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# Filter design of delayed static neural networks with Markovian jumping parameters

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ABSTRACT

This paper considers the $H_\infty$ filtering problem of static neural networks with Markovian jumping parameters and time-varying delay. A mode and delay dependent approach is presented to deal with it. By constructing a stochastic Lyapunov functional with triple-integral terms and employing a recently proposed integral inequality, a design criterion is derived under which the resulting filtering error system is stochastically stable with a guaranteed $H_\infty$ performance. Based on it, the proper gain matrices and optimal $H_\infty$ performance index can be efficiently obtained via solving a convex optimization problem subject to some linear matrix inequalities. An advantage of this approach is that most of the Lyapunov matrices are distinct with respect to system mode and thus the choice of these matrices becomes much flexible. Finally, an example is provided to illustrate the application and effectiveness of the developed result.



## 1. Introduction

As been known, the phenomenon of information latching frequently appears in recurrent neural networks. One of the promising means to tackle it is attempt to extract a finite state representation such that the neural network under consideration has finite modes [1,2]. Furthermore, as discussed in [3–5], the switching between these modes is dominated by a Markov chain. Consequently, a kind of hybrid neural network models was established and named as recurrent neural networks with Markovian jumping parameters (or Markovian jumping recurrent neural networks). On the other hand, time delay is inevitable in real systems due to signal transmission and processing between components [6–11]. In recurrent neural networks, time delay should be also taken into account [42,12–15]. More details can be found in the recent survey paper [16]. During the past few years, the study of delayed neural networks with Markovian jumping parameters has attracted a great deal of attention. Many interesting results have been reported in the literature (see, e.g., [17–20], etc.).

Generally, it could be very difficult and expensive to acquire the complete information of states of all neurons in a relatively large-scale delayed neural networks, especially, when Markovian jumping parameters are involved. However, it is found that many neural-network-based applications heavily rely on these information [21,22]. That is to say, in order to achieve special objectives, it is very necessary to know the neurons' state information in advance. Therefore, in this sense, we are required to propose a feasible approach to implementing the estimation of the states of all neurons. Then, in place of the true states of neurons, the estimated states can be utilized in practice. Since the seminal work in [23], the problem of state estimation of delayed recurrent neural networks has been widely investigated [24–27].

More recently, the state estimation problem was considered for delayed local field neural networks with Markovian jumping systems [28–31]. In [29], the authors presented a mode-dependent approach to studying this problem for a class of local field neural networks with Markovian jumping parameters and mixed delays. It was proved that the design criterion in [29] includes the one in [30] as a special case. The authors in [31] discussed the state estimation problem for discrete-time delayed Markovian jumping neural networks. A delay-dependent condition was provided by means of linear matrix inequalities (LMIs), which can be easily solved by some available algorithms [32].

It should be noted that almost all the above results are on local-field-type neural networks. It has been recognized that static neural networks, another kind of important recurrent neural networks, are distinct from local field ones [33]. It means that the design criteria in [28–31] are not able to be directly applied to delayed static neural networks with Markovian jumping parameters. This is the first motivation of the current study.

Meanwhile, in the theory of state estimation of delayed recurrent neural networks, performance analysis should be taken into account. In this framework, we can define an index to evaluate the

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performance of various approaches in the literature. As a matter of fact, noise disturbances could be introduced during the hardware implementation process of a recurrent neural network [34–36]. From this point of view, it is of great significance to consider the performance analysis in the state estimation theory of delayed static neural networks with Markovian jumping parameters. This also motivates our study.

In this paper, our attention focuses on the study of $H_\infty$ filtering problem for a class of delayed static neural networks with Markovian jumping parameters. Firstly, the mathematical model of such kind of neural networks is presented. Then, inspired by [29], a mode and delay dependent approach is proposed to resolve this issue. By defining a suitable stochastic Lyapunov functional with triple-integral terms and employing Wirtinger inequality [37] to cope with a cross term in the weak infinitesimal operator, a condition is derived such that the filtering error system is stochastically stable with a guaranteed $H_\infty$ performance. We can see that the optimal $H_\infty$ performance index and gain matrices are efficiently worked out via solving an LMI-based convex optimization problem. The advantage of this approach is that most of the Lyapunov matrices to be determined are district with respect to system mode. It is thus that the choice of these matrices is more flexible than common ones. A numerical example is finally given to demonstrate the effectiveness of our result.

