Available online at www.sciencedirect.com
ScienceDirect
Neurocomputing 71 (2008) 1032–1038
NEUROCOMPUTING
www.elsevier.com/locate/neucom

Asymptotic behavior of discrete solutions to delayed neural networks with impulses ☆

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Received 19 May 2006; received in revised form 6 September 2006; accepted 11 November 2006
Communicated by J. Cao
Available online 20 February 2007

Abstract

In this paper, we formulate and study a new class of discrete neural networks with time dependent delay and impulses. Sufficient conditions for the asymptotic stability of a unique equilibrium of the networks with Lipschitizian activation functions are established. Also when the impulsive jumps are absent the results reduce to those of non-impulsive networks.

MSC: primary 34K45; 39A11; 92B20

Keywords: Hopfield networks; Discrete-time analogues; Impulse; Asymptotic stability

1. Introduction

Mathematical modelling in neural networks has been based on neurons that are different both from real biological neurons and from the realistic functioning of simple electronic circuits. The neuronal model is made up of four basic components: an input vector, a set of synaptic weights, summing function with an activation, or transfer function, and an output. From the view point of mathematics, an artificial neural network corresponds to a nonlinear transformation of some inputs into certain outputs. In the past three decades, neural networks architectures have been extensively researched and developed [2,4–13,15–24,26,29–33,36–39,41–49]. Among the many classes of neural networks studied intensively during the last two decades, the Hopfield-type network [21,22] is an important one because of its potential for applications in associative memory, pattern recognition, signal processing, systems control, data compression, optimization problem, etc.

Most neural networks can be classified as either continuous and discrete. Recently, there have been many nice works on the continuous or piecewise continuous neural networks [1,15,46,12,23,31,29,32]. However, there are still many networks existing in the real world which display some kind of dynamics in between the two groups. These include for example many evolutionary processes, particularly some biological systems such as biological neural networks and bursting rhythm models in pathology. Other examples include optimal control models in economics, frequency-modulated signal processing systems, and flying object motions. All these real-world systems and natural processes behave in discrete or continuous style interlaced with instantaneous and abrupt changes. Systems with impulsive effects describe evolution processes which at certain moments rapidly change their state [28,3]. Impulsive differential equations are found in almost every domain of applied science, see for example [14,27,35,40,34,25].

It is important and, in fact, necessary to study impulsive systems. This paper is an attempt toward this goal. Specifically, in this work we research a new type of neural

☆This work was supported by the Natural Science Foundation of China under Grant 10471117.
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0925-2312/$ - see front matter
doi:10.1016/j.neucom.2006.11.022

networks—impulsive neural networks—as an appropriate description of such phenomena of abrupt dynamical changes of essentially discrete systems. The dynamical characteristics of the Hopfield network are assumed to be governed by the dynamics of the following ordinary differential equations:

$$\begin{cases} \mu_i \dfrac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -\dfrac{x_i(t)}{R_i} + \sum_{j=1}^{m} T_{ij} f_i(x_j(t)) + I_i, \\ \quad t > t_0 \in \mathbb{R}, \quad i = 1, 2, \ldots, m, \\ x(t_0) = x_0 \in \mathbb{R}^m, \end{cases} \tag{1}$$

where m is the number of neurons in the network; $x_i(t)$ is the average membrane potential of the ith neuron at time t; μ_i denotes the capacitance in the ith sub-circuit; f_i is the activation function of the ith neuron; T_{ij} is the connection strength from the jth neuron to the ith neuron; I_i is a constant external input current to the ith neuron; R_i denotes the resistance defined by $1/R_i = 1/r_i + \sum_{j=1}^{m} |T_{ij}|$ where r_i denotes the resistance representing the cell membrane impedance.

In recent years, neural networks supplemented with impulse conditions in the continuous-time case or discrete-time case have been investigated [2,15,29,42–45]. In the following we consider the system:

$$\begin{cases} \dfrac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -a_i u_i(t) + \sum_{j=1}^{m} b_{ij} g_j(u_j(t - \tau(t))) + c_i, \\ \quad t > 0, \quad i = 1, 2, \ldots, m, \\ u_i(0) = u_{i0} \in \mathbb{R}^m. \end{cases} \tag{2}$$

