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Neurocomputing 69 (2006) 1665–1669
NEUROCOMPUTING
www.elsevier.com/locate/neucom

Letters

An improved discrete Hopfield neural network for Max-Cut problems

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Received 15 October 2005; received in revised form 21 January 2006; accepted 7 February 2006
Communicated by R.W. Newcomb
Available online 11 May 2006

Abstract

In this paper, we proposed an improved discrete Hopfield neural network (DHNN) for Max-Cut problems. By introducing a nonlinear self-feedback term to the motion equation of the DHNN, the DHNN can escape from local minima and therefore get better solutions. Simulation results show that the proposed algorithm has superior ability for Max-Cut problems within reasonable number of iterations.

Keywords: Discrete Hopfield neural network; Max-Cut problem; Nonlinear dynamics

1. Introduction

One of the most important combinatorial optimization problems in graph theory is the Max-Cut problem [9]. Given a weighted, undirected graph $G=(V,E)$, the goal of this problem is to find a partition of vertices of the graph G into two disjoint sets, such that the total weight of the edges from one to the other is as large as possible. For example, Fig. 1(b) shows one of the Max-Cut solutions of Fig. 1(a). Many optimization problems can be formulated to find a Max-Cut in a network or a graph. Besides its theoretical importance, the Max-Cut problem has applications in the design of VLSI circuits, the design of communication networks, circuit layout design and statistical physics [1,4,10]. This problem is one of Karp's original NP-complete problems [9].

Since the Max-Cut problem is NP-complete for which the exact solution is difficult to obtain, different heuristic or approximation algorithms have been proposed, for example, algorithms based on the SDP relaxation proposed by Goemans et al. [6], Helmberg et al. [7], Benson et al. [2] and Choi et al. [5], and heuristic algorithm proposed by Burer et al. [3]. Furthermore, Wu et al. [11] gave new convergence conditions for discrete Hopfield neural networks (DHNN) [8] and proposed a DHNN with negative diagonal weight matrix for the Max-Cut problem, which improved the solution quality dramatically.

However, like the traditional Hopfield neural network [8], the DHNN with negative diagonal weight matrix converges to the first stable state it encounters, which is a local minima problem. In order to prevent a network to be trapped in a stable state corresponding to a local minimum, we introduce a nonlinear self-feedback term to the motion equation of the DHNN. We apply the proposed algorithm to the Max-Cut problem. A number of instances have been simulated to verify the proposed algorithm.

2. DHNN for Max-Cut problems

Let M be a DHNN with n neurons, where each neuron is connected to all the other neurons. The input, output state and threshold of neuron i is denoted by u_i, v_i and t_i, respectively, for $i=1,\ldots,n$; w_{ij} is the interconnection weight between neurons i and j, where symmetric weights are considered, for $i,j=1,\ldots,n$. Therefore, $W=(w_{ij})$ is a $n\times n$ weight matrix and $T=(t_1,\ldots,t_n)^{\mathrm{T}}$ is a threshold vector. The Lyapunov function of the DHNN is given by [8]

$$E=-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}w_{ij}v_iv_j+\sum_{i=1}^{n}t_ix_i. \qquad (1)$$

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0925-2312/$ - see front matter
doi:10.1016/j.neucom.2006.02.001

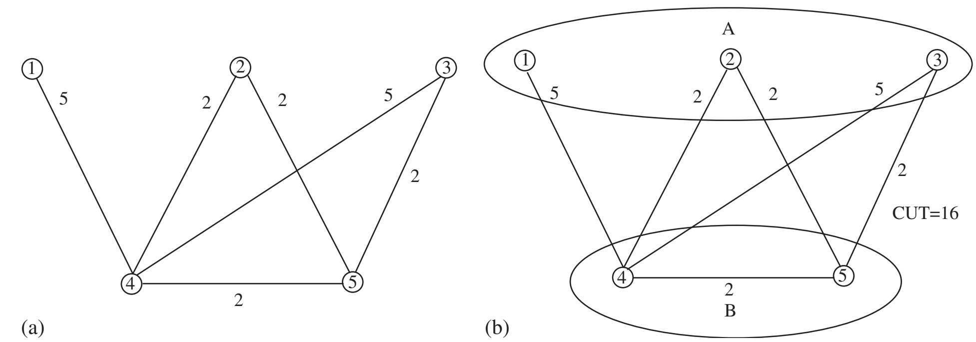

Fig. 1. (a) A simple undirected graph composed of five vertices and six edges, (b) one of its Max-Cut graphs.

