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Consensus of discrete-time third-order multi-agent systems in directed networks

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ABSTRACT

This paper studies the consensus of discrete-time third-order multi-agent systems in directed networks. By using algebraic graph theory and matrix theory, necessary and sufficient conditions for consensus of discrete-time third-order multi-agent systems are established. Compared with existing results, we focus on illustrating how scaling strengths and nonzero eigenvalues of the involved Laplacian matrix have an effect on consensus of the system. Finally, numerical simulations are given to illustrate the theoretical results.

1. Introduction

Multi-agent system is a complex system developed in recent years, which belongs to interdisciplinary subjects involving control, mathematics, biology, physics, computer, robot, communication and artificial intelligence, etc. So far, multi-agent systems have attracted many researchers who are interested in addressing some issues such as consensus, formation control, and flocking, partly due to its broad application in many fields.

Consensus, which can be seen as a collective behavior, is that the state of all agents converges to a common value by implementing an appropriate agreement. Consensus problem has been investigated for a long time in different research fields such as computer science [1], system and control [2,3], formation control [4], flocking [5,6], distributed sensor networks [7], networked Euler–Lagrange systems [8] and so on. The phenomena of consensus are very common in nature such as bird migration, fish cruise, traffic flow. Therefore, consensus problems have become one of the core subjects in social research for the moment.

In the past decade, consensus for first-order multi-agent systems [9–15] was studied extensively. By now, many interesting results have been established. In [9], the authors established one necessary and sufficient condition for consensus of first-order multi-agent systems. Consensus for first-order linear multi-agent systems was studied in [10] where some basic results were given. Under dynamically changing interaction topologies, sufficient conditions for consensus were presented for both continuous-time and discrete-time first-order multi-agent systems in [11]. Shang [12] considered continuous-time average consensus under dynamically changing topologies and multiple time-varying delays. Xiao et al. [13,14] studied asynchronous consensus for continuous-time multi-agent systems and consensus protocols for discrete-time multi-agent systems with time-varying delays. Recently, Xie et al. [15] investigated the group consensus issue of multi-agent systems, where the agents can reach more than one consistent value asymptotically.

For the case of second-order multi-agent system, there are also many interesting results in the literature. For example, Xiao and Chen [16] discussed sampled-date consensus for multiple double integrators with arbitrary sampling. Consensus of second-order discrete-time multi-agent systems with fixed topology was studied in [17]. Ren [18] extended the second-order consensus algorithms to the case of switching topologies and reference models. In [19], Gao et al. studied the consensus problem of discrete-time second-order agents with time-varying topology and time-varying delays, and proposed a method to design controller gains. In addition, some consensus results for second-order discrete-time multi-agents systems with nonuniform time-delays and dynamically changing topologies were presented in [20].

Recently, some people have paid attention to the consensus problem of agents with higher-order dynamics [21–23]. In [21], the authors established necessary and sufficient conditions for consensus in terms of eigenvalues of the system matrix for higher-order multi-agent systems. Consensus of multi-agent systems with

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linear higher-order agents was considered in [22], where the authors answered whether the group converges to consensus and what consensus value it eventually reaches. In [23], the authors proposed a new consensus protocol based on self-state-feedback and neighbor-output-feedback and designed a nonlinear heading consensus protocol for a multi-vehicle model. In [24], the authors considered the consensus problem of continuous-time third-order nonlinear multi-agent systems with a fixed communication topology.

Although some necessary and sufficient conditions for consensus of higher-order multi-agent systems have been established in the literature, it is still not clear how scaling strengths and nonzero eigenvalues of the involved Laplacian matrix have an effect on consensus. Recently, we solved the above problem for the continuous-time third-order multi-agent system in [25]. However, to the best of our knowledge, such a problem is still unresolved for the discrete-time case. Therefore, the main purpose of this paper is to study consensus for discrete-time third-order multi-agent systems in directed network. We will clearly show the equivalent relationship among consensus, scaling strengths and nonzero eigenvalues of the Laplacian matrix when the involved topology graph has a directed spanning tree.

Third-order multi-agent systems have profound physical and engineering meaning. In practice, the derivative (difference) of accelerated speed is usually called Jerk, which is a physical quantity to describe the changing rate of acceleration. A Jerk is often required in engineering, especially in transportation design and materials. Automotive engineers use the Jerk as the index of discomfort degree of passenger. Lorenz turns the butterfly effect into a similar one-dimensional sudden variant motion (that is, the movement which is described by a third-order differential equation with one independent variable) after a special deformation. Consequently, the study on third-order multi-agent systems is necessary and significant. What is more, many control systems are almost high-order systems in control engineering. Especially, third-order systems are very common, whose performance indexes may be very complex. In actual engineering, we need the system should work in a consistent state. Therefore, the results of this paper can be used to sustain agents consensus in engineering.

The rest of this paper is organized as follows. In Section 2, we formulate the problem and give some preliminaries. In Section 3, we present the main results of this paper. Necessary and sufficient conditions for consensus of discrete-time third-order multi-agent systems are presented. In Section 4, numerical simulations are given to illustrate the main result. Finally, conclusions are drawn in Section 5.