Next we formulate two discrete-time analogues of the continuous-time system (2). Let h be an arbitrary positive fixed real number. The semi-infinite interval $[0, \infty)$ is replaced by the interval $[[0/h]h, \infty)$ where $[r]$ denotes the integer part of $r \in \mathbb{R}$. We let $n_0 = [0/h]$ and consider the sequence $\{n_0 h, (n_0+1)h, (n_0+2)h, \ldots\}$ as a discrete set of points of the interval $[0, \infty)$. We replace the system (2) by the following differential equation with piecewise constant arguments

$$\begin{cases} \dfrac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -a_i u_i(t) + \sum_{j=1}^{m} b_{ij} g_j\left(u_j\left(\left[\dfrac{t}{h}\right]h - \left[\tau\left(\left[\dfrac{t}{h}\right]h\right)\right]\right)\right) + c_i, \\ \quad t > \left[\dfrac{0}{h}\right]h, \ i = 1, 2, \ldots, m, \\ u_i(0) = u_{i0} \in \mathbb{R}^m. \end{cases} \tag{3}$$

We let $n = [t/h]$, and use the notation $u_j(n) = u_j(nh)$. Thus we consider the following system:

$$\frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -a_i u_i(t) + \sum_{j=1}^{m} b_{ij} g_j(u_j(n - [\tau(n)])) + c_i,$$
$$t \in [nh, (n+1)h), \quad n = 1, 2, \ldots. \tag{4}$$

By using a semi-implicit Euler type approximation, we derive the following difference equation:

$$u_i(n+1) = \frac{1}{1 + a_i h} u_i(n) + \frac{h}{1 + a_i h} \sum_{j=1}^{m} b_{ij} g_j(u_j(n - [\tau(n)])) + \frac{c_i h}{1 + a_i h}, \quad n > 0, \quad i = 1, \ldots, m. \tag{5}$$

In the following we use another approximation and put (4) in the form:

$$\frac{\mathrm{d}}{\mathrm{d}t}[u_i(t) \mathrm{e}^{a_i t}] = \mathrm{e}^{a_i t} \left(\sum_{j=1}^{m} b_{ij} g_j(u_j(n - [\tau(n)])) + c_i \right).$$

Integrate both sides of this equation over the interval $[nh, t)$, let $t \to (n+1)h$ and simplify the resulting equation to obtain

$$u_i(n+1) = \mathrm{e}^{-a_i h} u_i(n) + \frac{1 - \mathrm{e}^{-a_i h}}{a_i} \left(\sum_{j=1}^{m} b_{ij} g_j(u_j(n - [\tau(n)])) + c_i \right),$$
$$n > 0, \quad i = 1, 2, \ldots, m. \tag{6}$$

In particular we consider the following impulsive difference systems:

$$\begin{cases} u_i(n+1) = \dfrac{1}{1 + a_i h} u_i(n) + \dfrac{h}{1 + a_i h} \sum_{j=1}^{m} b_{ij} g_j(u_j(n - [\tau(n)])) \\ \qquad + \dfrac{c_i h}{1 + a_i h}, \quad n \neq n_k, \quad i = 1, \ldots, m, \\ u_i(n_k + 1) - u_i(n_k) = I_{n_k}(u_i(n_k)), \quad k = 1, 2, \ldots \end{cases} \tag{7}$$

and

$$\begin{cases} u_i(n+1) = \mathrm{e}^{-a_i h} u_i(n) + \dfrac{1 - \mathrm{e}^{-a_i h}}{a_i} \left(\sum_{j=1}^{m} b_{ij} g_j(u_j(n - [\tau(n)])) + c_i\right), \\ \quad n \neq n_k, \quad i = 1, 2, \ldots, m, \\ u_i(n_k + 1) - u_i(n_k) = I_{n_k}(u_i(n_k)), \quad k = 1, 2, \ldots. \end{cases} \tag{8}$$

Using the similar method in [15], we suppose that $g_j(\cdot)$ are globally Lipschitz continuous such that

$$|g_j(u_j) - g_j(v_j)| \leqslant L_j |u_j - v_j|, \quad j = 1, 2, \ldots, m,$$

$$a_i > L_i \sum_{j=1}^{m} |b_{ji}|, \quad i = 1, 2, \ldots, m$$

and the impulsive jumps $I_{n_k}(\cdot)$ as assumed to satisfy $I_{n_k}(x_i^*) = 0$ $(i = 1, 2, \ldots, m)$ where $(x_1^*, x_2^*, \ldots, x_m^*)$ is the equilibrium of the above difference equation (5) (or (6)). Then system (7) (or (8)) exists a unique equilibrium.

Let $y_i(n) = u_i(n) - u_i^*$ $(i = 1, 2, \ldots, m)$ where $(u_1^*, u_2^*, \ldots, u_m^*)$ is the unique equilibrium of system (7) (or (8)),