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Hybrid synchronization of coupled fractional-order complex networks

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ABSTRACT

This paper investigates the hybrid synchronization problem of two coupled complex networks with fractional-order dynamical nodes. Based on the fractional-order Lyapunov stability theorem, some sufficient conditions are derived to realize the hybrid synchronization of two coupled fractional-order networks. Under suitable condition, two coupled networks can reach the hybrid synchronization, i.e., the outer synchronization between the drive and response networks, and the inner synchronization in each networks. Numerical simulations demonstrate the effectiveness and feasibility of the proposed synchronization protocol.

1. Introduction

Nowadays, there are numerous natural or man-made complex networks. For examples, the World Wide Web, communication networks, social networks, food webs, and so on. All the above networks can be represented in terms of nodes and edges indicating connections between nodes. Due to its tremendous potentials in real applications, the research of complex networks has become a hot topic in modern scientific research [1–4].

In recent years, synchronization in complex network, as collective behavior, has received increasing attention and been extensively investigated due to its potential applications in many fields, including secure communization, image processing, neural networks, information science, etc. [5–8]. Till now there are many works about the synchronization of the complex networks. For example, Wang and Chen studied synchronization in two specific kinds of networks: scale-free networks and small-world [9,10]. Lü and Chen investigated a time-varying dynamical network and further investigated its synchronization [11]. Wang and Cao investigated the dynamics of a system of linearly coupled identical connected neural networks with time-varying delay [12]. However, the above articles just concentrate on synchronization in a network which was called "inner synchronization" as it is a collective behavior within a network. In reality, there is another interesting network synchronization scheme, synchronization between two or more complex networks regardless of the inner synchronization, which was termed "outer synchronization". Li, Sun and Kurths investigated the outer synchronization between two unidirectionally coupled networks and a criterion for the synchronization between two networks with identical topological structures [13]. Tang, Chen, Lu and Tse studied the outer synchronization between two networks with different topological structures by the adaptive control method [14]. Wu and Lu investigated the outer synchronization of uncertain complex delayed networks with adaptive coupling [15].

It is well known that the fractional calculus is a classical mathematical notion, and is a generalization of ordinary differentiation and integration to arbitrary order. However, the fractional calculus did not attract much attention for a long time due to lack of application background. Nowadays, that many known systems can described by fractional-order systems, such as viscoelasticity, dielectric polarization, electromagnetic waves [16–18]. Compared with the classical integer-order models, fractional-order derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes. Therefore, it may be more accurate to model by fractional-order derivatives than by integer-order ones. It is demonstrated that many fractional-order differential systems behave chaotically or hyperchaotically, such as the fractional-order Chua circuit [19], the fractional-order Lorenz system [20], the fractional-order chaotic and hyperchaotic Rössler system [21], etc. Following these findings, synchronization of chaotic fractional-order differential systems becomes a challenging and interesting problem due to the potential applications in secure communication and control processing.

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Although there exist many results on synchronization of complex networks, most efforts have been devoted to complex networks whose nodes are constructed by integer-order ordinary differential equations. As we know, the well-studied integer-order complex networks are the special cases of the fractional-order complex network. However, due to the limited theories for the coupled fractional-order dynamical systems, the synchronization between fractional-order complex networks is still a challenging research topic and studied about it are still few [22–28]. For example, Chai, Chen et al. [22,23] investigated synchronization of general fractional-order complex dynamical networks by adaptive pinning method and cluster synchronization of complex dynamical networks with fractional-order dynamical nodes. Si, Sun and Zhang [24] have researched of identification fractional-order complex network with unknown system parameters and network topologies. Wu and Lu [25] investigated outer synchronization between two different fractional-order general complex networks. Yang and Jiang [26] have researched the adaptive synchronization in drive-response fractional-order dynamical networks with uncertain parameters. However, the aforementioned studies only concentrated on inner synchronization or outer synchronization. But recently, coexistence of outer synchronization and inner synchronization in chaotic systems are investigated intensively [27–29]. Yang and Jiang [27] have studied the hybrid projective synchronization for fractional-order chaotic systems with time delay. Wang, Yu and Diao [28] investigated the hybrid projective synchronization of different dimensional fractional order chaotic systems. The hybrid synchronization of two coupled networks based on the Lyapunov stability theory and Lasalle's invariance principle is investigated in [29]. To the best of our knowledge, most of these works were concerned with the hybrid synchronization only in two coupled chaotic systems, the hybrid synchronization of two coupled fractional-order networks has not been explicitly considered and studied.

Motivated by the above discussions, in this paper, we will study the hybrid synchronization of two coupled fractional-order complex networks. The outer and inner synchronization are investigated simultaneously, which is more reasonable in the practical application. Based on the fractional-order Lyapunov stability theorem, some sufficient conditions for the hybrid synchronization of the drive and response fractional-order networks are derived.

This paper is organized as follows: In Section 2, some fractional-order definitions, lemmas are given; in Section 3, fractional-order complex networks model and hybrid synchronization definitions are given; in Section 4, the criteria for the hybrid synchronization of the fractional-order complex networks are obtained; example is given in Section 5, followed by conclusions in Section 6.

