Neurocomputing 157 (2015) 306–314

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

H_∞ model approximation for discrete-time Takagi–Sugeno fuzzy systems with Markovian jumping parameters

Xunyuan Yin [c], Xu Zhang [c], Lixian Zhang [a,b,c,*], Changhong Wang [c], Maryam Al-Yami [b], Tasawar Hayat [b,d]

[a] State Key Laboratory of Urban Water Resources and Environment, Harbin Institute of Technology, Harbin 150090, China
[b] Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[c] School of Astronautics, Harbin Institute of Technology, Harbin 150080, China
[d] Department of Mathematics, Quaid-I-Azam University, Islamabad 44000, Pakistan

ARTICLE INFO

Article history:
Received 2 October 2014
Received in revised form
4 December 2014
Accepted 26 December 2014
Communicated by Yang Tang
Available online 11 February 2015

Keywords:
H_∞ performance
Markov jump systems
Projection approach
Takagi–Sugeno (T–S) fuzzy systems
Model order approximation
Disturbances

ABSTRACT

This paper is concerned with the H_∞ model approximation problem for a class of discrete-time Takagi–Sugeno (T–S) fuzzy Markov jump systems. The systems involve stochastic disturbances and nonlinearities that can be described by T–S fuzzy models. The problem to be solved in the paper is to find a reduced-order model, which is able to approximate the original T–S fuzzy Markov jump system with comparatively small and acceptable errors. Specifically, the corresponding error system is guaranteed to be asymptotically stable in the mean square with a prescribed H_∞ performance index. By using convex optimization approach and projection approach, respectively, sufficient conditions on the existence for such model with reduced-order are obtained and presented in the form of linear matrix inequalities. Finally, a numerical example is provided to demonstrate the effectiveness of the obtained results.

1. Introduction

It is constantly encountered that many practical engineering systems, regardless of the fields of their applications, are characterized by mathematical models with relatively high order, which will inevitably introduce difficulty and computational complexity to stability analysis, synthesis and simulation [1,2]. Therefore, an effective solution to the underlying problem is to reduce the order of the system models to a comparatively low order. Thus, model approximation for engineering systems is playing a significant role in the process of control system analysis and design. Given a full-order engineering system, the objective in terms of model approximation is to achieve a new model with reduced order that is able to approximate the original system such that the errors are small and acceptable. In order to achieve these goals, effective approaches have been proposed, such as the aggregation method [3], the optimal Hankel norm approximation method [4], and the balanced truncation method [5]. During the past several decades, much attention has been paid on model reduction for different kinds of systems, such as switched hybrid systems [2,6], Markov systems [7] and fuzzy systems [8].

On the other hand, great importance has been attached to the areas in terms of practical engineering systems with switching dynamics during the past several decades. These systems may experience inevitable changes or disturbances, such as unpredictable environmental changes, random component failures or other influences brought by interactions of subsystems [9]. The aforementioned systems that involve stochastic mode transitions can be represented by Markov jump systems, which have been considered to be a frequently addressed topic due to the widely practical applications in terms of power engineering systems, communication systems, irrigation systems, aerospace engineering, networked control systems and other manufacturing systems [7,10–13]. Involving stochastic processes, Markov jump systems are hybrid in essence, in which the continuous and discrete dynamics are, respectively, described by a series of differential equations and Markov chains to govern the transitions among each subsystem [14]. The phenomenon is known as a stochastic behaviour, which makes Markov systems vastly different from other kinds of hybrid systems, such as the nondeterministic switched systems. Markov jump system has attracted much attention mainly because of its promising prospect in many application areas. For instance, it can be adopted to describe the abrupt phenomenon such as random failures, repairs of the components and sudden

* Corresponding author at: State Key Laboratory of Urban Water Resources and Environment, Harbin Institute of Technology, Harbin 150090, China.
E-mail address: lixianzhang@hit.edu.cn (L. Zhang).

http://dx.doi.org/10.1016/j.neucom.2014.12.075
0925-2312/

environmental changes [13]. So far, many significant results on the stability analysis, filter design, optimal control, robust control, model reduction and other issues for Markov jump linear systems (MJLSs) have been obtained, such as [7,15–18], and the references therein.

