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Dissipativity analysis of singular systems with Markovian jump parameters and mode-dependent mixed time-delays

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ABSTRACT

In this paper, the problem of delay-dependent α-dissipativity analysis is investigated for a general class of singular systems with Markovian jump parameters and mode-dependent mixed time-delays. By using the novel Lyapunov–Krasovskii functional and stochastic analysis technology, the new criteria are derived to guarantee that the addressed singular systems are stochastically admissible and strictly (Q,R,S)-α-dissipative. Numerical examples are presented to illustrate the effectiveness and less conservativeness of the proposed theoretical results.

1. Introduction

Singular systems are also referred to as generalized systems, descriptor systems, differential-algebraic systems, or implicit systems, which are also a natural representation of dynamic systems and describe a larger family of systems than the normal linear systems [1]. There has been an increasing research interest on singular systems due to their extensive applications in electrical circuits, power systems, economics and other areas in the past decades [2]. It should be pointed out that the study on singular systems is much more complicated than that for state-space systems, because it needs to consider not only stability, but also regularity and impulse free (for continuous singular systems) or causality (for discrete singular systems) simultaneously, while the latter two do not occur in the regular ones [6–8].

It is well known that time delay is frequently encountered in a variety of industrial and engineering systems, and it becomes one of the main sources for causing instability and poor performance of singular systems [14–17]. Furthermore, as a combination of both discrete and distributed delays, the so-called mixed time-delays have gained much research attention and many relevant results have been reported in the literatures [26,27]. And mode-dependent mixed time-delays are of practical significance since the signal may switch between different modes and also propagate in a distributed way during a certain time period with the presence of an amount of parallel pathways [23–25]. Meanwhile, Markovian jump systems are referred to as a special family of hybrid systems and stochastic systems and have extensively been applied to economics, manufacture and communication [30–33]. And many significant results on H_∞ control, stochastical admissibility and H_∞ stability of Markovian jump singular systems with time delays have been available in the literatures [18–21,28].

On the other hand, dissipative systems, introduced in [5,34], are very useful for a wide range of fields such as system, circuit, network and control theory [3,4,9]. And the passivity theorem, the bounded real lemma and the circle criterion are generalized by dissipativity theory, which gives strong links among physics, systems theory and control engineering simultaneously. As pointed out in [11], dissipativity is also an important concept in singular systems. So far, the problem of dissipativity analysis for singular systems with time delays has been investigated in [10,12,13]. However, it is noted that the considered time delays of [10,12] are all time invariant, which limits the scope of applications of the proposed results. In [13], the delay is only discrete time-varying and not mode-dependent mixed delay, which will bring more conservativeness to the actual applications. For the case of Markovian jump singular systems with mode-dependent mixed delays, very little research has addressed dissipativity. To the best of the authors' knowledge, there is no result on the α-dissipativity problem for Markovian jump singular systems with mode-dependent mixed delays.

Motivated by the above discussion, we study α-dissipativity problem for Markovian jump singular systems with mode-dependent mixed delays in this paper. Note that the mixed time delays comprise both the discrete and distributed delays that are both dependent on the Markovian jump mode. The main

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contributions of this paper can be listed as follows: (1) a new Lyapunov–Krasovskii functional is proposed; (2) dissipativity problem for Markovian jump singular systems with mode-dependent mixed time-delays is studied; (3) the result developed in this paper is less conservativeness about delay-dependent than those in the literature. We derive sufficient condition to guarantee that the addressed singular systems are stochastically admissible and strictly (Q,R,S)-α-dissipative. Numerical simulations are presented to further demonstrate the effectiveness of the proposed approach.

Notation: The notation used throughout the paper is standard. $\star$ stands for the symmetric terms in a symmetric matrix and $sym\{A\}$ is defined as $A+A^T$; real matrix M, we define $\langle x,My\rangle_\tau = \int_0^\tau x^T(t)My(t)\,dt$; $\|\cdot\|$ refer to the Euclidean vector norm. $\varepsilon[\cdot]$ stands for the mathematical expectation. Matrices are assumed to be compatible for algebraic operations if their dimensions are not explicitly stated.