## 2. Notations and problem description

Let $\mathbb{R}$ be the set of real numbers, $\mathbb{R}^n$ the $n$-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ the set of all $n\times m$ real matrices. For $t\ge 0$, $r_t$, taking values from a finite set $\mathcal{M}=\{1,2,\ldots,M\}$, is a right-continuous Markov chain defined on a complete probability space $(\Omega,\mathcal{F},\mathcal{P})$. Its transition probabilities between different modes are given by

$$\mathcal{P}\{r_{t+h}=j|r_t=i\}=\begin{cases}\pi_{ij}h+o(h), & i\neq j\\ 1+\pi_{ii}h+o(h), & i=j\end{cases}$$

where $h>0$, $o(h)$ is a high-order infinitesimal of $h$ (i.e., $\lim_{h\to 0^+} o(h)/h=0$), $\pi_{ij}\ge 0$ for $j\neq i$, and for each $i\in\mathcal{M}$,

$$\pi_{ii}=-\sum_{j=1,j\neq i}^{M}\pi_{ij}.$$

$\mathcal{C}([-d,0];\mathbb{R}^n)$ is the family of continuous functions from $[-d,0]$ to $\mathbb{R}^n$, and $L_2[0,\infty)$ is the space of square-integrable vector functions over the interval $[0,\infty)$. For real square matrices $X$ and $Y$, $X>Y$ ($X<Y$, $X\ge Y$ and $X\le Y$) means that $X-Y$ is symmetric and positive definite (negative definite, positive semi-definite and negative semi-definite, respectively). $\mathrm{diag}\{Z_1,Z_2,\ldots,Z_p\}$ is a block diagonal matrix with diagonal blocks $Z_1$, $Z_2$, …, $Z_p$. $I$ is an identity matrix with appropriate dimension. The superscripts "$\top$" and "$-1$" respectively represent the transpose and the inverse of a matrix or vector if applicable. $*$ stands for the symmetric block in a symmetric matrix (e.g., $\begin{bmatrix} X & Y\\ * & Z\end{bmatrix}=\begin{bmatrix} X & Y\\ Y^\top & Z\end{bmatrix}$). Matrices, if not explicitly stated, are supposed to have compatible dimensions.

It is known that a delayed static neural network is expressed by

$$\dot{x}(t)=-Ax(t)+f(Wx(t-d(t))+J), \tag{1}$$

where $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^\top\in\mathbb{R}^n$ is a state vector of the neural network with $x_k(t)$ being the state of the $k$th neuron; $A=\mathrm{diag}(a_1,a_2,\ldots,a_n)$ is a diagonal matrix with each element $a_k>0$ being the firing rate of neuron $k$; $W=[w_{jk}]_{n\times n}$ is a delayed connection weight matrix; $f(x(t))=[f_1(x_1(t)),\ f_2(x_2(t)),\ \ldots,\ f_n(x_n(t))]^\top$ is a continuous activation function; $J=[J_1,J_2,\ldots,J_n]^\top$ is an external input vector, and $d(t)$ is a time-varying delay.