Compared to the literatures on MJLSs, fewer papers can be found on control issues for Markov jump nonlinear systems (MJNSs) due to the complexity generated from the nonlinear dynamics. However, the problems in terms of MJNSs are of vital importance and should be combated successfully because nonlinear systems are ubiquitously found in chemical process, power engineering systems, robotic systems, automotive systems, and many other manufacturing systems [8]. Problems on approximating nonlinear systems with acceptable errors were deeply investigated. Due to the nature of high efficiency, the T–S fuzzy system has become very important for the analysis and synthesis of complex nonlinear systems [19]. The work on stability analysis and controller design for T–S fuzzy systems with complex structures, time delay, disturbances, etc. So far, T–S fuzzy model has presented excellent performance in facilitating stability analysis and controller design [20,21], thus has attracted great attention. Some typical results on the stability analysis and controllability of T–S fuzzy systems are reported in [19,21–27]. Because of this advantage of the T–S fuzzy model, the concept of fuzzy Markov jump systems (FMJSs) was proposed and has attracted much attention during the past few years. By utilizing a family of IF–THEN rules that represent local linear input–output relations of each model of the MJNSs, the problem on the nonlinear dynamics can be efficiently addressed. To name a few, a robust H_∞ output feedback controller was designed in [28]. A mode-based method for FMJSs which can be found in [29] was proposed and verified. Stability analysis and stabilization issues of discrete-time FMJSs with time delays were investigated in [30,31]. A mode-independent fuzzy controller design method was reported in [32]. The stability analysis and a novel controller design technique for FMJSs was studied in [23], where slack variables were introduced to separate Lyapunov matrices from system matrices. Note that though the studies on control of FJMSs have been launched, the model approximation issue has seldom been addressed, which motivates us for this study.

In this paper, the problem of H_∞ model approximation for a class of discrete-time FMJSs is studied. Specifically, both convex linearization approach and projection approach are introduced to derive sufficient conditions of the existence of the reduced-order model and the corresponding solutions to system matrices of the reduced-order model such that the resulting error system is said to be stochastically stable with a guaranteed H_∞ performance index. Since the results based on the projection approach are presented with the form of LMIs with a non-convex constraint, cone complementary linearization (CCL) is employed to cope with the sequential minimization issue. Finally, a numerical example is presented to illustrate the validity and effectiveness of the proposed model reduction approach. The remained part of the paper is organized as follows. In Section 2 the formulation of the problems and preliminaries are presented. Main results on H_∞ model reduction of FMJSs are presented in Section 4, while simulation results are obtained in Section 5 to demonstrate the effectiveness of the main results. Finally, the conclusion is drawn in Section 6.

Notation: The notation used throughout this paper is fairly standard. The subscripts "T" and "-1" stand for matrix transposition and inverse, respectively. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The notation $P>0$ means that P is real symmetric positive definite. I and $\mathbf{0}$ represent identity matrix and zero matrix, respectively. In symmetric block matrices or complex matrix expressions, the symbol "$*$" is used to represent an ellipsis for the terms that are introduced by symmetry and diag$\{\cdots\}$ stands for a block-diagonal matrix. $\|\cdot\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix. $\|\cdot\|_2$ stands for the typical $l_2[0,\infty)$ norm while $\|\cdot\|_\infty$ represents the l_2-induced norm of a transfer function matrix or a general operator. For a matrix $U\in\mathbb{R}^{m\times n}$ with rank k, we denote $U^\perp$ as the orthogonal complement, which is possibly non-unique, such that $U^\perp U=0$. U^+ is defined as the Moore–Penrose inverse of the U, and U_L, U_R are defined as any full rank factors of U, i.e. $U_LU_R=U$.

2. Problem formulation and preliminaries

2.1. Physical plant

Consider the following discrete-time FMJSs.

