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Brief Papers

Discussions on observer design of nonlinear positive systems via T–S fuzzy modeling

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ABSTRACT

In this paper, the problems of state estimation for nonlinear positive systems based on T–S fuzzy modeling will be investigated. It will be shown that some new issues naturally arise when designing observers for positive nonlinear systems by using T–S fuzzy modeling approach. The results are developed in the following two cases: first, for systems that can be modeled by T–S fuzzy model composed of all positive local subsystems, a novel quadratic Lyapunov function is proposed to reduce the conservativeness by some other existing methods; second, for derived T–S fuzzy model comprising non-positive subsystems, the observer design is converted into the problem of stability for a positive linear system, upon which, a new algebraic algorithm for constructing a observer is given. Two numerical examples are given to demonstrate the effectiveness and applicability of the obtained theoretical results.

1. Introduction

A system is said to be positive if their state vector only takes nonnegative values whenever the initial conditions and input are nonnegative. Positive systems are of great practical importance, since the nonnegative quantity frequently occurs in numerous applications and in nature, e.g. positive systems are visible in chemical process, economics, biology, and communication networks, etc. The positivity will bring some interesting properties to positive systems. Meanwhile, the positivity will also lead to some difficulties in analysis and synthesis of such systems.

In recent yeas, positive systems have been of great interest to many researchers. Among these studies, many efforts have been devoted to the behavior analysis and property characterization such as the controllability/reachability [2], the realizability [5], and particularly the stability [6]. In the Lyapunov stability framework, the diagonal quadratic Lyapunov function, the quadratic Lyapunov function and the copositive Lyapunov function have been investigated and used for positive systems. On the other hand, some problems relating to control synthesis, such as model reduction, H_∞ filtering and stabilization, have also been solved for positive systems.

It is noted that a basic problem in control theory, concerning the observer design, has been recently studied for positive systems in the literature. By structural decomposition of system matrices, the authors in [10] design positive observers for compartmental systems. Based on a coordinates transformation method and the theory of positive realization, some existence conditions of positive Luenberger-type observers for positive linear systems are proposed in [1]. By common quadratic Lyapunov function (CQLF) approach, sufficient and necessary conditions for the existence of positive observers with general structure are derived in [8]. Also by the CQLF method, positive interval observers are designed in [7] for uncertain positive systems, which can estimate the bounds on the real states.

However, these results are only applicable to positive linear systems whereas few results on observer design have been reported for positive nonlinear systems [3]. Thanks to the T–S fuzzy model proposed by Takagi and Sugeno, which can approximate a nonlinear system and provides an efficient method to tackle some problems of nonlinear systems. Through such a modeling approach, a nonlinear system can be transformed into a framework composed of several linear subsystems [11]. As a result, some existing approaches developed for observer design for linear systems can be applied to nonlinear systems [12]. But, it should be pointed out that previous observer design approaches developed for general T–S fuzzy systems cannot be readily used for positive nonlinear systems represented by positive T–S fuzzy model. The reason lies in that those approaches do not take the positivity into account, which makes them inapplicable or inefficient for observer design of positive nonlinear systems based on positive T–S fuzzy modeling.

The above observations motivate us to carry out present work. In this paper, the problems of observer design are investigated for positive nonlinear systems by using the T–S fuzzy modeling approach. The main contributions of the paper lie in that (1) a novel quadratic Lyapunov function is explored to give less conservative conditions

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for observer design for a class of positive T–S fuzzy systems; (2) for systems whose local subsystems are not all positive, a new method for observer design is also established in the single-output case. The remainder of the paper is organized as follows. Section 2 recalls some definitions about T–S fuzzy systems and positive systems, and reviews some useful results. In Section 3, the conditions of observer design for positive T–S fuzzy systems will be established by proposing some efficient approaches. Two numerical examples are given in Section 4 to demonstrate the effectiveness and correctness of our theoretical findings. Finally, some conclusions are addressed in Section 5.

