



Novel results on robust finite-time passivity for discrete-time delayed neural networks



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ABSTRACT

This paper presents some novel results on robust finite-time passivity for a class of uncertain discrete-time neural networks (DNNs) with time varying delays. Using the Lyapunov theory together with the zero inequalities, convex combination and reciprocally convex combination approaches, we propose the sufficient conditions for finite-time boundedness and finite-time passivity of DNN for all admissible uncertainties. The results are achieved by using a new Lyapunov-Krasovskii functional (LKF) with novel triple summation terms, several delay-dependent criteria for the DNN are derived in terms of linear matrix inequalities (LMIs) which can be easily verified via the LMI toolbox. Finally, numerical example with simulation scheme have been presented to illustrate the applicability and usefulness of the obtained results.

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1. Introduction

Artificial neural networks (ANNs) such as Hopfield neural networks, cellular neural networks, Cohen–Crossberg neural networks and bidirectional associative memory neural networks have been extensively studied in the fast few decades. ANNs have attracted great attention due to its due to their potential applications in various practical engineering problems such as image processing, signal processing, pattern recognition, combinatorial optimization, and associative memory design. These applications heavily depend on determining the equilibrium and qualitative behaviors of the equilibrium point of the designed neural networks (NNs). A great deal of attention has been devoted to NNs and various types of dynamical behaviors like stability, passivity, control design are discussed in [2,13,17,19] and references therein. In the recent years, research on discrete-time ANNs has received much attention of researchers because it will be helpful to formulate an analogue of the continuous-time delayed neural network for the purposes of computer simulation and digital implementation, for details see [4,32].

In practice, it is well known that time delays are inevitable in practical systems, could degrade the performance of the system

and even cause sustained oscillations, such as bifurcation or chaos [7]. Therefore, it is necessary to consider time delays to ANNs so that the mathematical models under investigation may provide way to investigate many more practical applications. Further, the connection weights in the ANN model are inherent dependent on certain resistance and capacitance values that brings the uncertainty in the course of parameter identification process. In real world systems uncertainties often exist in most biological and engineering systems and may cause undesirable dynamic network behaviors. So, it is important to study the dynamical behaviors of ANNs by taking the time delay and uncertainty into account, see [1,15,23]. In [12], the authors studied the robust global exponential stability of recurrent neural networks subjected to time delays and random disturbances. The existence results and exponential stability of anti-periodic solutions of BAM neural networks with inertial term and delay has been investigated in [27].

On the other hand, the passivity theory is one of the effective tools to prove the stability analysis of nonlinear system. The passivity context has its roots in the circuit theory; the main idea of passivity theory is that the internal stability of system under investigation can be achieved by the passivity properties. Also, it is noted that using different supply functions, the passive theory can be established for various approaches including \mathcal{H}_∞ control, strictly input (or output) passivity and positive realness [25]. Passivity results for various systems like neural networks, chaotic systems, linear systems, switched systems have been proposed in

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the existing literature, for example; Exponential passivity results for memristor based neural networks with time delays has been analyzed in [22]. The delay-dependent exponential passivity of uncertain cellular neural networks with discrete and distributed time-varying delays was proposed in [6]. Some relaxed passivity conditions for neural networks with time-varying delays has been studied in [29]. Delay-dependent passivity and passification for uncertain Markovian jump switched systems with time-varying delay has been investigated in [28].

However, it should be mentioned that all these existing studies about the passivity analysis are performed using the conventional Lyapunov asymptotic stability theory, which is defined over the infinite-time interval. But in many practical applications, the transient behavior of system is concerned over a fixed finite-time interval, in which the system states need to grip below a prescribed upper bound and larger values are not permitted during this time-interval [31]. In these circumstances, it is important to check that these unacceptable values are not attained by the system state; for these purposes finite-time analysis would be very much useful. The concept of finite-time (or short-time) analysis problem was first proposed by Dorato in 1961 [5]. The examples of systems which work in a short time interval are communication network system, missile system and robot control system and etc. In recent years; many biologists are focusing on the transient values of the actual network states. Many interesting results for finite-time stability of various types of systems can be found in [9,18,24,30]. Recently, extended finite-time \mathcal{H}_∞ control problem for uncertain switched linear neutral systems with time-varying delays has been investigated in [21]. In [8] the finite-time \mathcal{H}_∞ control problem is investigated for discrete-time genetic regulatory networks with random delays. The results for finite-time stabilization of neural networks with discontinuous activations has been proposed in [10]. The sufficient conditions for stability, practical stability and boundedness of impulsive differential systems was studied in [20].

