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Pinning exponential synchronization of complex networks via event-triggered communication with combinational measurements

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ABSTRACT

In this paper, we consider the pinning exponential synchronization of complex networks via event-triggered communication. By using the combinational measurements, two classes of event triggers are designed, one depends on continuous communications between the agents, the other avoids the continuous communications. The controller updates when triggering function reaches certain threshold. For such classes of two event triggers, the exponential synchronization as well as the convergence rate of the controlled complex networks are studied, respectively, by employing the *M*-matrix theory, algebraic graph theory and the Lyapunov method. Two simulation examples are provided to illustrate the effectiveness of the proposed two classes of event triggering strategies. It is noteworthy that the event trigger with combinational measurements avoids decoupling the actual state of the nodes, which is more effective than the error-based event trigger.

1. Introduction

Complex networks that consist of vast interconnected nodes are ubiquitous in the real world. What we often encounter such as social networks, biological networks, electrical power grids, the Internet and World Wide Web all can be modeled and analyzed by complex networks [1,2]. The past decade has witnessed the dramatic progresses in studying synchronization problems of complex networks, including global synchronization [3], local synchronization [4], exponential synchronization [5,6], anticipation synchronization [7,8], cluster synchronization [9], to name just a few.

In practice, autonomous agent such as mobile robots are often equipped with digital microprocessors which coordinate the data acquisition, communication with other agents, and control actuation. Thus, it is necessary to implement controllers on a discrete time instants. There are mainly two classes of discrete time updated controllers: the time-triggered controller; the event-triggered controller. The updates of the former are determined by the preset triggering time instants, and the updates of the latter are determined by the preset triggering event instants. The impulsive controller [10–12] and the sampled data controller [13–15] are two typical types of the time-triggered controllers. However, the time-triggered controller may lead control wastes, a typical example is that the time-triggered controller will keep updating even when the control goal is achieved. To overcome this conservativeness of the time-triggered controller, the event-triggered controller has been proposed [16,17], where controller updates are determined by certain events that are triggered depending on the agents' behavior. The event-based controllers have been adopted for control engineering applications [18–20,22].

Zeno behavior often occurs in the event-based control systems. It describes the phenomenon that the event-based controller undergoes an unbounded number of updates in a finite and bounded length of time. This can happen, for example, when a controller unsuccessfully attempts to satisfy an invariance specification by updating the measurement error and the measurement threshold faster and faster [21]. It should be pointed out that the Zeno behavior is harmful to the event-based control systems: when Zeno behavior occurs, the event-based controller updates continuously, that is, the event-based controller degenerates to the continuous-time controller, which leads to the control wastes. Hence, Zeno behavior should be avoided when designing the event-based controllers.

Tabuada [22] seminally presented a triggering condition based on norms of the state and the state error. Zhu et al. [23] extended the work of Tabuada. They studied the event-based consensus problem for general LTI systems, where the triggering function depends on the measurement errors, the states of the neighboring agents and arbitrary small constants. However, this error-dependent nature of the event triggers makes the network executions decoupled from the actual state of the agents. Fan et al. [24] proposed the combinational measurement-based triggering function, which is a combination of its own state and its neighbors' states. This type of triggering controllers

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avoids the decoupling of the actual state of the agents. They also provided a new iterative event-triggered algorithm that can avoid the continuous measurement between the agents.

For large-scale networks, to reduce the control consumption, one often employ the pinning control strategy. Namely, controllers only apply control actions to a very few fraction of nodes [25–35]. However, most of the existing works are based on continuous-time communications between nodes that have a obvious insurmountable deficiency: the designed control law requires real-time updates, which promotes network nodes to be equipped with high performance processors and high-speed communication channels [36–39]. Gao et al. considered the pinning controllability of complex networks with a distributed event-triggered mechanism [40]. They proposed an error-based event trigger, and proved that the pinning controlled complex network can achieve synchronization under certain sufficient conditions. However, they did not prove that the Zeno behavior can be excluded for the designed event-trigger.

In this paper, we consider the pinning exponential synchronization problem for N identical nodes via event-triggered communication with combinational measurements. The controller updates of each agent are driven by properly defined events, which depends only on the combinational measurements. The primary contributions are stated as follows. Firstly, novel event-triggered pinning control approaches using combinational measurements are applied to the exponential synchronization problem. Here, the Laplacian matrix of the communication graph is no longer assumed to be symmetric. Second, two classes of event triggers for pinning exponential synchronization are designed. One requires continuous communications, the other avoids the continuous communications. The exponential synchronization as well as the convergence rate of the controlled complex networks are studied. Last, for such two event triggers, we prove that the Zeno behavior can be excluded, respectively.

Notations: Let $\|x\|$ and $\|A\|$ be the Euclidean norm of a vector x and a matrix A, respectively. Let I_n be the $n\times n$ identity matrix. Let 1_n be the vector whose elements are 1. Let $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ be the smallest and largest eigenvalue of symmetric matrix A. Let $\rho(A)$ be the spectral radius of matrix A. For a matrix A, $A>0$ (≤ 0) means all elements in A is positive (nonpositive). For a symmetric matrix A, $A\succ 0$ $(\prec 0)$ means A is positive definite (negative definite). diag$\{\cdots\}$ stands for a block-diagonal matrix. The superscript "T" represents the vector and matrix transpose. Let $\otimes$ be the Kronecker product.

2. Mathematical preliminaries

2.1. Algebraic graph theory

Several definitions and notations in the graph theory are introduced in the following which will be used in the later analysis. The interested readers refer to some textbooks of graph theory [41] for more details.

