Neurocomputing 177 (2016) 594–599

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Optimal topology for consensus of heterogeneous multi-agent systems ☆

Huaizhu Wang [a,b], Jingying Ma [a]

[a] School of Mathematics and Computer, Ningxia University, Yinchuan 750021, PR China
[b] Key Laboratory of Communication and Information System, Beijing Jiaotong University, Beijing 100044, PR China

A R T I C L E I N F O

Article history:
Received 7 August 2015
Received in revised form
9 October 2015
Accepted 23 November 2015
Communicated by Yang Tang
Available online 12 December 2015

Keywords:
Heterogeneous multi-agent system
Consensus
Linear-quadratic regulator
Optimal topology

A B S T R A C T

Consensus can be achieved under various topologies for multi-agent systems. Then, which one is the optimal? In this paper, we consider the problem of optimal topology for consensus of heterogeneous multi-agent systems. We assume that the system consists of a leader and several followers with heterogeneous dynamics. Firstly, we define a quadratic cost function composed of consensus error and control effort for the heterogeneous multi-agent system. Secondly, by using linear-quadratic regulator theory and matrix theory, we prove that the optimal position topology and the optimal velocity topology are star graphs. Finally, simulations are carried out to illustrate the effectiveness of the theoretical results.

1. Introduction

In the last decade, more and more attentions have been paid on distributed control of multi-agent systems (MASs) from different fields including system control, physics, biology, communication, computer science and social science [1–3], etc. Consensus means that a group of agents achieve an agreement or reach a common value. As a fundamental issue, consensus problem of MASs has attracted increasing attentions. Consensus problem of first-order multi-agent systems is primarily studied. Vicsek et al. [1] proposed a system consisting of n agents and demonstrated by simulation that all agents asymptotically moved to one direction with the same speed. Jadbabaie et al. [4] gave a theoretical explanation for the consensus behavior in Vicsek model by using graph theory. With the rapid development of issue, extensive studies and results were provided with different models and protocols by single integrator, to name but a few, consensus problem with time-delay [5], consensus of switched multi-agent systems [6], finite-time consensus problem [7] and leader-following consensus problem [8]. Since second-order dynamics exist widely in practice, Xie and Wang [9] studied the consensus problem of second-order multi-agent systems with fixed and switching topologies. In [10], finite-time consensus problem was investigated for second-order multi-agent systems. Some sufficient conditions of achieving finite-time consensus were obtained.

The above results considered homogeneous multi-agent systems where all agents have the same dynamics behavior. However, the dynamics of agents coupled with each others are different because of various restrictions or the common goals with mixed agents in practical systems. This triggers researchers to focus on distributed control of heterogeneous multi-agent systems where the dynamics of agents are different. In [11], Zheng et al. investigated the consensus problem of the heterogeneous multi-agent system which is composed of first-order and second-order integrator agents. For linear consensus protocol and saturated consensus protocol, they obtained some sufficient conditions of asymptotically solving consensus problem. Moreover, necessary and sufficient condition of solving the consensus was developed for heterogeneous multi-agent systems with directed networks [12]. Finite-time consensus problem and containment control problem of heterogeneous multi-agent systems were also investigated in [13] and [14], respectively. Liu et al. [15] studied second-order consensus of multi-agent systems with heterogeneous nonlinear dynamics and time-varying delays.

Existing results have shown that consensus can be achieved under various topologies [4,9]. Therefore, it is natural to investigate reaching consensus in an optimal way. So far, fast consensus problem and finite-time consensus problem have been studied by many researchers [5,7,13,16,17]. In addition, the optimal consensus problem was also studied from linear-quadratic regulator (LQR)

☆This work is supported by NSFC (Grant nos. 61562069 and 61563043) and the Natural Science Foundation of Ningxia (Grant no. NZ14022).
E-mail address: majy1980@126.com (J. Ma).

http://dx.doi.org/10.1016/j.neucom.2015.11.076
0925-2312/

perspective. Based on LQR theory, Cao and Ren [18] proved that the optimal consensus protocol for leaderless multi-agent systems corresponds to a complete graph. For multi-agent systems with a leader, Ma et al. [19] obtained the optimal topology of the consensus problem for the first-order MASs and second-order MASs, respectively. Meanwhile, optimal topology problem for containment control was further investigated. In [20], the authors investigated the optimal topology problem of multi-agent systems with two competitive leaders from a zero-sum game perspective. However, the existing studies only considered the optimal topology problem of multi-agent systems where all agents have the same dynamics behavior.

Motivated by above results, we consider the optimal topology problem for the heterogeneous multi-agent system. In other words, which interaction topology is the optimal under a distributed linear protocol of heterogeneous multi-agent systems? Optimality here refers to minimization of a linear quadratic cost function which is a balance between the global consensus error and the control effort. Since the system consists of first-order and second-order integrator followers and a leader, there exists position and velocity information exchanging. As a result, we model the information interaction of system as a position topology and a velocity topology. By employing LQR theory and matrix theory, we prove that the optimal position topology and the optimal velocity topology are star graphs. The results of this paper give a theoretical explanation on this fact: when all followers can directly access the information from the leader, the information exchange among followers is inefficient and uneconomical. Moreover, we give a unified framework and obtain a generalized extension for the optimal topology of multi-agent systems proposed in [19].

This paper is organized as follows. In Section 2, we introduce graph theory and infinite-time LQR theory. The main results are given in Section 3. And in Section 4, numerical simulations are carried out to illustrate the effectiveness of our results. Some conclusions are drawn in Section 5.

