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Global exponential stability of high-order bidirectional associative memory (BAM) neural networks with time delays in leakage terms

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ABSTRACT

High-order bidirectional associative memory (BAM) neural networks with time delays in leakage terms play an increasingly important role in the design and implementation of neural network systems. Based on Lyapunov–Krasovskii functional and linear matrix inequality technique, issues of global exponential stability for high-order BAM neural networks with time delays in leakage terms are investigated. The proposed results, which do not require the boundedness, differentiability and monotonicity of the activation functions. In addition, the stability criteria that depend on the leakage time delays and our results are presented in terms of LMIs, which can be efficiently solved via a standard numerical package. Three numerical examples are provided to demonstrate the effectiveness of the proposed approach.

1. Introduction

The bidirectional associative memory (BAM) neural networks, first proposed and researched by Kosko [1–3], consist of neurons in two layers, the ***U***-layer and the ***V***-layer. The neurons in one layer are fully interconnected to the neurons arranged in the other layer, while there are no interconnect among neurons in the same layer. Through iterations of forward and backward information flows between the two layers, the BAM neural networks perform a two-way associative search for stored bipolar vector pairs. BAM neural networks have been extensively studied and applied to many applications such as signal processing, pattern recognition, associative memories, and optimization problems, to name a few [4–12]. Since the stability is the first requirement in modern control theories, most applications of neural networks heavily depend on the stability of the equilibrium point of BAM neural networks [13–15].

It is well known that first-order neural networks are shown to have low storage capacity and weak error-correcting capability when used in pattern recognition problems and optimization problems [16]. Therefore, Simpson [17] introduced higher-order BAM neural networks, especially second-order BAM neural networks which can increase the storage capacity at the expense of more connections. From then on, different architectures with high-order interactions have been used to design desirable neural networks with stronger approximation property, faster convergence rate, greater storage capacity and higher fault tolerance than low-order neural networks [18–20]. In the past years, high-order neural networks have been successfully applied in many areas, such as biological science, pattern recognition and optimization. These applications are largely dependent on the stability of the equilibrium point of high-order neural networks. There is increasing need to study the stability of the equilibrium point of high-order neural networks. Many results on high-order neural networks have been reported in the literature [21–27].

The time delays are inevitably encountered in both neural processing and signal transmission due to the limited bandwidth of neurons and amplifiers [28]. This may cause undesirable dynamic network behaviors such as oscillation and instability [29]. Moreover, it should be likewise noted that a typical time delay called as Leakage (forgetting) delay may exist in the negative feedback terms of the neural network system and it has a great impact on the dynamic behaviors of delayed neural networks [30]. Time-delay in the leakage term has a great impact on the dynamics of neural networks since time-delay in the stabilizing negative feedback term has a tendency to destabilize a system. Therefore leakage delays must have a destabilizing influence on the dynamical behaviors of high-order neural networks [31–36]. This is to say, it is necessary to consider the effect of leakage delays when studying the stability of high-order neural networks. In [30],

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investigators obtained stability criteria for BAM neural networks with leakage delays and probabilistic time-varying delays in terms of the Lyapunov–Krasovskii functional and stochastic analysis approach. In [33], the global exponential stability is investigated for a class of BAM neural networks with time-varying delays in the leakage terms by using Lyapunov functional theory. In [34], researchers studied the stability of mixed recurrent neural networks with time delay in the leakage terms under impulsive perturbations. Some sufficient conditions to ensure the system to be stable were derived. In [36], the investigators focus on the existence and exponential stability of recurrent neural networks with time delays in the leakage terms under impulsive perturbations by utilizing Lyapunov–Krasovskii functional and *M*-matrix theory. However, few researches investigate the exponential stability of high-order BAM neural networks with leakage delays. There exists a high demand for further characterization of the dynamics of high-order BAM neural networks with leakage delays. On the other hand, in most papers [21], the activation functions have to meet the requirements of bounded. However, in some applications, unbounded of activation functions are needed. This leads to our present research.

Motivated by the above discussions, this paper aims to analyze the exponential stability for high-order BAM neural networks with leakage delays by constructing suitable new Lyapunov functional which combining double integral term and leakage term. It is worth noting that, the proposed results, which do not require the boundedness, differentiability and monotonicity of the activation functions. Hence, the results are less conservative.

The rest of this paper is organized as follows. The model description and some preliminaries are given in Section 2. In Section 3, sufficient criteria are proposed for the global exponential stability of the equilibrium point of the high-order BAM neural networks with leakage delays. In Section 4, three examples are given to demonstrate the effectiveness of the proposed approach. Finally, conclusions are given in Section 5.

