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Finite-time recurrent neural networks for solving nonlinear optimization problems and their application

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ABSTRACT

This paper focuses on finite-time recurrent neural networks with continuous but non-smooth activation function solving nonlinearly constrained optimization problems. Firstly, definition of finite-time stability and finite-time convergence criteria are reviewed. Secondly, a finite-time recurrent neural network is proposed to solve the nonlinear optimization problem. It is shown that the proposed recurrent neural network is globally finite-time stable under the condition that the Hessian matrix of the associated Lagrangian function is positive definite. Its output converges to a minimum solution globally and finite-time, which means that the actual minimum solution can be derived in finite-time period. In addition, our recurrent neural network is applied to a hydrothermal scheduling problem. Compared with other methods, a lower consumption scheme can be derived in finite-time interval. At last, numerical simulations demonstrate the superiority and effectiveness of our proposed neural networks by solving nonlinear optimization problems with inequality constraints.

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1. Introduction

Nonlinear constrained optimization problems are widely applied in the fields of science and engineering [1], for example, electrical networks planning, optimal control, mechanical design, hydrothermal scheduling, structure design and so on. In the past decades, numerical methods have been proposed to solve nonlinear convex optimization problems and approximate optimal solutions could be derived [2–4]. However, it is usual imperative to require real-time solutions in many applications. Because of parallel-distributed character and hardware-realization amenity, recurrent neural networks are capable of real-time computing and have been applied in solving nonlinear optimization problems with inequality constraints.

Different from traditional numerical optimization algorithms, recurrent neural networks governed by dynamic equations can apply directly the techniques of the numerical ODE and the dynamic to solve constrained optimization problems effectively. In what follows, we give a detailed introduction for the development of recurrent neural networks. In 1986, Hopfield firstly presented a neural network to solve linear programming problems [5]. In order to solve nonlinear convex programming problems, Kennedy proposed a neural network with a finite penalty parameter [6]. Although the Kuhn–Tucker optimal conditions hold for Kennedy's and Hopfield's, this neural network with a finite penalty parameter cannot find the exact optimal solution. In addition, it is difficult to implement the neural network with a very large penalty parameter [7]. In order to overcome the imperfection, some scholars attempted to design neural networks without penalty parameter. For example, Rodriguez designed a switched-capacitor neural network to solve a class of optimization problems with nonlinear convex constraints [8]. For convex quadratic optimization problems with bounded constraints, Bouzerdoum presented a recurrent neural network to solve them [9]. Zhang proposed a Lagrange neural network to deal with nonlinear programming problems with equality constraints [10]. Especially, a projection neural network is developed to solve monotone variational inequality problems with limit constraints [11,12]. By extending the projection neural network, two recurrent neural networks for solving strictly convex programming with nonlinear inequality constraints and linear constraints were presented in [13,14], respectively. The projection neural network was also used to solve pseudomonotone variational inequality problems in [15]. The above recurrent neural networks are proposed to solve linear or nonlinear convex problems. But, many optimization problems may be nonconvex or nonsmooth in the engineering application fields, so it is interesting to study recurrent neural networks to solve nonconvex or nonsmooth

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optimization problems. Recently, some neural network models dealing with these optimization problems are proposed in [16–21].

It should be noted that all the above mentioned neural networks are asymptotically stable or exponentially stable, which means that the exact solutions can be obtained when time goes to infinity. In order to accelerate the convergent speed, some neural networks with finite-time convergent properties are explored. For example, Cheng et al. presented a recurrent neural network with a specific nonlinear unit to deal with convex optimization problems [22]. The proposed neural network converges to the optimal solution in a finite-time interval. The computation efficiency is increased dramatically as well. A finite-time convergent neural network has also been proposed to solve quadratic programming problems [23]. The actual optimal solutions can be derived in finite time interval. However, the neural networks proposed in [23] only deal with convex optimization problems. In this paper, in order to overcome these problems, we design a finite-time recurrent neural network based on the authors' previous work [24–26], to solve nonlinear nonconvex optimization problems. Under the condition that the Hessian matrix of the associated Lagrangian function is positive definite, we prove that the proposed recurrent neural network is globally finite-time stable. Then, the actual minimum solution can be derived in finite-time period. In addition, our recurrent neural network is applied to a hydrothermal scheduling problem. It is shown the superiority and effectiveness of our proposed neural networks by solving nonlinear optimization problems with inequality constraints.

This paper is organized as follows. In Section 2, definition of finite-time stability and finite-time convergence criteria are reviewed. In Section 3, we present a finite-time recurrent neural network to solve nonlinearly constrained optimization problems. In Section 4, our method is applied to solve a short-time hydrothermal scheduling problem. Numerical simulations and a test hydrothermal system are given to show the effectiveness of our methods in Section 5. Section 6 concludes the paper.

2. Preliminaries

For the system:

$$\dot{x}(t) = f(x(t)), \quad f(0) = 0, \quad x \in R^n, \quad x(0) = x_0, \tag{1}$$

where $f : D \to R^n$ denotes a continuous function defined on an open neighborhood D such that $0 \in D$, we give the following definition of finite-time stability and finite-time stable criterion.

