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A variational based smart segmentation model for speckled images ☆

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ABSTRACT

In this paper, we propose a new variational model in the fuzzy framework to achieve the task of partitioning speckled images, such as the synthetic aperture radar (SAR) images. The model is partly derived by using the so-called maximizing a posteriori (MAP) estimation method. The novelties of the model are that (1) the Gamma distribution rather than the classical Gaussian distribution is used to simulate the gray intensities in each homogeneous region of the images; (2) a smart regularization term with respect to fuzzy membership functions is designed. The newly designed regularization term equals to an adaptive weighted total variation (TV) regularizer. Compared with the classical TV regularizer, the proposed regularization term not only has a sparser property, but also protects the segmentation results from degeneration (being over-smoothed). In addition, a new algorithm based on the alternative direction iteration algorithm is proposed to solve the model. The algorithm is efficient since it integrates the split Bregman method and Chambolle's projection method. Numerical examples are given to verify the promising efficiency of our model.

1. Introduction

In the variational framework, images are considered as functions, and the task of image segmentation refers to partitioning the domain of an image into several meaningful homogeneous regions. To achieve the task, both edge/boundary- and region-based models are proposed. Edge-based models [1–3] drive active contours towards the boundaries of regions. As some operations on images such as the gradient and the Laplace are usually used in edge-based models, the models have a common drawback that they are not quite robust to strong noise since the computation of the operations is negatively influenced by the noise. To overcome the drawback, region-based models are proposed.

Region-based image segmentation models are usually roughly divided into the following three categories: the level-set based models [4–7], the phase-field based models [8] and the fuzzy based models [9,10]. The models in the first two categories use level set function and sinusoidal potential function, respectively, to represent the regions of the images to be segmented. Then, the models are usually considered as hard-type or 0/1-type models since the regions estimated from the models can automatically avoid vacuum or overlap. The models in the third category use overlapping but usually not heavily fuzzy membership functions (FMFs) to indicate different regions. Here, the FMFs range from the interval [0, 1], and for a certain region in images, a point can be classified (resp. not classified) into the region when the value of a FMF at the point approximates to 1 (resp. 0). It has been proved that fuzzy-type image segmentation models can show better performance [9] and are more efficient [10] than hard-type models. For this reason, we only discuss our segmentation model in the fuzzy framework in this paper. However, as the regions estimated from fuzzy-type models tend to be overlapping with each other, the overlap degree among regions should be reduced so that the true boundaries of regions can be simply obtained. Therefore, in fuzzy-type models, the geometrical structure of the FMFs should be well preserved. Then, the true boundaries of image regions can be directly obtained from the estimated FMFs.

Traditionally, for region-based image segmentation models, the following assumptions are usually used. For the case that the gray intensities of pixels in a region do not uniformly vary with points, which means the intensities are homogeneous in the region, the gray intensity of each pixel in the region can be assumed to be a sample independently drawn from a Gaussian distribution with the same mean and variance. For the case that the gray intensities of pixels in a region vary with points uniformly, which means the

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intensities in the region are not rigidly homogeneous, the fitting methods [11,12] and the bias correction methods [13,14] are usually introduced in image segmentation, which is equivalent to assume that the intensity of a pixel in small local parts of a region follows a Gaussian distribution in a weighting sense [13]. However, for a speckled image, due to the coherent nature of the image acquisition process [15], the intensities in each region or local parts of the region cannot be simply assumed to follow a Gaussian distribution. Instead, the Rayleigh and Gamma distributions are more suitable [16,17]. Another common problem for classical image segmentation models is that the total variation (TV) regularizer is usually used to protect the FMFs from degeneration. However, for images contaminated by strong speckle noise, the TV regularizer tends to blur edges of the FMFs with the iteration number of the algorithms of models increasing. This point will be discussed in detail in our numerical experiments.

In brief, to obtain relatively good segmentation results for speckled images, in this paper we propose a new fuzzy-type variational based image segmentation model. The model is partly derived by using an estimation method called as the maximizing a posteriori (MAP). The derived model contains a fidelity term and a regularization term. The fidelity term is derived from the assumption that the gray intensities in a homogeneous region follow a Gamma distribution. The regularization term can be seen as a weighted TV regularizer which overcomes the blurring drawback of the TV regularizer. As the weight is self-adaptive, the proposed regularization term has smart properties. It is worth noting that our model does not take the images with intensity inhomogeneity into consideration since in this paper we mainly focus on the problem of the construction of a non-Gaussian framework for the image segmentation task, and, compared with the Gaussian assumption, the superiority of the Gamma assumption in our model can be highlighted. As to the problem of partitioning the speckled images with intensity inhomogeneity, we will solve it in our future work. In addition, in order to solve the model efficiently, we design a new algorithm which is based on an alternative direction iteration process. The algorithm can also be seen as a generalized iteratively reweighting algorithm [18,19]. Furthermore, in the algorithm, the following efficient minimization methods are integrated: the split Bregman method [20,21] and Chambolle's projection method [22].

The rest of the paper is organized as follows. Section 2 gives a detailed discussion on the proposed model. During the discussion, we also compare our model with some classical image segmentation models. The implementation details of the model are presented in Section 3. In Section 4, we demonstrate the efficiency of the model with both synthetic and authentic speckled images.

