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An optimized nonlinear grey Bernoulli model and its applications

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ABSTRACT

Nonlinear grey Bernoulli model (NGBM) is a recently developed grey forecasting model, and this investigation will propose an optimized NGBM(1,1) (ONGBM). ONGBM takes the n-th component of 1-AGO series as initial condition, which is also optimized with a boundary value correction method. Meanwhile, ONGBM adopts a simultaneous optimization approach to calculate the unknown parameters aiming at achieving the minimization of simulated average relative error. One actual forecasting example of foreign exchange rates is chosen for practical test of ONGBM, and the results are encouraging. Subsequently, ONGBM is applied to forecast the traffic flow with nonlinear small sample characteristics. The simulation results demonstrate that ONGBM is feasible and efficient, and it can improve the prediction accuracy and adaptability.

1. Introduction

Time series prediction refers to the process by which the future values of a system is forecasted based on the information obtained from the past and current data points [1], and it has been successfully used in many areas such as stock market forecasting [2], exchange rate forecasting [3], traffic state prediction [4–6], wind farm power prediction [7], power load forecasting [8], and so on. Much effort has been devoted to develop time series forecasting models, and a variety of prediction approaches have been brought forward, which are generally categorized as follows. Traditional statistical models including moving average, exponential smoothing, and AR (Auto Regressive), MA (Moving Average), ARMA (Auto Regressive Moving Average), ARIMA (Auto Regressive Integrated Moving Average) are linear in that predictions of the future values are constrained to be linear functions of past observations. The other category of time series models are nonlinear models which can overcome the linear limitation of time series models [9], including the threshold autoregressive (TAR) model [10], autoregressive conditional heteroscedasticity (ARCH) [11], bilinear model [12], chaotic prediction methods [13,14], fuzzy systems [15], support vector machines (SVMs) [16], etc. In addition, some hybrid models are also proposed to overcome the limitations of each single model and to improve forecasting performances in the literatures [2,9,17].

Accurate forecast is the premise and foundation of scientific decision-making, and most traditional forecasting models are on the basis of a great deal of sample data. But sometimes it is difficult to collect large samples in system studies, and this would restrict the applications of traditional prediction approaches. Grey prediction theory, which was firstly introduced in early 1980 by Prof. Deng, can overcome this limitation as grey models require only a limited amount of data to estimate the behavior of unknown systems. Grey prediction theory is an interdisciplinary scientific area and has rapidly become an excellent tool to deal with systems that have partially unknown parameters during the last three decades [18]. GM(1,1) model and grey Verhulst model are the two most frequently used models in grey predictions, and GM(1,1) model is suitable for the observed data with exponential distributing while the grey Verhulst model is suitable for the observed data with "S" distributing (i.e., saturated distributing) [19]. The two prediction models have drawn sufficient attention, and many improvements on the two models have been published in the literatures [19–26], which mainly focus on modifying parameter estimation methods, optimizing background values and optimizing initial conditions.

Besides, the recently developed nonlinear grey Bernoulli model (NGBM) [27,28] is an important extension of traditional grey predictions. NGBM(1,1) was named by Chen [28], firstly appeared in the book [29]. If the power exponent γ could be selected properly, NGBM(1,1) can reflect the feature of non-linearity in data well and can be used to make predictions for oscillatory sequences. Traditional GM(1,1) model and grey Verhulst model are both special forms of NGBM(1,1) while

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power exponent γ being 0 and 2 respectively. Some scholars have shown great interest in NGBM(1,1). Dr. Wang educed the estimate arithmetic of parameter γ based on the basic principle of information overlap in the grey system [30], and also studied the optimization methods about model parameters [31–34]. Particle swarm optimization (PSO) algorithm and Genetic algorithm were used to solve the power exponent in [35,36] respectively. Pao et al. employed NGBM (1,1) to predict CO_2 emissions, energy consumption, and economic growth in China and proposed a numerical iterative method to optimize the parameter of NGBM(1,1) [37]. All these studies indicate that the forecasting ability of NGBM(1,1) is significantly superior to traditional grey predictions because of its nonlinear adjustable parameter – power exponent γ. Further to improve the forecasting precision, Chen et al. adopted NGBM(1,1) together with the Nash equilibrium concept, thus a novel model named Nash nonlinear grey Bernoulli model (NNGBM) is proposed, and it has two adjustable parameters – power exponent and the background value parameter [38]. In addition, initial condition is also an important factor determining the grey modeling accuracy as it is a part of the predictive function [34]. To adequately apply new information contained in terms, Dang et al. chose the n-th component $x^{(1)}(n)$ of 1-AGO series $\boldsymbol{X}^{(1)}$ as the initial conditions to improve GM(1,1) model and grey Verhulst model [21]. Wang et al. used an undetermined constant to minimize the error in the summed squares for ascertaining the initial condition of NNGBM and also presented a method of using the weighted sum of $x^{(1)}(n)$ and $x^{(1)}(1)$ as the initial condition [34]. Zhang et al. optimized the initial condition $x^{(1)}(1)$ of GM(1,1) with boundary value correction by minimizing the sum of the squared error [23]. Here drawing lessons from [21], we will choose $x^{(1)}(n)$ as the initial condition in NGBM(1,1). And considering the complexity and uncertainty involved, we will further employ boundary value correction method to correct $x^{(1)}(n)$ in NGBM(1,1).

