Neurocomputing 73 (2009) 542–551

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Letters

Robust stability of stochastic Cohen–Grossberg neural networks with mixed time-varying delays ☆

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ARTICLE INFO

Article history:
Received 25 November 2008
Received in revised form
1 April 2009
Accepted 1 July 2009
Communicated by H. Zhang
Available online 4 August 2009

Keywords:
Stochastic Cohen–Grossberg neural networks
Robust exponential stability
Linear fractional uncertainty
Time-varying delays
LMI approach

ABSTRACT

In the paper, the problem of robust exponential stability analysis is investigated for stochastic Cohen–Grossberg neural networks with both interval time-varying and distributed time-varying delays. By employing an augmented Lyapunov–Krasovskii functional, together with the LMI approach and definition on convex set, two delay-dependent conditions guaranteeing the robust exponential stability (in the mean square sense) of addressed system are presented. Additionally, the activation functions are of more general descriptions and the derivative of time-varying delay being less than 1 is released, which generalize and further improve those earlier methods. Numerical examples are provided to demonstrate the effectiveness of proposed stability conditions.

1. Introduction

In the past few decades, there have been tremendous developments on dynamics research of neural networks. Various neural network models including cellular neural network, bidirectional associative neural network and Cohen–Grossberg neural network have been widely investigated and successfully applied in many fields. Among the systems above, the Cohen–Grossberg neural network [1] has especially gained particular research attention since it is quite general to include several well-known neural networks as its special cases and has promising application potentials for tasks of classification, associative memory, parallel computation and so on.

In many practical cases, time-delays are unavoidably encountered in the implementation of neural networks, and they may induce the undesirable dynamic network behaviors such as oscillation, instability or other poor performances. Time-delay occurs due to the finite speeds of the switching and transmission of signals in a network, which leads to delayed neural networks that were firstly introduced in [2]. Since then, the dynamics of delayed neural networks have been widely studied. In the recent years, there has been an increasing research interest on the stability analysis on delayed Cohen–Grossberg neural networks and many results have been reported. Large numbers of conditions, either delay-dependent or delay-independent, have been proposed to guarantee the asymptotic and exponential stability for Cohen–Grossberg neural networks with time-delays [3,4,8–13,23–27].

Although it has been realized that discrete time-delays can be introduced into communication channels since they are ubiquitous in both the neural processing and signal transmission, a neural network usually has a special nature due to the presence of an amount of parallel pathways with a variety of axon sizes and lengths. Such an inherent nature can be suitably modeled by distributed delays [5], because the signal propagation is distributed during a certain time period. As a matter of fact, a realistic neural network should involve both discrete and distributed delays [7]. Recently, the stability analysis problems of Cohen–Grossberg neural networks with distributed delays have received much attention [8,9]. It is worth noting that, most recently, the asymptotic or exponential stability problems have been investigated in [10–13] for the neural networks with both discrete and distributed delays, in which LMI approach has been employed to derive the stability criteria.

During the past years, there are the emergences of the dynamical behaviors of stochastic neural networks as a new subject of research topic. In real nervous systems, the synaptic transmission is a noisy process brought on by random fluctuations from the release of

☆ This work is supported by the national Natural Science Foundation of China under Grant nos. 60764001 and 60875035, the Science Foundation of Guizhou Province under Grant no. 20072208, the Specialized Research Fund for Doctoral Program of Higher Education of China under Grant no. 20030286013 and Jiangsu Planned Projects for Postdoctoral Research Funds No. 0901005B.

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doi:10.1016/j.neucom.2009.07.007