Notations: Throughout this paper, let $\mathbb{R}$ denote the set of real numbers. A^{T} and A^{-1} represent the transpose and inverse of the matrix of A, respectively. For matrix $P>0(<0)$ means that P is a positive- (negative-) definite matrix. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ represent the largest eigenvalue and smallest eigenvalue of matrix $(\cdot)$, respectively.

2. Model description and preliminaries

Consider the high-order BAM neural networks with time delays in the leakage terms for the entire time interval described by the following delay differential equations:

$$\begin{cases} \dot{u}_i(t) = -a_i u_i(t-h) + \sum_{j=1}^{m} h_{ij}\tilde{g}_j(v_j(t-\tau_j)) + \sum_{j=1}^{m}\sum_{l=1}^{m} e_{ijl}\tilde{g}_j(v_j(t-\tau_j))\tilde{g}_l(v_l(t-\tau_l)) + I_i, \\ \dot{v}_j(t) = -d_j v_j(t-\rho) + \sum_{i=1}^{n} p_{ji}\tilde{f}_i(u_i(t-\delta_i)) + \sum_{i=1}^{n}\sum_{l=1}^{n} q_{jil}\tilde{f}_i(u_i(t-\delta_i))\tilde{f}_l(u_l(t-\delta_l)) + J_j. \end{cases} \quad (1)$$

where $i=1,2,\cdots,n; j=1,2,\cdots,m; u_i(t)$ and $v_j(t)$ are activations of the neuron i and j, respectively; $\tilde{f}_i$ and $\tilde{g}_j$denote the activation functions of the neurons; τ_j and δ_i correspond to the transmission delays and satisfy $0 \le \tau = \max_{j=1,2,\cdots,m}\{\tau_j\}$ and $0 \le \delta = \max_{i=1,2,\cdots,n}\{\delta_i\}$, respectively; the leakage delays $h \ge 0$, $\rho \ge 0$ are constants; $a_i>0$ and $d_j>0$ denote the rates with which the neuron i and j reset their potentials to the resting state when isolated from the other neurons and inputs; h_{ij}, p_{ji}, e_{ijl} and q_{jil} are the first- and the second-order connection weights of the neural networks, respectively. I_i and $J_j$are the ith and jth component of an external input source introduced from outside network to the neuron i and j at time t, respectively.

The initial conditions associated with system (1) are given as

$$\begin{cases} \mathrm{u_i(s)} = \phi_{ui}(\mathrm{s}), & \mathrm{s} \in [-\tau^*, \ 0], \ i=1,2,\cdots,n, \\ \mathrm{v_j(s)} = \phi_{vj}(\mathrm{s}), & \mathrm{s} \in [-\delta^*, \ 0], \ j=1,2,\cdots,m, \end{cases} \quad (2)$$

where $\tau^* = \max\{\tau, h\}, \delta^* = \max\{\delta, \rho\}$, $\phi_{ui}(\cdot)$ and $\phi_{vj}(\cdot)$ denote the real-valued continuous functions defined on $[-\tau^*, 0]$ and $[-\delta^*, 0]$, respectively.

One assumption which system (1) meets is reported as follows.

Assumption 1. : There exist constants l_i^-, l_i^+, z_j^-, z_j^+, such that the neuron activation function satisfies:

$$l_i^- \le \frac{\tilde{f}_i(x) - \tilde{f}_i(y)}{x-y} \le l_i^+, \quad z_j^- \le \frac{\tilde{g}_j(x) - \tilde{g}_j(y)}{x-y} \le z_j^+.$$

for any $x, \ y \in \mathbb{R}, \ x \ne y, \ i=1,2,\cdots,n, \quad j=1,2,\cdots,m.$

Remark 1. : The constants l_i^-, l_i^+, z_j^-, z_j^+ are allowed to be positive, negative or zero. Hence, activation functions may be non-monotonic. Thus, the assumptions on activation functions in this paper are weaker than those generally used in the literature [21,24–27,35].

From Assumption 1, it is not difficult to prove the existence of equilibrium point of the system (1) by using Brouwer's fixed point theorem [37,38]. In the following, we shall analyze the asymptotic stability of the equilibrium point, which implies the uniqueness of the equilibrium point.

Let (u^*, v^*), $(u^* = (u_1^*, u_2^*, \cdots, u_n^*)^{\mathrm{T}}, v^* = (v_1^*, v_2^*, \cdots, v_m^*)^{\mathrm{T}})$ be the equilibrium point of (1). For simplicity, we can shift (u^*, v^*) to the origin by letting $x_i(t) = u_i(t) - u_i^*$, $y_j(t) = v_j(t) - v_j^*$, $g_j(y_j(t)) = \tilde{g}_j(y_j(t) + v_j^*) - \tilde{g}_j(v_j^*)$ and $f_i(x_i(t)) = \tilde{f}_i(x_i(t) + u_i^*) - \tilde{f}_i(u_i^*)$. The system (1) can then be written