The remainder of this paper is organized as follows. In Section 2, the modeling procedures of NGBM(1,1) is reviewed. Section 3 introduces the optimization method of using boundary value corrected $x^{(1)}(n)$ as initial condition for NGBM(1,1). In Section 4, a simultaneous optimization approach of power exponent and background value is presented. Section 5 adopts an actual forecasting example of foreign exchange rates to demonstrate the improvement in the accuracy of ONGBM. And in Section 6, ONGBM is used to forecast the traffic flow, and the experimental findings are discussed. Finally, Section 7 concludes this paper.

2. Overview of nonlinear grey Bernoulli model NGBM(1,1)

This section reviews the procedures of deriving NGBM(1,1) briefly.

Step 1: Assume that the original nonnegative time series with n entries is:

$$\boldsymbol{X}^{(0)}=(x^{(0)}(1),\quad x^{(0)}(2),\ \cdots,\ x^{(0)}(k)\cdots,\ x^{(0)}(n)),\ n\geq 4, \tag{1}$$

where $x^{(0)}(k)$ is the value of the behavior series at k, $k=1,\ 2,\ \cdots,\ n$.

Step 2: Apply one time accumulated generating operation (1-AGO) to construct $\boldsymbol{X}^{(1)}$:

$$\boldsymbol{X}^{(1)}=(x^{(1)}(1),\quad x^{(1)}(2),\ \cdots,\ x^{(1)}(k)\cdots,\ x^{(1)}(n)), \tag{2}$$

where $x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i)$, $k=1,\ 2,\ \cdots,\ n$.

Step 3: The grey differential equation of the NGBM(1,1) is defined as:

$$x^{(0)}(k)+az^{(1)}(k)=b(z^{(1)}(k))^{\gamma},\ \gamma\neq 1,\ k=2,\ 3,\ \cdots,\ n, \tag{3}$$

where $x^{(0)}(k)$ is the grey derivative, $z^{(1)}(k)=0.5x^{(1)}(k-1)+0.5x^{(1)}(k)$ is referred to the background value of the grey derivative, and a, b are model parameters. γ is the power exponent which belongs to real numbers, and serves as an adjustable parameter [27] which endows NGBM(1,1) model with excellent flexibility.

Step 4: Choose the optimal power exponent. Based on the basic principle of information overlap in the grey system, a calculation method for power exponent γ is presented in [30] as follows:

$$\gamma=\frac{1}{n-2}\sum_{k=2}^{n-1}\gamma(k), \tag{4}$$

$$\begin{aligned}\gamma(k)=\{&[x^{(0)}(k+1)-x^{(0)}(k)]\cdot z^{(1)}(k+1)\cdot z^{(1)}(k)\cdot x^{(0)}(k)\\&-[x^{(0)}(k)-x^{(0)}(k-1)]\cdot z^{(1)}(k+1)\cdot z^{(1)}(k)\cdot x^{(0)}(k+1)\}/\\&\{[x^{(0)}(k+1)]^2\cdot z^{(1)}(k)\cdot x^{(0)}(k)-[x^{(0)}(k)]^2\cdot z^{(1)}(k+1)\cdot x^{(0)}(k+1)\},\\&k=2,\ 3,\ \cdots,\ n-1.\end{aligned} \tag{5}$$

Step 5: Adopt least square method to estimate the model parameters $[a,b]^{\mathrm{T}}$:

$$[a,b]^{\mathrm{T}}=(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B})^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{Y}, \tag{6}$$

where $\boldsymbol{B}=\begin{bmatrix}-z^{(1)}(2) & (z^{(1)}(2))^{\gamma}\\ -z^{(1)}(3) & (z^{(1)}(3))^{\gamma}\\ \vdots & \vdots\\ -z^{(1)}(n) & (z^{(1)}(n))^{\gamma}\end{bmatrix}$, $\boldsymbol{Y}=\begin{bmatrix}x^{(0)}(2)\\ x^{(0)}(3)\\ \vdots\\ x^{(0)}(n)\end{bmatrix}$.

Table 1
The accuracies of forecasting foreign exchange rates.

	RPE(%), Taiwan (NTD/USD)		*RPE*(%), China (mainland) (CNY/USD)		*RPE*(%), Japan (JPY/USD)		*RPE*(%), Euro (EUR/USD)	
	NGBM, $\gamma=0.341$	ONGBM, $\gamma=0.326$, $m=9.9539$	NGBM, $\gamma=0$	ONGBM, $\gamma=-0.0003$, $m=6.4879$	NGBM, $\gamma=0.584$	ONGBM, $\gamma=0.6224$, $m=12.168$	NGBM, $\gamma=0.42$	ONGBM, $\gamma=0.4182$, $m=7313.5$
1999	0	0.0941	0	0.00074	0	0.3014	0	0.42
2000	0.73	0	0.01	0.003	0	0	0.09	0
2001	0.26	0.0022	0.01	0.0034	0.52	0	0.67	0.7082
2002	0	0	0.003	0	2.09	1.6828	2.61	2.6932
2003	0.22	0.3474	0	0.0018	1.32	1.1595	3.78	3.6836
2004	0.02	0.1439	0.003	0.00045	0.10	1.1576	1.45	1.5239
ARPE(%)	**0.25**	**0.0979**	**0.005**	**0.0016**	**0.81**	**0.7169**	**1.72**	**1.5475**