Throughout this paper, the following notations will be used: let $\mathbb{R}$ be the set of real numbers. $\mathbb{R}^{n\times m}$ is the set of $n\times m$ real matrices. Denote by $\mathbf{0}_{n\times m}$ an $n\times m$-dimensional matrix with all entries equal to zeros. I_n denotes an n-dimensional identity matrix. For a column vector $\mathbf{b}=[b_1,b_2,\ldots,b_n]^T$, diag$\{\mathbf{b}\}$ is a diagonal matrix with b_i, $i=1,\ldots,n$, on its diagonal. For two sets S_1 and S_2, denote $S_1\backslash S_2=S_1-S_2$.

2. Preliminaries

In this section, we give some basic notions of algebraic graph theory and infinite-time LQR theory which will be used in this paper. For more details, please refer to [21] and [22], respectively.

2.1. Graph theory

Let $G=\{V,E,A\}$ be a weighted directed graph consisting of a vertex set $V=\{1,2,\ldots,n\}$, an edge set $E=\{(i,j)\in V\times V\}$ and an adjacency matrix $A=[a_{ij}]\in\mathbb{R}^{n\times n}$. The adjacency matrix A of G is defined such that for all $i\in V$, $a_{ii}=0$ and for all $i\neq j$, $(j,i)\in E$ if and only if $a_{ij}>0$, while $a_{ij}=0$ otherwise. Denote by $\mathcal{N}_i=\{j:a_{ij}>0\}$ the neighbor set of the agent i. The degree matrix $D\in\mathbb{R}^{n\times n}=$ diag$\{\sum_{j\in\mathcal{N}_1}a_{1j},\ldots,\sum_{j\in\mathcal{N}_n}a_{nj}\}$ and the Laplacian matrix $L=D-A$. A graph is called star graph if its edge set $E=\{(i_0,j),i_0\in V,j\in V\backslash\{i_0\}\}$. It is obvious to see that the Laplacian matrix of a star graph has the following structure:

$$L=\begin{bmatrix}\mathrm{diag}\{\mathbf{b}_1\} & -\mathbf{b}_1 & 0_{(i_0-1)\times(n-i_0)}\\ 0_{1\times(i_0-1)} & 0 & 0_{1\times(n-i_0)}\\ 0_{(i_0-1)\times(n-i_0)} & -\mathbf{b}_2 & \mathrm{diag}\{\mathbf{b}_2\}\end{bmatrix}\ \text{the } i_0-\text{th row},\quad(\text{the } i_0-\text{th column})$$

where $\mathbf{b}_1$ and $\mathbf{b}_2$ are (i_0-1)–dimensional and $(n-i_0)$–dimensional positive vectors, respectively.

2.2. Infinite-time linear quadratic regulator theory

Consider a linear system as follows:

$$\dot{X}(t)=GX(t)+HU(t)\tag{1}$$

where $X(t)\in\mathbb{R}^n$, $U(t)\in\mathbb{R}^m$, $G\in\mathbb{R}^{n\times n}$ and $H\in\mathbb{R}^{n\times m}$. Let $Q\in\mathbb{R}^{n\times n}$ and $R\in\mathbb{R}^{m\times m}$ be symmetric, and be nonnegative and positive definite, respectively. Define the optimal control problem

$$\min_{U(t)}J(U(.),X(0))=\int_0^\infty\left[X^T(t)QX(t)+U^T(t)RU(t)\right]dt \quad \text{subject to}\quad \dot{X}(t)=GX(t)+HU(t).\tag{2}$$

Optimal control problem (2) is to find an optimal control $U^*(t)=-K^*X(t)$ minimizing $J(U(.),X(0))$, where K^* is called the optimal feedback gain matrix [22]. In other words, optimal control can drive the state close to the equilibrium state without any excessive cost.

Lemma 1 (*Anderson and Moore [22]*). *Suppose that system (1) is completely controllable or completely stabilizable. Then the algebraic Riccati equation (ARE)*

$$G^TP+PG+Q-PHR^{-1}H^TP=0$$

has a unique positive-definite solution. Furthermore, the optimal control of (2) $U^*(t)=-R^{-1}H^TPX(t)$ minimizes the cost function $J(U(.),X(0))$ and drives the system to achieve asymptotic stability.

3. Main results

Consider a heterogeneous multi-agent system which is composed of a leader, m second-order integrator followers and $n-m$ first-order integrator followers. Let $V^1=\{1,2,\ldots,m\}$ be the set of second-order integrator followers and $V^2=\{m+1,\ldots,n\}$ be the set of first-order integrator followers. Each second-order follower dynamics is given as follows:

$$\begin{cases}\dot{x}_i(t)=v_i(t),\\ \dot{v}_i(t)=u_i(t),\quad i\in V^1,\end{cases}\tag{3}$$

where $x_i\in\mathbb{R}$, $v_i\in\mathbb{R}$ and $u_i\in\mathbb{R}$ are the position, velocity and control input, respectively, of agent i. Each first-order follower dynamics is given as follows:

$$\dot{x}_i(t)=u_i(t),\quad i\in V^2\tag{4}$$

where $x_i\in\mathbb{R}$ and $u_i\in\mathbb{R}$ are the position and control input of agent i, respectively. The leader's dynamics is

$$\dot{x}_{n+1}(t)=0.\tag{5}$$

Denote the followers' position states and the velocity states by $x=[x_1,\ldots,x_n]^T$ and $v=[v_1,\ldots,v_m]^T$, respectively.

Definition 1. Heterogeneous multi-agent system (3)–(5) is said to reach consensus if for any initial states of $x(0)$, $v(0)$ and $x_{n+1}(0)$, we have

$$\lim_{t\to\infty}|x_i(t)-x_{n+1}(t)|=0,\quad \text{for } i\in V^1\cup V^2,$$