Throughout this paper, the following notations are used. $\|\cdot\|$ is the Euclidean norm of a vector; A^T means the transpose of the matrix A; $I \in \mathbb{R}^{n\times n}$ denotes the identity matrix with dimension n; $\otimes$ representation Kronecker product of two matrices and A^s denotes $(A+A^T)/2$.

2. Model description and preliminaries

2.1. Fractional calculus

The fractional-order integer-differential operator is the generalization concept of an integer-order integer-differential operator, which could be denoted by a fundamental operator as follows:

$$ {}_aD_t^q = \begin{cases} \frac{d^q}{dt^q}, & R(q)>0 \\ 1, & R(q)=0 \\ \int_a^t (d\tau)^{-q}, & R(q)<0 \end{cases} \tag{1}$$

where q is the fractional-order calculus operator which can be a complex number, a and t are the limits of the operation. The commonly used definitions are Grunwald–Letnikov (GL), Riemann–Liouville (RL), and Caputo (C) definitions. In the rest of this paper, the notation (d^q/dt^q) is chosen as the Caputo fractional derivation operator.

Definition 1. The Caputo fractional derivative is define as follows:

$$ {}_a^cD_t^q x(t) = \begin{cases} \frac{1}{\Gamma(n-q)}\int_a^t (t-\tau)^{n-q-1}x^{(n)}(\tau)d\tau, & n-1<q<n \\ \frac{d^n}{dt^n}x(t), & q=n \end{cases} \tag{2}$$

where $\Gamma(\cdot)$ is the Gamma function which is defined by $\Gamma(z)=\int_0^\infty e^{-z}t^{z-1}dt$.

It should be noted that the advantage of the Caputo approach is that the initial conditions for fractional differential equations with Caputo derivatives take on the same form as those for integer-order ones, which have well understood physical meaning.

The following lemmas are needed to derive our main results.

Lemma 1. *[30,31]. Let $\mathbf{x}(t)=[x_1(t),\ldots,x_n(t)]^T \in \mathbb{R}^n$ be a real continuous and differentiable vector function. Then $D^q[\mathbf{x}^T(t)P\mathbf{x}(t)] \le 2\mathbf{x}^T(t)PD^q\mathbf{x}(t)$, where $0<q<1$ is the fractional order and $P=\mathrm{diag}[p_1,\ldots,p_n]>0$.*

Lemma 2. *[32]. Consider the fractional-order nonautonomous system*

$$ {}_{t_0}D_t^\alpha x(t) = f(t,x) \tag{3}$$

with initial condition $x(t_0)$. Let $x=0$ be an equilibrium point for the system (3) and $\mathbb{D}\subset\mathbb{R}^n$ be a domain containing the origin. Let $V(t,x(t)) : [0,\infty)\times\mathbb{D}\to\mathbb{R}$ be a continuously differentiable function and locally Lipschitz with respect to x such that

$$ \alpha_1\|x\|^a \le V(t,x(t)) \le \alpha_2\|x\|^{ab} \tag{4}$$

$$ {}_0^cD_t^\beta V(t,x(t)) \le -\alpha_3\|x\|^{ab} \tag{5}$$

where $t\ge 0$, $x\in\mathbb{D}$, $\beta\in(0,1)$, α_1, α_2, α_3, a and b are arbitrary positive constants. Then $x=0$ is Mittag-Leffler stable. If the assumptions hold globally on $\mathbb{R}^n$, then $x=0$ is globally Mittag-Leffler stable.

2.2. Network model

In this paper, fractional-order complex network model consisting N nodes are described as follows

$$ \frac{d^q x_i(t)}{dt^q} = f(x_i(t)) + c\sum_{j=1}^N a_{ij}\Gamma x_j(t), \quad i=1,2,\ldots,N, \tag{6}$$

where $0<q<1$ is the fractional order, $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))\in\mathbb{R}^n$ is the state variable of the ith node, $f: R^+\times R^n$ is a smooth nonlinear continuous map, $A=(a_{ij})\in\mathbb{R}^{N\times N}$ denotes the coupling configuration matrix; c is the coupling strength, and $\Gamma=\mathrm{diag}(\zeta_1,\zeta_2,\ldots,\zeta_n)\in\mathbb{R}^{n\times n}$ is the inner coupling matrix. The matrix A is defined as follows: if there exist a connection between node i and node $j(i\ne j)$, then $a_{ij}>0$; otherwise, $a_{ij}=0$ and he diagonal elements of matrix A are defined as

$$ a_{ii} = -\sum_{j=1, j\ne i}^N a_{ij}, \quad i=1,2,\ldots,N \tag{7}$$

We refer to model (6) as the drive complex networks. Correspondingly the response complex network with the control inputs $u_i(t)\in\mathbb{R}^n$ $(i=1,2,\ldots,N)$ can be rewritten as

$$ \frac{d^q y_i(t)}{dt^q} = f(y_i(t)) + c\sum_{j=1}^N a_{ij}\Gamma y_j(t) + u_i(t), \quad i=1,2,\ldots,N \tag{8}$$

where $y_i(t)=(y_{i1},y_{i2},\ldots,y_{in})^T\in\mathbb{R}^n$ is the response state vector of the ith node; f, c, Γ have the same meaning as those in Eq.(6), $u_i(t)$ are the controllers to be designed.