Plant rule i: IF $s_1(k)$ is μ_{i1} and $s_2(k)$ is μ_{i2} and … and $s_h(k)$ is μ_{ih}, THEN

$$\begin{cases} x(k+1)=A_i(r_k)x(k)+B_i(r_k)u(k) \\ y(k)=C_i(r_k)x(k)+D_i(r_k)u(k) \end{cases} \tag{1}$$

where $x(k)\in\mathbb{R}^n$ is the state vector, $u(k)\in\mathbb{R}^p$ which belongs to $l_2[0,\infty)$ is the input and $y(k)\in\mathbb{R}^q$ is the measured output; $i\in\mathbb{S}=\{1,2,\ldots,w\}$, w is the number of IF–THEN rules, $\mu_{i1},\mu_{i2},\ldots,\mu_{ih}$ is the fuzzy set, $s_1(k)$, $s_2(k),\ldots,s_h(k)$ are the premise variables; $\{r_k\}$ is a discrete-time Markov process that takes values in a finite set $\mathbb{T}=\{1,2,\ldots,g\}$ with mode transition probability matrix $\Psi=\{\pi_{lm}\}$ that is listed as follows:

$$\pi_{lm}=\Pr(r_{k+1}=m\,|\,r_k=l)$$

where π_{lm} represents the transition probability from mode l at time k to mode m at time $k+1$, for all $l\in\mathbb{T}$, $m\in\mathbb{T}$, $\pi_{lm}\geq 0$ and $\sum_{i=1}^{g}\pi_{lm}=1$. The system matrices of different modes are denoted by $A_i(r_k)$, $B_i(r_k)$, $C_i(r_k)$ and $D_i(r_k)$, which are real known matrices with appropriate dimensions.

For notational convenience, in the subsequent sections of this paper, for $r(k)=l$, $l\in\mathbb{T}$, we will denote the system matrices that are related to the lth mode by $A_{il}=A_i(r_k)$, $B_{il}=B_i(r_k)$, $C_{il}=C_i(r_k)$, $D_{il}=D_i(r_k)$.

It is assumed that the premise variables are not dependent on the input variables $u(k)$. Then if given a pair of $(x(k),u(k))$, for any $r(k)=l$, $l\in\mathbb{T}$, the final model of the fuzzy Markov jump system in (1) can be easily obtained as follows:

$$\begin{cases} x(k+1)=\sum_{i=1}^{w}h_i(s(k))\{A_{il}x(k)+B_{il}u(k)\} \\ y(k)=\sum_{i=1}^{w}h_i(s(k))\{C_{il}x(k)+D_{il}u(k)\} \end{cases} \tag{2}$$

where

$$h_i(s(k))=\frac{v_i(s((k))}{\sum_{i=1}^{w}v_i(s((k))},\quad v_i(s((k))=\prod_{f=1}^{h}\mu_{if}(s_f(k))$$

In (2), $h_i(s(k))$ is the fuzzy basis function, $\mu_{if}(s_f(k))$ is the grade membership of $s_f(k)$ in μ_{if}. Suppose $v_i(s(k))\geq 0$ and $\sum_{i=1}^{w}v_i(s(k))>0$ for all k. Therefore, we have $h_i(s(k))\geq 0$ and $\sum_{i=1}^{w}h_i(s(k))=1$ for all k.

In this paper, we are interested in finding an effective method by which the original system (2) can be accurately approximated by a reduced-order mathematical model that is of the following structure:

$$\begin{cases} \hat{x}(k+1)=\sum_{i=1}^{w}h_i(s(k))\{\hat{A}_{il}\hat{x}(k)+\hat{B}_{il}u(k)\} \\ \hat{y}(k)=\sum_{i=1}^{w}h_i(s(k))\{\hat{C}_{il}\hat{x}(k)+\hat{D}_{il}u(k)\} \end{cases} \tag{3}$$

where $\hat{x}(k)\in\mathbb{R}^t$ is the state vector of the reduced-order system with $t<n$, $\hat{A}_{il}$, $\hat{B}_{il}$, $\hat{C}_{il}$ and $\hat{D}_{il}$, for all $l\in\mathbb{T}$, are matrices with appropriate dimensions that are of compatible manipulations. These matrices in the reduced-order model are to be determined later.

By denoting $\tilde{x}(k)\triangleq[x^T(k)\ \hat{x}^T(k)]^T$, $e(k)\triangleq y(k)-\hat{y}(k)$, we are able to augment the model of (2) to include (3), the error system is