neurotransmitters and other probabilistic causes, and it has been realized that a neural network could be stabilized or destabilized by certain stochastic inputs [17] which leads to the research on dynamics of stochastic neural networks. Particularly, the stability of delayed stochastic neural networks including the stochastic Cohen–Grossberg ones has become an attractive research problem of prime significance and many elegant methods have been proposed [17–27] by employing various methods including the LMI technique. However, the stability criteria in [23] are not presented in terms of LMIs, which makes them checked inconveniently by resorting to Matlab LMI Toolbox. The methods in [24–26] cannot be employed to tackle the stochastic Cohen–Grossberg models with time-varying delays. In [27], the stability criteria cannot tackle the case that derivative of delay has an upper bound being greater than 1. Moreover, the derivative of $\varrho_m \int_{-\varrho_m}^{0} \int_{t+\theta}^{t} f^T(x(s))Tf(x(s))\,ds\,d\theta$ is estimated by $\varrho_m^2 f^T(x(t))Tf(x(t)) -[\int_{t-\varrho(t)}^{t} f(x(s))\,ds]^T T[\int_{t-\varrho(t)}^{t} f(x(s))\,ds]$ and the term $-[\int_{t-\varrho_m}^{t-\varrho(t)} f(x(s))\,ds]^T T[\int_{t-\varrho_m}^{t-\varrho(t)} f(x(s))\,ds]$ is ignored in the present literature, which may lead to some conservatism. On the other hand, the range of variable delay considered in [23–27] is from 0 to an upper bound. In practice, the range of delay may vary in a range for which the lower bound is not restricted to be 0. In the case, the criteria in these works are conservative because they do not take into account the information of the lower bound of delay. To the best of authors' knowledge, as for that restriction on derivative of delay less than 1 is removed and stability criteria are presented in terms of LMIs, few authors have investigated the robust exponential stability of stochastic Cohen–Grossberg neural networks with both interval time-varying and distributed delays yet, which remains important and challenging.

Motivated by above discussions, the objective of this paper is to study the exponential stability of uncertain stochastic Cohen–Grossberg neural networks with mixed time-varying delays by employing a novel Lyapunov–Krasovskii functional, in which the variable delay belongs to an interval and the activation function is varies in general, assuming neither differentiability nor strict monotonicity. Through introducing some free-weighting matrices and equivalent transformation of considered system, the delay-dependent criteria are established in the forms of LMIs. Finally, the effectiveness of the proposed stability criteria is illustrated with the help of two numerical examples.

Notations. For the symmetric matrix X, $X>0$ (respectively, $X\geq 0$) means that $X>0$ ($X\geq 0$) is a positive-definite (respectively, positive-semidefinite) matrix; A^T, A^{-T} represent the transposes of matrices A and A^{-1}, respectively. For $\tau>0$, $\mathscr{C}([-\tau,0];\mathbf{R}^n)$ denotes the family of continuous functions φ from $[-\tau,0]$ to $\mathbf{R}^n$ with the norm $\|\varphi\| = \sup_{-\tau\leq\theta\leq 0}|\varphi|$. Let $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geq 0},P)$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions; $L^p_{\mathscr{F}_0}([-\tau,0];\mathbf{R}^n)$ is the family of all $\mathscr{F}_0$-measurable $\mathscr{C}([-\tau,0];\mathbf{R}^n)$-valued random variables $\xi=\{\xi(\theta): -\tau\leq\theta\leq 0\}$ such that $\sup_{-\tau\leq\theta\leq 0}\mathbf{E}|\xi(\theta)|^p<\infty$ where $\mathbf{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P; I denotes the identity matrix with an appropriate dimension; the symmetric term in a symmetric matrix is denoted by $*$.

2. Problem formulations

We consider uncertain stochastic Cohen–Grossberg neural networks with mixed time-varying delays described by

$$dx(t) = -\alpha(x(t))\left[\beta(x(t)) - A(t)f(x(t)) - B(t)f(x(t-\tau(t))) - D(t)\int_{t-\varrho(t)}^{t} f(x(s))\,ds\right]dt + \sigma(t,x(t),x(t-\tau(t)))\,d\omega(t), \tag{1}$$

where $x(t)=[x_1(t),\ldots,x_n(t)]^T\in\mathbf{R}^n$ is the neuron state vector; $\alpha(x(t))=\mathrm{diag}\{\alpha_1(x_1(t)),\ldots,\alpha_n(x_n(t))\}$ represents the amplification function and $\alpha_i(x_i(t))(i=1,\ldots,n)$ is assumed to be positive, bounded and locally Lipschitz continuous and $\beta(x(t))=[\beta_1(x_1(t)),\ldots,\beta_n(x_n(t))]^T$ is the behaved function; $f(\cdot)=[f_1(\cdot),\ldots,f_n(\cdot)]^T\in\mathbf{R}^n$ stands for the neuron activation function; $\omega(t)=[\omega_1(t),\ldots,\omega_m(t)]^T\in\mathbf{R}^m$ is an m-dimensional Brownian motion defined on $(\Omega,\mathscr{F},P)$; and $A(t)=A+\Delta A(t)$, $B(t)=B+\Delta B(t)$, $D(t)=D+\Delta D(t)$ are the uncertain matrices of appropriate dimensions. Here, $\tau(t),\varrho(t)$ denote the interval time-varying delay and distributed one satisfying

$$0\leq\tau_0\leq\tau(t)\leq\tau_m,\quad \dot{\tau}(t)\leq\mu,\quad 0\leq\varrho(t)\leq\varrho_m, \tag{2}$$

in which τ_m, τ_0, μ, ϱ_m are constants.