Let $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ be a weighted digraph of order N with the set of nodes $\mathcal{V}=\{1,2,\ldots,N\}$, set of directed edges $\mathcal{E}=\mathcal{V}\times\mathcal{V}$, and a weighted adjacency matrix $\mathcal{A}=(a_{ij})_{N\times N}$. An edge in network $\mathcal{G}$ is denoted by (i,j) where i and j are called the terminal and initial nodes, respectively, which means that node i can receive information from node j. The set of neighboring set of agent i is denoted by $N_i=\{j\in\mathcal{V}:(j,i)\in\mathcal{E}\}$. The adjacency elements associated with the edges of the graph are positive and the others are zero, i.e., $a_{ij}>0$ if and only if $(j,i)\in\mathcal{E}$, and $a_{ij}=0$ otherwise. Moreover, we assume that the graph contains no self-loops, i.e., $a_{ii}=0$ for all $i=1,2,\ldots,N$. Let $\mathcal{D}=\mathrm{diag}\{d_1,d_2,\ldots,d_N\}$, where $d_i=\sum_{j=1}^N a_{ij}$. The Laplacian of the weighted digraph $\mathcal{G}$ is defined as $L=\mathcal{D}-\mathcal{A}$, whose elements are presented as follows:

$$l_{ij}=\begin{cases}-a_{ij}, & i\neq j,\\ \sum\limits_{j=1,j\neq i}^{N} a_{ij}, & i=j.\end{cases}$$

A directed path from node j to node i is a sequence of edges of the form $(i,i_1),(i_1,i_2),\ldots,(i_k,j)$ in the directed network with distinct nodes $i_l\in\mathcal{V}$, $l=1,2,\ldots,k$. We say that a digraph $\mathcal{G}$ has a spanning tree if there exists a vertex called root which can access all other vertices. Denote the root set by root. We say a digraph $\mathcal{G}$ is strongly connected, if for any vertex pair (i,j) there exist a directed path from vertex j to vertex i and another directed path from vertex i to vertex j. We say a node i is globally reachable, if there exists a path from node i to every node j in $\mathcal{G}$.

2.2. M-matrices

In this section, some results on M-matrices are provided to investigate the pinning control of complex networks [42,43].

Definition 1. Let $A=(a_{ij})_{N\times N}$ be nonsingular and $a_{ij}\le 0$ if $i\neq j$, $i,j=1,2,\ldots,N$. Then A is called an M-matrix if and all the eigenvalues of A have positive real parts.

Lemma 1. *If A is a nonsingular M-matrix, then the following statements are equivalent.*

(1) *Matrix A can be expressed by $A=\gamma I_N-B$, where $B\geqslant 0$ and $\gamma>\rho(B)$.*
(2) *A^{-1} exists, and $A^{-1}\ge 0$.*
(3) $\mathrm{Re}(\lambda_i(A))>0$ *for all $i=1,2,\ldots,N$.*
(4) *There exists a positive definite diagonal matrix $\Xi=\mathrm{diag}\{\xi_1,\xi_2,\ldots,\xi_N\}$ such that $\Xi A+A^T\Xi\succ 0$.*
(5) *There is a positive vector $x\in\mathbb{R}^N$ such that $Ax>0$.*

Remark 1. In this paper, Lemma 1 plays a key role in analyzing the pinning synchronization of complex networks.

2.3. Pinning complex networks via event-triggered communication

Consider a linearly coupled complex network composed of N identical nodes, in which each node is an n-dimensional dynamical system described by

$$\dot{x}_i(t)=f(t,x_i(t))+c\sum_{j=1}^{N}a_{ij}\Gamma(x_j(t)-x_i(t)),\quad i=1,2,\ldots,N, \tag{1}$$

where $x_i(t)\in\mathbb{R}^n$ is the state of node i, and $f(t,x_i(t))\in\mathbb{R}^n$ is a continuous vector-valued function. The positive constant c represents the coupling strength, the inner coupling matrix $\Gamma\in\mathbb{R}^{n\times n}$ is positive definite and describes an inner-coupling between the subsystems, the matrix $\mathcal{A}=(a_{ij})_{N\times N}\in\mathbb{R}^{N\times N}$ represents the coupling configuration. We consider the N subsystem in an induced graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$, the vertices are labeled by $\mathcal{V}=\{1,2,\ldots,N\}$, corresponds to N subsystems. The edge is defined as $a_{ij}>0$ if and only if $(j,i)\in\mathcal{E}$; $(j,i)\overline{\in}\mathcal{E}$ if and only if $a_{ij}=0$.

The isolated node of network (1), labeled by node 0, is given by $s(t)$ whose dynamics is described as follows:

$$\dot{s}(t)=f(t,s(t)). \tag{2}$$

where $s(t)=(s_1(t),s_2(t),\ldots,s_n(t))^T\in\mathbb{R}^n$.

Definition 2. Synchronization is said to be reached exponentially, if there exists positive constants $\kappa>0$, $\gamma>0$ and $T>0$ such that

$$\|x_i(t)-s(t)\|\le\kappa e^{-\gamma t},\quad i=1,2,\ldots,N,$$

for all $t>T$. γ is called the convergence rate.

Our control goal is to synchronize complex network (1) to the homogenous trajectory (2). The previous most common models are investigated [25–35], however, their insurmountable deficiency is very obvious: each node's controller implementation must realize its all neighbors, current states at every time instant, that means continuous