The inputs of the neurons are computed by

$$u_i(t+1) = \sum_{j=1}^{n} w_{ij} v_j(t) + t_i, \tag{2}$$

and the output state of the neurons are computed by

$$v_i(t+1) = \mathrm{sgn}(u_i(t+1)), \tag{3}$$

where sgn is the signum function (or bipolar binary function) defined by

$$\mathrm{sgn}(x) = 1 \quad \text{if } x \geqslant 0; \quad -1 \text{ otherwise}. \tag{4}$$

Given initial input values for $u_i(0)$ $(t=0)$, the DHNN, which runs in the asynchronous mode, will always converge to a stable state if its diagonal element of weight matrix W is nonnegative (that is $w_{ii} \geqslant 0$) [8]. There were few applications of the DHNN in combinatorial optimization because the condition of nonnegative diagonal element of weight matrix W is not always satisfied.

Recently, Wu et al. [11] gave new convergence conditions for the DHNN and constructed a DHNN with negative diagonal weight matrix to solve the Max-Cut problem. They proved that the DHNN can converge to a stable state or a stable cycle by rearranging the weight matrix W to even negative diagonal element. That is

$$\text{Let } d_i = \min\left\{ \left| \sum_{j \neq i}^{n} w_{ij} v_j + t_i \right| : \sum_{j \neq i}^{n} w_{ij} v_j + t_i \neq 0, \; v_j \in \{-1, 1\} \right\} \quad \text{if } w_{ii} > -d_i, \quad i = 1, \ldots, n,$$

then the DHNN can converge to a stable state or a stable cycle.

Furthermore, they pointed out that smaller diagonal elements may result in fewer stable states of a network, then reduce the possibility to be trapped in a state corresponding to a local minimum or poor quality solution and may approach to a better solution.

The n vertices Max-Cut problem can be mapped onto the DHNN with n neuron representation [11]. The output $v_i = 1$ means vertex i is assigned to one set, the $v_i = -1$ means vertex i is assigned to another set. The energy function for the Max-Cut problem is given by

$$E = -\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} e_{ij} \frac{1 - v_i v_j}{2}, \tag{5}$$

where e_{ij} is weight on edge whose endpoints are vertex i and j. For example, the matrix for Fig. 1(a) can be written as

$$[e_{ij}] = \begin{bmatrix} 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 5 & 2 \\ 5 & 2 & 5 & 0 & 2 \\ 0 & 2 & 2 & 2 & 0 \end{bmatrix}.$$

3. Proposed algorithm for Max-Cut problems

Wu et al. [11] gave new convergence conditions for the DHNN and constructed a DHNN with negative diagonal weight matrix to solve the Max-Cut problem. However, like the traditional Hopfield neural network, the DHNN with negative diagonal weight matrix converges to the first stable state it encounters, which is a local minima problem [11]. In order to prevent network to be trapped in a stable state corresponding to a local minimum, we introduce a nonlinear self-feedback term to the motion Eq. (2) of the DHNN as defined below:

$$u_i(t+1) = \sum_{j=1}^{n} w_{ij} v_j(t) + t_i + g(u_i(t) - u_i(t-1)). \tag{6}$$

The self-feedback term in Eq. (6) is switched on at $t=1$, so that only $u_i(0)$ is sufficient to initialize the system. In the Eq. (6), g is a nonlinear function as defined below:

$$g(x) = p_1 x \exp(-p_2 |x|), \tag{7}$$

where p_1 and p_2 are adjustable parameters, which determine he chaotic dynamics of the function $g(x)$. Fig. 2 depicts the figure of function $g(x)$ with $p_1 = 15$, and $p_2 = 5$, where $m = 1/p_2$, and $g_m = g(m) = p_1/ep_2$. The introduction of $g(x)$ with suitable parameters p_1 and p_2 will cause the chaotic phenomenon, and more details can be