2. Proposed model

2.1. Model derivation

Let $\mathbf{I}$ be a speckled image defined in an open bounded domain Ω. Suppose $\mathbf{I}$ contains N homogeneous regions such that $\Omega = \bigcup_{i=1}^{N} \Omega_i$ and $\Omega_i \bigcap \Omega_j = \varnothing$ for $i \neq j$ and $i, j = 1 : N$. Furthermore, for $\forall x \in \Omega_i$, the gray intensity $I(x)$ is a sample independently drawn from a Gamma distribution whose probability density function (PDF) $p(I(x) | \mu_i, \Omega_i)$ is defined by

$$p(I(x) | \mu_i, \Omega_i) = \left(\frac{L}{\mu_i}\right)^L \frac{1}{\Gamma(L)} (I(x))^{L-1} \exp\left(-\frac{LI(x)}{\mu_i}\right), \tag{1}$$

where the constant L, being called as the equivalent number of looks, can be estimated beforehand [15], $\Gamma(\cdot)$ is the usual Gamma function, and the parameter μ_i is the intensity mean of the region Ω_i. Then, the joint PDF $p(\mathbf{I} | \mu, \{\Omega_i\}_{i=1}^N)$ of Eq. (1) is given by

$$p(\mathbf{I} | \mu, \{\Omega_i\}_{i=1}^N) = \prod_{i=1}^{N} \prod_{x \in \Omega_i} p(I(x) | \mu_i, \Omega_i), \tag{2}$$

where the denotation $\mathbf{I}$ represents the entirety of $I(x)$ for $x \in \Omega$, $\mu = (\mu_1, \mu_2, \dots, \mu_N)$, and the denotation $\{\Omega_i\}_{i=1}^N$ represents a segmentation mode. Further, by introducing the membership functions

$$u_i(x) = \mathbf{1}_{x \in \Omega_i} = \begin{cases} 1, & x \in \Omega_i, \\ 0, & \text{otherwise}, \end{cases} \tag{3}$$

we can write Eq. (2) in the following form

$$p(\mathbf{I} | \mu, \mathbf{u}) = \prod_{i=1}^{N} \prod_{\{x | u_i(x) \neq 0\}} p(I(x) | \mu_i, u_i(x)), \tag{4}$$

where the notation $\mathbf{u}$ represents the entirety of $u_i(x)$ for $x \in \Omega$ and $i = 1 : N$. The purpose of the introduction of $u_i(x)$ is to convert the problem of estimating $\{\Omega_i\}_{i=1}^N$ in optimizing $\mathbf{u}$.

Further, we assume $u_i(x)$ to be a random variable. In order to construct a probability prior on $u_i(x)$, we consider the following fact: if $u_i(x) = 1$ (resp. 0), then $u_i(y) = 1$ (resp. 0) will be true in a great probability, where y is in a small neighborhood of x. The fact is equivalent to restrict $u_i(x) \approx u_i(y)$ in a neighborhood of x. In fact, in its discrete form, the above fact has been widely discussed in the framework of the Markov random field (MRF) [23], in which, for $\forall x \in \Omega$, $u_i(x)$ is only related to the x-neighborhood values $u_i(y)$. Then, following the work in [24] and according to the Hammersley–Clifford theorem on the MRF, we can assume a continuous Gibbs-type prior on $u_i(x)$, namely

$$p(\mathbf{u} | \Lambda) = \frac{1}{Z} \prod_{i=1}^{N} \prod_{x \in \Omega} \lambda_i(x) \exp(-\lambda_i(x) | \nabla u_i(x) |), \tag{5}$$

where Z is a normalizing constant defined by

$$Z = \prod_{i=1}^{N} \int_\Omega \lambda_i(x) \exp(-\lambda_i(x) | \nabla u_i(x) |) \, dx,$$

and the notation Λ represents the entirety of all positive parameters $\lambda_i(x)$ for $x \in \Omega$ and $i = 1 : N$. In Eq. (5), the denotation $|\nabla u_i(x)|$ represents the module of the gradient of $u_i(x)$. Then, according to the nature of the discrete gradient operator ∇, the value $|\nabla u_i(x)|$ inherently describes the differences between $u_i(x)$ and $u_i(y)$, where y is in the neighborhood of x. For example, as defined in [22], only the values $u_i(y)$ at the four pixels y adjoining x are used to compute the discrete gradient of $u_i(x)$. Furthermore, for a given Λ, the probability $p(\mathbf{u} | \Lambda)$ becomes larger when $|\nabla u_i(x)|$ decreases. It means that a larger value of $p(\mathbf{u} | \Lambda)$ implies $u_i(x) \approx u_i(y)$. It is worth noting that in the work [9,10,16], all $\lambda_i(x)$ are simply assigned to be the same constant number. Obviously, our assumption on Eq. (5) is more general and more self-adaptive since $\lambda_i(x)$ varies with x.

Applying the Bayesian rule on $\mathbf{u}$, we can find that the posteriori PDF $p(\mathbf{u} | \mathbf{I}, \mu, \Lambda)$ satisfies

$$p(\mathbf{u} | \mathbf{I}, \mu, \Lambda) \propto p(\mathbf{I} | \mu, \mathbf{u}) p(\mathbf{u} | \Lambda). \tag{6}$$

Note that the expression in Eq. (6) is a reduced form. Its complete form as well as its corresponding demonstration can be found in Fig. 1.

Based on the above assumptions , we use the MAP estimation method to optimize $\mathbf{u}$. For given μ and Λ, the ideal of the membership functions $\mathbf{u}$ is to minimize the negative log $p(\mathbf{I} | \mu, \mathbf{u}) p(\mathbf{u} | \Lambda)$. Then, we need to solve the following minimization problem

$$\min_{\mathbf{u}} \left\{ \Phi_{\mu, \mathbf{I}}(\mathbf{u}) + \Psi_\Lambda(\mathbf{u}) - \sum_{i=1}^{N} \int_\Omega \log \lambda_i(x) \, dx + C \right\}, \tag{7}$$

where the constant C is independent of μ, Λ and $\mathbf{u}$. The terms $\Phi_{\mu, \mathbf{I}}(\mathbf{u})$ and $\Psi_\Lambda(\mathbf{u})$ are the fidelity term and the regularization term,