2. System description and preliminaries

Consider a class of Markovian jump singular system with disturbances and mode-dependent mixed time-delays described by

$$E\dot{x}(t) = [A(r(t))+\Delta A(r(t))]x(t)+[A_\tau(r(t))+\Delta A_\tau(r(t))]x(t-\tau_{1,r(t)}) + H(r(t))\int_{t-\tau_{2,r(t)}}^{t} f(x(s))\,ds+[B(r(t))+\Delta B(r(t))]\omega(t), \tag{1}$$

$$z(t) = [C(r(t))+\Delta C(r(t))]x(t)+[C_\tau(r(t))+\Delta C_\tau(r(t))]x(t-\tau_{1,r(t)}) + [D(r(t))+\Delta D(r(t)]\omega(t),$$

$$x(t)=\phi(t), t\in[-\tau,0],$$

where $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T\in R^n$ is the state vectors, $\omega(t)\in R^m$ is the exogenous disturbance input that belongs to $L_2[0,\infty)$,$z(t)\in R^p$ is the controlled output. $E,A(r(t)),A_\tau(r(t)),H(r(t)),B(r(t)),C(r(t)),C_\tau(r(t))$ and $D(r(t))$ are known real constant matrices with appropriate dimensions, $rank\, E = p\le n$. $f(\cdot): R^n\to R^n$ is a nonlinear vector function. In (1), $\tau_{1,r(t)}$ stand for the discrete mode-dependent time delays, while $\tau_{2,r(t)}$ describe the distributed mode-dependent time delays, $\tau := \max\{\tau_{i,j}|i=1,2,j=1,2,\ldots,N\}$, $\phi(t)\in L^2_{F_0}([-\tau,0];R^n)$ is the initial condition. Let $\{r(t),t\ge 0\}$ be a right-continuous Markov process on the probability space which takes values in the finite space $\Lambda=\{1,2,\ldots,N\}$, with transition probability matrix $\Pi=\{\delta_{ij}\},(i,j)\in\Lambda$ given by

$$P\{r(t+\Delta t)=j : r(t)=i\} = \begin{cases} \delta_{ij}\Delta t + O(\Delta t) & \text{if } j\ne i, \\ 1+\delta_{ij}\Delta t+O(\Delta t) & \text{if } j=i, \end{cases}$$

with $\Delta t>0$, and $\lim_{\Delta t\to 0} O(\Delta t)/\Delta t=0$. Here, $\delta_{ij}\ge 0$ is the transition rate from i to j if $j\ne i$, while $\delta_{ii}=-\sum_{j=1,j\ne i}^N \delta_{ij}$.

In this paper, we make the following assumption:

$$[f(x)-G_1x]^T[f(x)-G_2x]\le 0, \quad f(0)\equiv 0, \tag{2}$$

where G_1,G_2 are known constant matrices, and the uncertainties are assumed to be norm-bounded and for each $r(t)=i,i\in\Lambda$, can be described as

$$\begin{bmatrix} \Delta A_i(t) & \Delta A_{\tau i}(t) & \Delta B_i(t) \\ \Delta C_i(t) & \Delta C_{\tau i}(t) & \Delta D_i(t) \end{bmatrix} = \begin{bmatrix} M_{1i} \\ M_{2i} \end{bmatrix} F_i(t)[N_{1i}\ \ N_{2i}\ \ N_{3i}], \tag{3}$$

where $M_{1i},M_{2i},N_{1i},N_{2i},N_{3i}$ and N_{4i} are known real constant matrices with appropriate dimensions. The uncertain matrix $F_i(t)$ satisfies $F_i^T(t)F_i(t)\le I,\forall i\in\Lambda,\forall t$.

