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# Periodic solution for nonautonomous bidirectional associative memory neural networks with impulses

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**Abstract**

By using the continuation theorem of coincidence degree theory and constructing suitable Lyapunov functions, we study the existence, uniqueness and global exponential stability of periodic solution for nonautonomous bidirectional associative memory (BAM) neural networks with impulsive effect. The results extend earlier ones where impulses are absent. Further the numerical simulation shows that our system can occur in many forms of complexities including periodic oscillation and chaotic strange attractor.
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## 1. Introduction

The stability characteristics of equilibria of additive bidirectional neural networks of the type

$$\begin{cases} \dfrac{dx_i}{dt} = -x_i(t) + \sum\limits_{j=1}^{m} b_{ij} S(y_j(t)) + I_i(t), \\ \dfrac{dy_j}{dt} = -y_j(t) + \sum\limits_{i=1}^{m} b_{ji} S(x_j(t)) + J_j(t), \end{cases} \tag{1}$$

for $i,j = 1,2,\ldots,m,\ t>0$ and some of their generalizations have been investigated by Kosko [21–23]. The equilibria of (1) are referred to as patterns or memories associated with temporally uniform external inputs $I_i$, $J_i$. System (1) consists of two sets of $m$ neurons (or units) arranged on two layers, namely, $I$-layer and $J$-layer. On the $I$-layer, the set of $m$ neurons whose membrane potentials denoted by $x_i(\cdot)$ receive external inputs $I_i$; whereas on the $J$-layer, the other set of $m$ neurons whose membrane potentials denoted by $y_i(\cdot)$ receive external inputs $J_i$. The synaptic connectivity between neurons from these two layers are possible through the summing junctions $\sum_{j=1}^{m} b_{ij} S(y_i(t))$ and $\sum_{j=1}^{m} b_{ij} S(x_i(t))$.

Recently, in Refs. [2,4–10,19,28,29,31,36], the authors discussed the problem of stability for BAM networks with and without axonal signal transmission delays, and some sufficient conditions are obtained for the BAM networks. It is well known that studies on neural dynamical systems not only involve discussion of stability property, but also involve other dynamics behaviors such as periodic oscillatory, bifurcation and chaos. In many applications, the property of periodic oscillatory solutions are of great interest. For example, the human brain has been in periodic oscillatory or chaos state, hence it is of prime importance to study periodic oscillatory and chaos phenomenon of neural networks. As it is well known, nonautonomous phenomena often occur in many realistic systems. Particularly when we consider a long-term dynamical behaviors of a system, the parameters of the system usually will change with time [11,20,30,33,37].

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Some kind of dynamics behaving in a style with both continuous and discrete characteristics, for instance, many evolutionary processes, particularly some biological systems such as biological neural networks and bursting rhythm models in pathology, as well as optimal control models in economics, frequency-modulated signal processing systems, and flying object motions, are characterized by abrupt changes of states at certain time instants [3,18,24,32]. This is the familiar impulsive phenomena. Examples of impulsive phenomena can also be found in other fields of information science, electronics, automatic control systems, computer networking, artificial intelligence, robotics, and telecommunications, etc. Often, sudden and sharp changes occur instantaneously, in the form of impulses, which cannot be well described by using pure continuous or pure discrete models. Therefore, it is important and, in effect, necessary to introduce a new type of models-impulsive models as an appropriate description of these phenomena of abrupt qualitative dynamical changes of essentially continuous-time systems.

Most widely studied and used neural networks can be classified as either continuous or discrete. Recently, there has been a somewhat new category of neural networks, which is neither purely continuous-time nor purely discrete-time ones; these are called impulsive neural networks. This third category of neural networks display a combination of characteristics of both the continuous-time and discrete-time systems [1,14–17,25–27,34,35]. In this paper, we consider the following nonautonomous BAM neural networks model with impulses:

$$\begin{cases} \dfrac{\mathrm{d}x_i}{\mathrm{d}t} = -a_i(t)x_i(t) + \sum\limits_{j=1}^{m} p_{ji}(t) f_j(y_j(t)) + c_i(t), \\ \quad t>0, \;\; t \neq t_k, \;\; i = 1,2,\ldots,n, \\ \Delta x_i(t_k) = I_k(x_i(t_k)), \quad i = 1,2,\ldots,n, \;\; k = 1,2,\ldots, \\ \dfrac{\mathrm{d}y_j}{\mathrm{d}t} = -b_j(t)y_j(t) + \sum\limits_{i=1}^{n} q_{ij}(t) g_i(x_i(t)) + d_j(t), \\ \quad t>0, \;\; t \neq t_k, \;\; j = 1,2,\ldots,m, \\ \Delta y_j(t_k) = J_k(y_j(t_k)), \quad j = 1,2,\ldots,m, \;\; k = 1,2,\ldots, \end{cases} \tag{2}$$

where $\Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-), \Delta y_j(t_k) = y_j(t_k^+) - y_j(t_k^-)$ are the impulses at moments $t_k$ and $t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{k\to\infty} t_k = +\infty$.