Notations: In this paper, the notations used are standard. $A \succ 0$ and $x \succ 0$ (or $A \succcurlyeq 0$, $x \succcurlyeq 0$) mean that all elements of matrix A and x are positive (or nonnegative); the notation $P>0$ (≥ 0) means that P is a real symmetric and positive definite (semi-positive definite) matrix; $\mathbb{R}$ and $\mathbb{R}^n$ denote the field of real numbers and n-dimensional Euclidean space, respectively; $\mathcal{I}_m \in \mathbb{R}^{(n-m+1)\times n}$ is a matrix composed of the rows $m, m+1, \ldots, n$ of I_n; $x^{[2]} := [x_1^2 \; x_2^2 \; \cdots \; x_n^2]^T$; $\mathbb{R}_0^n$ denotes $\mathbb{R}^n \setminus \{0^n\}$; $\mathbb{R}_+^n$ is the nonnegative orthant in $\mathbb{R}_0^n$; $A \equiv [a_{ij}]_{n\times n}$, where a_{ij} is the ith line and jth column entry of A.

2. Problem formulation and preliminaries

This paper considers the following positive nonlinear system:

$$\dot{x}(t) = f(x(t), u(t), t), \quad x(t_0) = x_0 \succcurlyeq 0$$
$$y(t) = Cx(t) \tag{1}$$

which could be modeled by fuzzy rules [9], and can be represented as follows:

Model rule i : **IF** $\theta_1(t)$ is M_1^i and $\cdots \theta_p(t)$ is M_p^i **THEN**

$$\dot{x}(t) = A_i x(t) + B_i u(t), \quad i \in R = \{1, 2, \ldots r\}$$
$$y(t) = Cx(t) \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the input; M_j^i is the fuzzy set, and r stands for the number of IF–THEN rules; A_i, B_i, C, $i \in R$, are real matrices with appropriate dimensions; $\theta(t) = [\theta_1(t), \theta_2(t), \cdots, \theta_p(t)]^T$ is the premise variables vector. It is assumed that $\theta_j(t)$ depends on measured variables, and does not contain the input $u(t)$; through using a standard fuzzy inference method, a more compact presentation of system (1) is

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) = \sum_{i=1}^{r} h_i(\theta(t))(A_i x(t) + B_i u(t)),$$
$$x(t_0) = x_0 \succcurlyeq 0$$
$$y(t) = Cx(t) \tag{3}$$

where $h_i(\theta(t))$ are the normalized membership functions satisfying

$$h_i(\theta(t)) = \frac{\Pi_{j=1}^p M_j^i(\theta_j(t))}{\sum_{i=1}^r \Pi_{j=1}^p M_j^i(\theta_j(t))} \geq 0$$
$$\sum_{i=1}^r h_i(\theta(t)) = 1 \tag{4}$$

and $M_j^i(\theta_j(t))$ denoting the grade of membership of premise variable $\theta_j(t)$ in M_j^i.

Next, we recall some definitions and lemmas that will be used later.

Definition 1. System (1) is said to be positive if for any initial condition $x(0) \in \mathbb{R}_+^n$ and any input function $u(t) \in \mathbb{R}_+^n$, the corresponding state trajectory $x(t) \in \mathbb{R}_+^n$, $\forall t > 0$.

Definition 2. A linear system $\dot{x}(t) = Ax(t) + Bu(t)$ is said to be positive, if for any initial condition $x(0) \in \mathbb{R}_+^n$ and input function $u(t) \in \mathbb{R}_+^n$, the corresponding state trajectory $x(t) \in \mathbb{R}_+^n$.

Lemma 1. *System (3) is positive if* $\sum_{i=1}^r h_i(\theta(t)A_i$ *is a Metzler matrix,* $\sum_{i=1}^r h_i(\theta(t)B_i$ *and* C *are nonnegative matrices,* $\forall t \geq 0$.

Lemma 2. *For Metzler matrices* $A \in \mathbb{R}^{n\times n}$ *and* $B \in \mathbb{R}^{n\times n}$, *if* $A \succcurlyeq B$, *then* $\rho(A) \geq \rho(B)$.

Lemma 3. *(Kaczorek [4]) Let* $A \in \mathbb{R}^{n\times n}$. *Then* $e^{At} \succcurlyeq 0$, $\forall t \geq 0$, *if and only if* A *is a Metzler matrix.*

In this paper, we consider the problems of state observation for positive nonlinear systems that can be modeled by system (3) satisfying the conditions in Lemma 1. It is also noted by Lemma 1 that a sufficient condition guaranteeing the positivity of system (3) is that A_i is a Metzler matrix, B_i and C are nonnegative matrices, $\forall i \in R$ since $h_i(\theta(t)) \geq 0$.