With the rapid development of science and technology, it could not be a good choice to design one neural network to handle a practical problem with high nonlinearity and complexity. In some real applications, one needs to combine several neural networks to gain better performance. That is, among them, every neural network is designated to solve a specific subproblem in a comprehensive situation. It inevitably leads to designing and investigating hybrid neural networks. On the other hand, the so-called information latching phenomenon is frequently encountered in recurrent neural networks. One promising way to tackle this issue is to take a finite state representation from the underlying neural network. In this sense, it can be viewed as having finite modes, and the transition between these modes is governed by a Markov chain [1,4]. As a result, it is reasonable to introduce Markovian jumping parameters into recurrent neural networks with information latching. In recent years, much effort was spent to discuss delayed recurrent neural networks with Markovian jumping parameters. Many interesting results have been available on stability and passivity analysis, and state estimation of such kind of delayed neural networks [3,5,17–20,28–31]. While, almost all the results mentioned above are concerned with local-field-type neural networks. It is well known from [33] that static neural networks are generally not equivalent to local field ones. Based on these observations, in this study, we dedicate to considering delayed static neural networks with Markovian jumping parameters. Similar to [18,29,30], the mathematical model of a delayed static neural network with Markovian jumping parameters is given by

$$\dot{x}(t)=-A_{r_t}x(t)+f_{r_t}\left(W_{r_t}x(t-d(t))+J_{r_t}\right)+B_{1r_t}w(t), \tag{2}$$

$$y(t)=C_{r_t}x(t)+D_{r_t}x(t-d(t))+B_{2r_t}w(t), \tag{3}$$

$$z(t)=E_{r_t}x(t), \tag{4}$$

$$x(t)=\phi(t),\quad t\in[-d,0]\quad\text{and}\quad r_t=r_0, \tag{5}$$

where $x(t)\in\mathbb{R}^n$ and $y(t)\in\mathbb{R}^m$ are respectively the state vector and available output measurement. $z(t)\in\mathbb{R}^p$ is a linear combination of the states, which is to be estimated. $w(t)\in\mathbb{R}^q$ is a noise signal in the space $L_2[0,\infty)$. $d(t)$ is a time-varying delay with an upper bound $d>0$. For each $r_t\in\mathcal{M}$, $f_{r_t}$ is an activation function, $A_{r_t}$, $W_{r_t}$, $B_{1r_t}$, $C_{r_t}$, $D_{r_t}$, $B_{2r_t}$ and $E_{r_t}$ are real known matrices with compatible dimensions, and $J_{r_t}$ is an external input vector. $\phi(t)$ is an initial function belonging to $\mathcal{C}([-d,0];\mathbb{R}^n)$ and $r_0$ is an initial mode.

In fact, for a fixed $r_t$, (2) becomes a delayed static neural network. Therefore, the hybrid neural network model (2) can be regarded as a combination of $M$ different delayed static neural networks

$$\dot{x}(t)=-A_ix(t)+f_i\left(W_ix(t-d(t))+J_i\right)+B_{1i}w(t)$$

with $i=1,2,\ldots,M$. The switching between them follows the movement of the Markov chain $r_t$ ($t\ge 0$).

For convenience, for each $r_t=i\in\mathcal{M}$, we denote the matrix $X_{r_t}$ to be $X_i$ and the activation function $f_{r_t}(x(t))$ to be $f_i(x(t))$. For example, when $r_t=i$, the matrices $A_{r_t}$ and $B_{1r_t}$ in (2) are written as $A_i$ and $B_{1i}$, respectively. As defined before, the activation function $f_i(x(t))$ in (2) has the form $f_i(x(t))=[f_{i1}(x_1(t)),f_{i2}(x_2(t)),\ldots,f_{in}(x_n(t))]^\top$ with $i\in\mathcal{M}$. In this study, it is assumed that

**Assumption 1.** For each $i\in\mathcal{M}$ and $u\neq v\in\mathbb{R}$, there are constants $l_{ik}^-$ and $l_{ik}^+$ such that the activation function $f_{ik}(x(t))$ satisfies

$$l_{ik}^-\le\frac{f_{ik}(u)-f_{ik}(v)}{u-v}\le l_{ik}^+. \tag{6}$$

Here, $k=1,2,\ldots,n$.

**Assumption 2.** For $t>0$, there are constants $d>0$ and $\mu$ such that the time delay $d(t)$ satisfies

$$0\le d(t)\le d\quad\text{and}\quad\dot{d}(t)\le\mu. \tag{7}$$