As usual in the theory of impulsive differential equations, at the points of discontinuity $t_k$ of the solution $t \mapsto (x_1(t), x_2(t), \ldots, x_n(t), y_1(t), y_2(t), \ldots, y_m(t))^{\mathrm{T}}$, we assume that $(x_1(t_k), x_2(t_k), \ldots, x_n(t_k), y_1(t_k), y_2(t_k), \ldots, y_m(t_k))^{\mathrm{T}} \equiv (x_1(t_k^-), x_2(t_k^-), \ldots, x_n(t_k^-), y_1(t_k^-), y_2(t_k^-), \ldots, y_m(t_k^-))^{\mathrm{T}}$. It is clear that, in general, the derivatives $x_i'(t_k)$ and $y_j'(t_k)$ do not exist. On the other hand, according to the first two and the third equalities of (2) there exist the limits $x_i'(t_k^{\mp})$ and $y_j'(t_k^{\mp})$. According to the above convention, we assume $x_i'(t_k) \equiv x_i'(t_k^-)$ and $y_j'(t_k) \equiv y_j'(t_k^-)$.

Throughout this paper, we assume that

($H_1$) $a_i(t)>0, b_j(t)>0, p_{ji}(t), q_{ij}(t), c_i(t)$ and $d_j(t)$ are all continuous $\omega$-periodic functions.

($H_2$) $f_j$ and $g_i$ are Lipschitz-continuous on $\mathbb{R}$ with Lipschitz constants $L_j^f$ and $L_i^g$, that is,

$$|f_j(u_1) - f_j(u_2)| \leqslant L_j^f |u_1 - u_2|,$$
$$|g_i(u_1) - g_i(u_2)| \leqslant L_i^g |u_1 - u_2| \;\; \forall u_1, u_2 \in \mathbb{R}.$$

($H_3$) There exist positive constants $M_j>0$, $N_j>0$ such that $|f_j(y)| \leqslant M_j$, $|g_j(x)| \leqslant N_j$, for $j = 1,2,\ldots,n$, $x, y \in \mathbb{R}$.

($H_4$) There exists a positive integer $q$ such that $t_{k+q} = t_k + \omega, I_{k+q}(x) = I_k(x), J_{k+q}(x) = J_k(x)$, $k = 1,2,\ldots$ .

Let $r(t)$ be a $\omega$-periodic continuous function defined on $\mathbb{R}$. We define

$$r^- = \min_{0\leqslant t\leqslant\omega} |r(t)|, \quad r^+ = \max_{0\leqslant t\leqslant\omega} |r(t)|,$$
$$\bar{r} = \frac{1}{\omega}\int_0^{\omega} r(t)\,\mathrm{d}t, \quad \|r\|_2 = \left(\int_0^{\omega} |r(t)|^2\,\mathrm{d}t\right)^{1/2}.$$

As far as we know, there are few papers to deal with such a problem. By using continuation theorem of coincidence degree theory and some new analysis techniques, we obtain some new results on the existence of periodic solutions of system (2). The methods to estimate a priori bounds of periodic solution are different from those of [18,25–27,34,35].

The remaining part of this paper is organized as follows: in Section 2, we prove the existence of the periodic solution by using the continuation theorem of coincidence degree theory proposed by Gains and Mawhin [13]; in Section 3, we establish the results that periodic solutions are the globally exponentially stable by using Lyapunov functional method; in Section 4, by using numerical simulation method the influences of the impulsive perturbations on the inherent oscillation are investigated.

## 2. Existence of periodic solutions

In this section, based on the Mawhin's continuation theorem, we shall study the existence of at least one periodic solution of (2). To do so, we shall make some preparations.

Consider the impulsive system

$$\begin{cases} x'(t) = f(t,x), \quad t \neq t_k, \;\; k = 1,2,\ldots, \\ \Delta x(t)|_{t=t_k} = I_k(x(t_k^-)). \end{cases} \tag{3}$$

For system (3), finding the periodic solutions is equivalent to finding solutions of the following boundary value problem [25,26]:

$$\begin{cases} x'(t) = f(t,x), \quad t \neq t_k, \;\; k = 1,2,\ldots, \\ \Delta x(t)|_{t=t_k} = I_k(x(t_k^-)), \quad x(0) = x(\omega). \end{cases}$$

Let $\mathbb{X}$, $\mathbb{Y}$ be normed vector spaces, $L : \mathrm{Dom}\, L \subset \mathbb{X} \to \mathbb{Y}$ be a linear mapping, and $N : \mathbb{X} \to \mathbb{Y}$ be a continuous mapping. The mapping $L$ will be called a Fredholm mapping of index zero if $\dim \mathrm{Ker}\, L = \mathrm{codim}\, \mathrm{Im}\, L < +\infty$ and $\mathrm{Im}\, L$ is closed in $\mathbb{Y}$. If $L$ is a Fredholm mapping of

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