Definition 1 (*Bhat and Bernstein [27], Shen and Xia [28]*). If there exists an open neighborhood U such that $0 \in U$, and a function $T_x : U \backslash 0 \to (0, \infty)$, such that every solution $x(t, x_0)$ of (1) with the initial point $x_0 \in U \backslash 0$ is well-defined and unique in forward time for $t \in [0, T_x(x_0))$, $\lim_{t \to T_x(x_0)} x(t, x_0) = 0$, and $x(t, x_0) = 0$ for $t \geq T_x(x_0)$, then the system (1) finite-time converges to the equilibrium $x = 0$, $T_x(x_0)$ denotes the convergent time. The equilibrium of (1) is finite-time stable if it is Lyapunov stable and finite-time convergent. In addition, the origin is a globally finite-time stable equilibrium when $U = D = R^n$.

Lemma 1 (*Bhat and Bernstein [27]*). *If there exists a positive definite function V(x) defined on a neighborhood $U \subset R^n$ of the origin, two real numbers $k > 0$ and $0 < r < 1$ satisfying*

$$\dot{V}(x) \leq -kV(x)^r, \quad \forall x \in U, \tag{2}$$

then, the origin of system (1) is finite-time stable. Moreover, the upper bound of the convergence time T_1 satisfies

$$T_1 \leq \frac{V(x_0)^{1-r}}{k(1-r)}, \quad \forall x_0 \in U. \tag{3}$$

If $U = R^n$ and $V(x)$ is radially unbounded, the origin of system (1) is globally finite-time stable.

The following lemma will be used in the proof of our main results.

Lemma 2 (*Clarke [29]*). *Let $f : R^n \to R$ be continuously differentiable. Then $\max\{0, f(x)\}$ is a regular function, its Clarkes generalized gradient is given as follows:*

$$\partial \max\{0, f(x)\} = \begin{cases} \nabla f(x) & \text{if } f(x) > 0, \\ [0,1]\nabla f(x) & \text{if } f(x) = 0, \\ 0 & \text{if } f(x) < 0. \end{cases} \tag{4}$$

3. Finite-time recurrent neural network

In this section, we consider the following nonlinear optimization problem (NOP):

$$\text{minimize} \quad f(x), \tag{5a}$$

$$\text{subject to} \quad c(x) \leq 0, \; x \geq 0, \tag{5b}$$

where $f : R^n \to R$, $c(x) = [c_1(x), \ldots, c_m(x)]^T$, $x \in R^n$. In this paper, we assume that f and c_i are twice differentiable, and there exists at least a local optimal solution to the NOP.

It is clear that the NOP is a convex programming problem (CPP) if $f(x)$ and $c_i(x)$ are all convex, otherwise, the NOP is a nonconvex programming problem (NCPP). From [9], we have that if x^* is a local optimal solution to the NOP, then there exits $y^* \in R^m$ such that (x^*, y^*) is a Karush–Kuhn–Tucker (KKT) point satisfying the following conditions:

$$\begin{cases} y \geq 0, & c(x) \leq 0, \; x \geq 0, \\ \nabla f(x) + \nabla c(x) y \geq 0, & y^T c(x) = 0, \end{cases} \tag{6}$$

where $\nabla c(x)$ and $\nabla f(x)$ are the gradient of x. By the projection theorem [30], the following projection equations can be used to replace (6):

$$\begin{cases} (x - (\nabla f(x) + \nabla c(x) y))^+ - x = 0, \\ (y + c(x))^+ - y = 0, \end{cases} \tag{7}$$

where $x^+ = \max\{0, x\}$.

Let $z = (x, y)^T$, $z^+ = (x^+, y^+)^T$ and $\tilde{z} = (\tilde{x}, \tilde{y})^T = ((\nabla f(x^+) + \nabla c(x^+) y^+), -c(x^+))^T$. If and only if $z^* = (x^*, y^*)$ satisfies (7), then z^* is a KKT point of the NOP. Furthermore, z^* is a KKT point if z^* is also a solution to the variational inequality problem:

$$(z - z^*) F(z^*) \geq , \quad \forall z \geq 0. \tag{8}$$

where $F(z^*) = [\nabla f(x^*) + \nabla c(x^*) y^*; -c(x^*)]$.

Next, we design the following finite-time recurrent neural network to solve (5) by extending the ODE method in [31]:

$$\text{state equation} \quad \dot{z}(t) = -\varepsilon \mathcal{F}(z - z^+ + \tilde{z}), \tag{9a}$$

$$\text{output equation} \quad x(t) = x^+, \tag{9b}$$

where $\varepsilon > 0$ is a parameter and the activation function $\mathcal{F}(x)$ is defined as

$$\mathcal{F}(x) = \text{sign}(x) |x|^r, \tag{10}$$

where $0 < r < 1$ is a real number, for $x = (x_1, x_2, \ldots, x_n)^T$, $\mathcal{F}(x) = (\mathcal{F}(x_1), \mathcal{F}(x_2), \ldots, \mathcal{F}(x_n))^T$ and $\text{sign}(x) = (\text{sign}(x_1), \text{sign}(x_2), \ldots, \text{sign}(x_n))^T$.

We can use a recurrent neural network with a one-layer structure to realize the dynamical state equation expressed by (9). The circuit used to realize the neural network has $n+m$ integrators processors, $n+m$ continuous but nonsmooth activation functions processors for z, $n+m$ processors for z^+ and $n+m$

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