Although many literatures are available on passivity results for discrete-time neural networks, it is important to study the finite-time analysis, to make sure the large values of states cannot be accepted in the presence of saturations. Motivated by this, in this paper, we consider finite-time boundedness and passivity of uncertain discrete-time neural networks with time-delays. Different from the existing works, we used new type of LKF to handle the given range of time delay interval together with free weighting matrix approach to drive the main results. Our main contributions are highlighted as follows:

- The finite-time passivity result for discrete-time neural networks with uncertainty and time-varying delays is proposed for the first time.
- New type of zero inequalities are introduced and reciprocally convex combination approach is used to handle the triple summation terms for the first time.
- Delay-dependent results for finite-time boundedness and finite-time passivity are derived by using finite-time stability method and Lyapunov-Krasovskii functional approach.
- Sufficient conditions are proposed in terms of LMIs, which can be solved by using standard numerical packages.

The remainder of the paper is organized as follows. Section 2 presents the problem formulation and mathematical preliminaries. In Section 3, we present the finite-time boundedness and finite-time passivity of considered model. Section 4 presents a numerical example to validate the proposed results. Finally, the paper is concluded in Section 5.

Notations: The notations used in this paper are fairly standard. Throughout this paper, the superscripts “ T ” and “ $(-)$ ” stand for

matrix transposition and matrix inverse respectively. \mathbb{R}^n denotes the n -dimensional Euclidean space. $\mathbb{R}^{n \times n}$ denotes the set of all $n \times n$ real matrices. $P > 0$ means that P is positive definite. I_n and 0_n represent identity matrix and zero matrix with compatible dimension. In symmetric block matrices or long matrix expressions, we use an asterisk (*) to represent a term that is induced by symmetry. $\text{diag}\{\cdot\}$ stands for a block-diagonal matrix. $\text{sym}(A)$ is defined as $A + A^T$. $l_2(0, \infty)$ denotes the space of square summable infinite vector sequences. The notation $\|\cdot\|$ stands for the usual $l_2(0, \infty)$ norm.

2. Problem formulation and preliminaries

Consider the following uncertain discrete-time neural network with time-varying delays described by

$$x(k+1) = A(k)x(k) + B(k)f(x(k)) + C(k)f(x(k-\tau(k))) + v(k) \quad (1)$$

$$y(k) = B_y f(x(k)) + C_y f(x(k-\tau(k))), \quad (2)$$

$$x(k) = \phi(k) \quad \text{for every } k \in [-\tau_M, 0],$$

where $x(k) \in \mathbb{R}^n$ is the neural state vector, $v(k)$ is the exogenous disturbance input vector belongs to $\mathcal{L}_2[0, \infty)$ and $y(k)$ is the output vector of the neural network, $f(x(k))$ is the neuron activation function, the positive integer $\tau(k)$ denotes the time-varying delay satisfying $\tau_m \leq \tau(k) \leq \tau_M$ for all $k \in N$, where τ_m and τ_M are constant positive scalars representing the minimum and maximum delays respectively, $A(k) = A + \Delta A(k)$, $B(k) = B + \Delta B(k)$, $C(k) = C + \Delta C(k)$, in which $A = \text{diag}\{a_1, a_2, \dots, a_n\}$ represents the state feedback coefficient matrix with $|a_i| < 1$, $B = [b_{ij}]_{n \times n}$, $C = [c_{ij}]_{n \times n}$, respectively, the connection weights, and the delayed connection weights, the initial function $\phi(k)$ is continuous and defined on $[-\tau_M, 0]$. Further the parameter uncertainties $\Delta A(k)$, $\Delta B(k)$ and $\Delta C(k)$ are described as follows :

$$[\Delta A(k) \ \Delta B(k) \ \Delta C(k)] = EF(k)[E_a \ E_b \ E_c] \quad (3)$$

where E , E_a , E_b , and E_c are known constant matrices of appropriate dimensions and $F(k)$ is an unknown time-varying matrix with Lebesgue measurable elements bounded by $F^T(k)F(k) \leq I$.

In this paper, we make following assumptions:

(A1). For any $i = 1, 2, \dots, n$ there exist constants F_i^- and F_i^+ , such that

$$F_i^- \leq \frac{f_i(x_1) - f_i(x_2)}{x_1 - x_2} \leq F_i^+, \quad \text{for all } x_1, x_2 \in \mathbb{R}, \quad x_1 \neq x_2.$$

For presentation convenience, in the following, we denote $F_1 = \text{diag}\{F_1^- F_1^+, \dots, F_m^- F_m^+\}$, $F_2 = \text{diag}\left\{\frac{F_1^- + F_1^+}{2}, \dots, \frac{F_n^- + F_n^+}{2}\right\}$.

(A2). The disturbance input vector $v(k)$ is time-varying and for a given $\nu > 0$, satisfies $v^T(k)v(k) \leq \nu$.

Remark 2.1. Assumption (A1) on nonlinear activation function was first proposed by [11]. The constants F_i^- , and F_i^+ , $i = 1, 2, \dots, n$ in (A1) are allowed to be positive or negative or zero. Therefore, the resulting activation function could be non-monotonic, and are more general than the usual sigmoid functions and commonly used Lipschitz conditions in existing literature. These kind of functions will be useful in many real time systems, for example, in electronic circuits where the input-output functions of amplifiers may be neither monotonically increasing nor continuously differentiable.

The following definitions and lemmas will be used in the proof of main results.

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