Notations: For any complex number s, $\Re(s)$ and $\mathscr{I}(s)$ denote its real and imaginary part, respectively. $\otimes$ denotes Kronecker product. $I_N \in R^{N\times N}$ ($O_N \in R^{N\times N}$) is an $N\times N$-dimensional identity (zero) matrix, and $\mathbf{1}_N=(1,1,\dots,1)^T$ ($\mathbf{0}_N=(0,0,\dots,0)^T$) is the N-dimensional column vector whose elements are 1 (0). $\|\cdot\|$ denotes the Euclidean norm.

2. Preliminaries and problem statements

In this section, we will give the basic concept of algebraic graph, basic matrix theory, some definitions and lemmas. For details, refer to [27–29].

Let $\mathcal{g}=(\nu,\varepsilon,G)$ be a weighted directed graph (network), where the vertex set is $\nu=\{\vartheta_1,\vartheta_2,\dots,\vartheta_N\}$ with $N\ge 3$, the set of directed edges is $\varepsilon=\{\varepsilon_{ij}=(\vartheta_i,\vartheta_j):\vartheta_i,\vartheta_j\in\nu\}\subseteq\nu\times\nu$, and $G=(g_{ij})_{N\times N}$ is called a weighted adjacent matrix. We assume $g_{ii}=0$ for arbitrary $i=1,2,\dots,N$, $g_{ij}>0$ if $\varepsilon_{ji}\in\varepsilon$, and $g_{ij}=0$ otherwise. A directed spanning tree of $\mathcal{g}$ is a directed tree consisting of all the nodes.

As usual, we define the Laplacian matrix $L=(l_{ij})$ as follows:

$$l_{ii}=-\sum_{j\ne i} l_{ij},\quad l_{ij}=-g_{ij}, i\ne j, i,j=1,2,\dots,N.$$

It is obvious that matrix L satisfies $l_{ii}\ge 0$, $l_{ij}\le 0$, $i\ne j$, and $\sum_{j=1}^N l_{ij}=0$ $(i=1,2,\dots,N)$. Therefore, L has at least a zero-eigenvalue whose eigenvector is $\mathbf{1}_N$. In the sequel, μ_i $(i=1,\dots,N)$ denote the eigenvalues of the Laplacian matrix L with $\mu_1=0$.

Now, let us consider a directed network $\mathcal{g}$ with N agents which update their states based on the topology of information exchange among agents described by a directed graph. Each agent in the network satisfies the following discrete-time third-order equation

$$\begin{cases} x_i(k+1)=x_i(k)+v_i(k),\\ v_i(k+1)=v_i(k)+z_i(k),\\ z_i(k+1)=z_i(k)+u_i(k),\end{cases} \tag{2.1}$$

where $k=0,1,2,\dots,x_i(k)\in R^n$, $v_i(k)\in R^n$ and $z_i(k)\in R^n$ represent the state of position, velocity and accelerated speed at time k for $i=1,2,\dots,N$, respectively, $u_i(k)\in R^n$ is the control input or protocol.

In this paper, we consider the following protocol:

$$u_i(k)=\alpha\sum_{j\ne i} g_{ij}(x_j(k)-x_i(k))+\beta\sum_{j\ne i} g_{ij}(v_j(k)-v_i(k))+\gamma\sum_{j\ne i} g_{ij}(z_j(k)-z_i(k)), \tag{2.2}$$

where $\alpha>0$, $\beta>0$ and $\gamma>0$ are scaling strengths to be determined.

Let

$$x(k)=(x_1^T(k),x_2^T(k),\dots,x_N^T(k))^T,$$
$$v(k)=(v_1^T(k),v_2^T(k),\dots,v_N^T(k))^T,$$
$$z(k)=(z_1^T(k),z_2^T(k),\dots,z_N^T(k))^T,$$

and

$$y(k)=(x^T(k),v^T(k),z^T(k))^T.$$

System (2.1) with protocol (2.2) can be equivalently written in a compact matrix form as

$$y(k+1)=(\Gamma\otimes I_n)y(k), \tag{2.3}$$

where

$$\Gamma=\begin{pmatrix} I_N & I_N & O_N\\ O_N & I_N & I_N\\ -\alpha L & -\beta L & I_N-\gamma L\end{pmatrix}.$$

System (2.1) with protocol (2.2) or system (2.3) is said to achieve consensus asymptotically, if $\lim_{k\to\infty}\|x_i(k)-x_j(k)\|=0$, $\lim_{k\to\infty}\|v_i(k)-v_j(k)\|=0$ and $\lim_{k\to\infty}\|z_i(k)-z_j(k)\|=0$ hold for any $i,j=1,2,\dots,N$.

Set

$$\phi(k)=(\phi_1^T(k),\phi_2^T(k),\dots,\phi_{N-1}^T(k))^T,$$
$$\varphi(k)=(\varphi_1^T(k),\varphi_2^T(k),\dots,\varphi_{N-1}^T(k))^T,$$
$$\psi(k)=(\psi_1^T(k),\psi_2^T(k),\dots,\psi_{N-1}^T(k))^T,$$

and

$$\omega(k)=(\phi^T(k),\varphi^T(k),\psi^T(k))^T$$

, where

$$\phi_i(k)=x_{i+1}(k)-x_1(k),$$
$$\varphi_i(k)=v_{i+1}(k)-v_1(k),$$
$$\psi_i(k)=z_{i+1}(k)-z_1(k),\quad i=1,2,\dots,N-1.$$

We can obtain the following reduced-order form of system (2.3):

$$\omega(k+1)=(F\otimes I_n)\omega(k), \tag{2.4}$$