3. Main results

The following observer will be used for state estimation of system (3):

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(\theta(t))A_i\hat{x}(t) + \sum_{i=1}^{r} h_i(\theta(t))B_i u(t) + \sum_{i=1}^{r} h_i(\theta(t))L_i(y(t) - \hat{y}(t))$$
$$\hat{x}(0) = 0$$
$$\hat{y}(t) = C\hat{x}(t) \tag{5}$$

where $L_i \succcurlyeq 0$. By defining the error state $\tilde{x}(t) = x(t) - \hat{x}(t)$, we can get the error dynamic system as follows:

$$\dot{\tilde{x}}(t) = (A(t) - L(t)C)\tilde{x}(t) \tag{6}$$

where $L(t) = \sum_{i=1}^r h_i(\theta(t))L_i$. On the other hand, observer (5) is equivalent to

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(\theta(t))(A_i - L_iC)\hat{x}(t) + \sum_{i=1}^{r} h_i(\theta(t))B_i u(t) + \sum_{i=1}^{r} h_i(\theta(t))L_i y(t) \tag{7}$$

Now, we first assume that the obtained A_i in (3) is Metzler, $\forall i \in R$. In this scenario, a design method of observer for system (3) is presented in the following theorem.

Theorem 1. *If there exist real matrices* $P \in \mathbb{R}^{n\times n}$, $\Theta \in \mathbb{R}^{n\times n}$, $\Theta' \in \mathbb{R}^{(\vartheta(n,2)-n)\times(\vartheta(n,2)-n)}$, $\Xi_p \in \mathbb{R}^{n\times n}$, *and* $\Xi_p' \in \mathbb{R}^{(\vartheta(n,2)-n)\times(\vartheta(n,2)-n)}$, *such that* $\forall p \in R$

$$-\Theta - P < 0 \tag{8}$$

$$\Theta' < 0 \tag{9}$$

$$\begin{bmatrix} -2P - \Xi_p - \epsilon_p P & \epsilon_p I + (A_p - L_pC + I)^T \\ \epsilon_p I + A_p - L_pC + I & -\epsilon_p P^{-1} \end{bmatrix} < 0 \tag{10}$$

$$\Xi_p' < 0 \tag{11}$$

$$-\mathcal{E}_k A_p \mathcal{E}_{k'}^T - \mathcal{E}_k L_p C \mathcal{E}_{k'}^T \leq 0, (k, k') \in \Bbbk \times \Bbbk = \{1, 2, \ldots, n\}, \quad k \neq k' \tag{12}$$

where $\Theta = [\theta_{ij}]_{n\times n}$ *with* $\theta_{ii} = 0$, $\Theta' = \mathrm{diag}\{\theta_1', \ldots, \theta_{n-1}'\}$, $\Xi_p = [\omega_{ij}^p]_{n\times n}$ *with* $\omega_{ii}^p = 0$, $\Xi_p' = \mathrm{diag}\{\omega_1^p, \ldots, \omega_{n-1}^p\}$, $\theta_k' = \mathrm{diag}\{\theta_{k(k+1)} + \theta_{(k+1)k}, \ldots, \theta_{kn} + \theta_{nk}\}$ *and* $\omega_k^p = \mathrm{diag}\{\omega_{k(k+1)}^p + \omega_{(k+1)k}^p, \ldots, \omega_{kn}^p + \omega_{nk}^p\}$, $k \in \{1, 2, \ldots, n-1\}$, *then, observer (5) is a positive observer such that error system (6) is asymptotically stable.*

Proof. By inequality (12), it is clear that $A_i - L_iC$, $\forall i \in R$, is a Metzler matrix. Then, due to the fact $0 \leq h_i(\theta(t)) \leq 1$, $\sum_{i=1}^r h_i(\theta(t))(A_i - L_iC)$ is Metzler, $\forall t \geq 0$. This together with Lemma 1, $\sum_{i=1}^r h_i(\theta(t))B_i \succcurlyeq 0$, implies that observer (7) is a positive observer. Also note that $\tilde{x}(0) \succcurlyeq 0$ since $\hat{x}(0) = 0$. Therefore, $\tilde{x}(t) \succcurlyeq 0$. Noting the positivity of $\tilde{x}(t)$, we choose the following Lyapunov function candidate for