Definition 1 (*Ma and Boukas [20]*). 1. The Markovian jump singular time-delay system

$$E\dot{x}(t) = A_i x(t)+A_{\tau i}x(t-\tau_{1,i}) \tag{4}$$

is said to be regular and impulse free if the pairs $(E,A_i),(E,A_i+A_\tau(i))$ are regular and impulse free, for each mode $i\in\Lambda$.

2. The system (4) is said to be stochastically stable, if for any $x(0)\in R^n$ and $r(0)\in\Lambda$, there exists a scalar $\tilde{M}(x(0),r(0))>0$ such that

$$\lim_{t\to\infty}\varepsilon\left\{\int_0^t x^T(s,x(0),r(0))x(s,x(0),r(0))\,ds\,|x(0),r(0)\right\}\le \tilde{M}(x(0),r(0)),$$

where $x(t,x(0),r(0))$ denotes the solution of the system (4) at time t under the initial conditions $x(0)$ and $r(0)$, $\varepsilon\{\}$ denotes the expectation operator.

3. The system (4) is said to be stochastically admissible, if it is regular, impulse-free and stochastically stable.

Definition 2. The uncertain Markovian jump singular system (1) is said to be robustly stochastically admissible, if the system with $\omega(t)=0$ is stochastically admissible for all admissible uncertainties $\Delta A(r(t))$ and $\Delta A_\tau(r(t))$.

The quadratic energy supply function Ψ associated with system (1) is defined by

$$\Psi(\omega,z,\tau)=\varepsilon\langle z,Qz\rangle_\tau+2\varepsilon\langle z,S\omega\rangle_\tau+\varepsilon\langle\omega,R\omega\rangle_\tau,$$

where Q, S and R are real matrices of appropriate dimensions with Q and R symmetric. Without loss of generality, it is assumed that $Q\le 0$.

Definition 3. Singular system (1) is said to be strictly (Q,R,S)-dissipative, if for any $\tau\ge 0$ and some scalar $\alpha>0$, under zero initial state, the following condition is satisfied:

$$\Psi(\omega,z,\tau)\ge\alpha\varepsilon\langle\omega,\omega\rangle_\tau.$$

Lemma 1 (*Xu and Lam [2]*). *The Markovian jump singular system (4) is stochastically admissible if and only if there exist matrices $P_i, i=1,2,\ldots,N$, such that*

$$\begin{cases} E^TP_i = P_i^TE\ge 0, \\ (A_i+A_{\tau i})^TP_i+P_i^T(A_i+A_{\tau i})+\sum_{j=1}^N\delta_{ij}E^TP_j<0. \end{cases} \tag{5}$$

Lemma 2 (*Xie and Souza [29]*). *For symmetric positive-definite matrix Q and matrices P, R with appropriate dimensions, the following inequality holds*:

$$PR^T+RP^T\le RQR^T+PQ^{-1}P^T.$$

Lemma 3 (*Petersen [22]*). *Given matrices Ω,Γ and Θ with appropriate dimensions and with Ω symmetrical, then*

$$\Omega+\Gamma F(t)\Theta+\Theta^TF^T(t)\Gamma^T<0,$$

for any $F^T(t)F(t)\le 0$ if and only if there exists a scalar $\epsilon>0$ such that

$$\Omega+\epsilon\Gamma\Gamma^T+\epsilon^{-1}\Theta^T\Theta<0.$$

Lemma 4 (*Wang et al. [23]*). *For any matrix $M>0$, scalar $\gamma>0$, vector function $\varphi:[0,\gamma]\to R^n$ such that the integrations concerned are well defined, the following inequality holds*:

$$\left(\int_0^\gamma\varphi(s)\,ds\right)^T M\left(\int_0^\gamma\varphi(s)\,ds\right)\le\gamma\left(\int_0^\gamma\varphi^T(s)M\varphi(s)\,ds\right).$$

3. Main results

Let us now consider the stochastically admissibility and strictly (Q,R,S)-α-dissipativity of the singular systems (1) with