Remark 1. The hypothesis on derivative of variable delay being less than 1 is imposed on the stability criteria proposed in [10,27]. However, in the paper, the restriction of delay's derivative being less than 1 is removed, which is more meaningful than the ones in [10,27].

The following assumptions are made throughout this paper.

Assumption 1. $\Delta A(t),\Delta B(t),\Delta D(t)$ are unknown matrices representing time-varying parametric uncertainties, and are of linear fractional forms

$$[\Delta A(t)\ \ \Delta B(t)\ \ \Delta D(t)] = F\Delta(t)[E_1\ \ E_2\ \ E_3], \tag{3}$$

$$\Delta(t)=\Lambda(t)(I-J\Lambda(t))^{-1},\quad I-J^TJ>0,$$

in which $F,J,E_i\ (i=1,2,3)$ are known constant matrices of appropriate dimensions and $\Lambda(t)$ is an unknown time-varying matrix function satisfying the following condition:

$$\Lambda^T(t)\Lambda(t)\leq I. \tag{4}$$

Assumption 2. Each $\alpha_i(\cdot)$ is a continuous function and satisfies $0<\underline{a}_i\leq\alpha_i(\cdot)\leq\overline{a}_i$ for all $i=1,\ldots,n$. And each function $\beta_i(\cdot):\mathbf{R}\to\mathbf{R}$ is locally Lipschitz and there exist π_i,γ_i such that $\pi_i\geq\dot{\beta}_i(\cdot)\geq\gamma_i>0$ for all $i=1,\ldots,n$. Here, we denote $\Lambda=\mathrm{diag}\{\underline{a}_1,\ldots,\underline{a}_n\}$, $\Psi=\mathrm{diag}\{\overline{a}_1,\ldots,\overline{a}_n\}$, $\Pi=\mathrm{diag}\{\pi_1,\ldots,\pi_n\}$, and $\Gamma=\mathrm{diag}\{\gamma_1,\ldots,\gamma_n\}$.

Assumption 3. Each activation function $f_i(\cdot)$ in (1) is bounded and satisfies

$$\sigma_i^-\leq\frac{f_i(x)-f_i(y)}{x-y}\leq\sigma_i^+,\quad \forall x,y\in\mathbf{R},\ \ x\neq y,\ \ i=1,2,\ldots,n, \tag{5}$$

and $f_i(0)=0$, where σ_i^+,σ_i^- are constants. We denote $\tilde{\Sigma}=\mathrm{diag}\{\sigma_1^+,\ldots,\sigma_n^+\}$, $\Sigma=\mathrm{diag}\{\sigma_1^-,\ldots,\sigma_n^-\}$, and

$$\Sigma_1=\mathrm{diag}\{\sigma_1^+\sigma_1^-,\ldots,\sigma_n^+\sigma_n^-\},\quad \Sigma_2=\mathrm{diag}\left\{\frac{\sigma_1^++\sigma_1^-}{2},\ldots,\frac{\sigma_n^++\sigma_n^-}{2}\right\}. \tag{6}$$

Assumption 4. $\sigma(t,\cdot,\cdot):\mathbf{R}^+\times\mathbf{R}^n\times\mathbf{R}^n\to\mathbf{R}^{n\times m}$ $(\sigma(t,0,0)=0)$ is locally Lipschitz continuous and satisfies the linear growth condition as well. Moreover, $\sigma(t,\cdot,\cdot)$ satisfies the following condition:

$$\mathrm{trace}[\sigma^T(t,x,y)\sigma(t,x,y)]\leq x^T\Pi_1^T\Pi_1x+y^T\Pi_2^T\Pi_2y+f^T(x)\Xi_1^T\Xi_1f(x)+f^T(y)\Xi_2^T\Xi_2f(y), \tag{7}$$