However, this MAM in not robust to image transformations which make it inadaptable to be directly applied in object recognition problems.

3. Behavior of $\mathbf{W_{XY}}$ under true-color noisy patterns

In this section a study of the behavior of $\mathbf{W_{XY}}$ under true-color noisy patterns is presented. Two types of experiments will be performed. In the first case we study the auto-associative version of $\mathbf{W_{XY}}$, in the second case we study the hetero-associative version.

For the case of the auto-associative version, first to all, we verified if the MAAM $\mathbf{W_{XY}}$ was capable to recall the complete set of associations. Then we verified the behavior of $\mathbf{W_{XY}}$ using noisy versions of the images used to train the MAAM. After that, we performed a study of how the number of associations influence the behavior of the MAAM $\mathbf{W_{XY}}$.

For the case of the hetero-associative version, first to all, we verified if the MHAM $\mathbf{W_{XY}}$ was capable to recall the complete set of associations. At last, we verified the behavior of $\mathbf{W_{XY}}$ using noisy versions of the images used to train the MHAM.

The benchmark used in this set of experiments is composed by 14 400 color images of 63×43 pixels and 24 bits in a bmp format. This benchmark contains 40 classes of flowers and animals. Per each class, there are 90 images altered with additive noise (0% of the pixels to 90% of the pixels), 90 images altered with subtractive noise (0% of the pixels to 90% of the pixels), 90 images altered with mixed noise (0% of the pixels to 90% of the pixels) and 90 images altered with Gaussian noise (0% of the pixels to 90% of the pixels). Fig. 1 shows some images which compose this benchmark.

Although it seems that there is not much difference between gray level images and true-color images, MAMs are not designed to cope with multivariable patterns (three channels per pixel) because they are based on gray level morphological operations. Instead of training one memory per color channel and then deciding how to combine the information recalled by each memory and finally restore the true-color image, we proposed to transform these three channels in one channel. However, it is important to notice that if we transform the RGB channels into one channel by means of computing the average of the three channels (in other words transform the true-color image into a gray level image), we will not able to recover the information of the RGB channels from the average channel.

For that reason, before the MAM $\mathbf{W_{XY}}$ was trained, each image had to be transformed into an image pattern. To build an image pattern from the bmp file, the image was read from left-right and up-down; each RGB pixel (hexadecimal value) was transformed into a decimal value and finally, this information was stored into an array. For example, suppose that the value of a RGB pixel is "0x3E53A1" (where R=3E, G=53 and B=A1) then by transforming into its decimal version, its corresponding decimal value will be "4084641". With this new procedure, we avoid training the MAM with multivariable patterns and we can recover the RGB channels by transforming the decimal value into its hexadecimal value.

Once trained the associative memory, we proceeded to evaluate the behavior of the two MAM $\mathbf{W_{XY}}$ versions. In order to measure the accuracy of the MAM we counted the number of pixels correctly recalled. We reported the percentage of pixels that can be recalled by the method because we wanted to empathize if the associative model really restores the altered pixels. However, for practical image processing purposes, we also used the normalized mean square error to measure the difference between the original image and the recalled image.

3.1. Auto-associative version of $\mathbf{W_{XY}}$

It is important to remark that even using true-color patterns the MAAM $\mathbf{W_{XY}}$ was capable to recall the complete set of associations. However, it is important to analyze the robustness that the model presents in the presence of noisy patterns.

In the next set of experiments, we study the behavior of the MAAM $\mathbf{W_{XY}}$ under different type of noises. For the case of additive noise (Fig. 2(a)), we can observe that if only the 2% of the pixels are altered, the MAAM $\mathbf{W_{XY}}$ is capable of correctly recalling only the 23.6% of the pixels. For the case of mixed and Gaussian noise (Figs. 2(c and d)), we can observe that if only the 2% of the pixels are altered, the MAAM $\mathbf{W_{XY}}$ is capable of correctly recall only the 29.7% and 46.8% of the pixels, respectively. These percentages decrease as the number of altered pixels increases. For the case of subtractive noise (Fig. 2(b)), we can observe that even when the 90% of the pixels are altered, the MAAM $\mathbf{W_{XY}}$ is capable of correctly recall the 77.4% of the pixels.