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# Second-order group consensus for multi-agent systems with time delays

Dongmei Xie [a,b,*], Teng Liang [a]

[a] Department of Mathematics, School of Sciences, Tianjin University, Tianjin, China
[b] Department of Mathematics, University of California, Irvine, CA, USA

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ABSTRACT

This paper studies the group consensus problem of second-order multi-agent systems (MASs) with time delays. First, by state transformation method, the group consensus problem of multi-agent systems can be equivalently transformed into the asymptotical stability of a time-delay system. Then, by Lyapunov first method and Hopf bifurcation theory, respectively, we aim to find the upper bound of time delay $\tau^*$ such that the multi-agent systems can achieve group consensus for $\tau \in [0, \tau^*)$. Finally, simulation examples are given to show the effectiveness of our theoretical analysis.



## 1. Introduction

Multi-agent system is a computerized system consisting of multiple interacting intelligent agents within an environment, where each agent may be a physical or abstract entity. Because of the characteristics of autonomy, distribution, coordination and the abilities of self-organization, multi-agent technology is widely used in cooperative control of unmanned air vehicles, flocking of multiple vehicles [1], formation control [2,3], scheduling of automated highway systems and other fields [4–6]. In these applications, a single agent is often required to be consistent with the rest of agents on some quantities (position, velocity, etc.), which is called the consensus problem.

Consensus problem is one of the important issues in coordination control of multi-agent systems and receives the researchers' attention [7–17]. Vicsek et al. [10] proposed a simple model for phase transition of a group of self-driven particles and numerically demonstrated complex dynamics of the model. Using graph theory, Jadbabaie et al. [11] provided a theoretical explanation for the consensus behavior of the Vicsek model. Based on the analysis in [11], Saber et al. [12] investigated the consensus problems for networks of dynamic agents with fixed and switching topologies by discussing three cases: directed networks with fixed topology, directed networks with switching topology, undirected networks with communication time-delays and fixed topology. Moreover, [12,13] gave some relationships between topology and Laplacian matrix. Ref. [14] established a necessary and sufficient second-order consensus criterion and proved that both the real and imaginary parts of the eigenvalues of the Laplacian matrix play key roles in reaching consensus. Ref. [16] studied the average-consensus problem in directed networks of agents with both switching topology and time delays. In [17], a leader-following consensus problem of second-order multi-agent systems with fixed and switching topologies as well as non-uniform time-varying delays is considered.

Recently, group/cluster consensus has been pointed out as an important elaboration of the classical consensus problem and attracted a great deal of attention within the control community [18–24]. Group consensus means that all the agents in a multi-agent system are divided into multiple subgroups and each group will reach a consistent state while no consensus appears among different groups. Moreover, group consensus is a fundamental phenomenon in nature and human society, such as the group formation of personal opinion, and birds coordination by interacting with peers in their own and other species [25]. Compared with traditional consensus problem, group consensus problem is more complex. Therefore, some necessary assumptions on the coupling matrix of the network were given to ensure the realization of group consensus in most papers. For example, [18] assumed that the influence from any other group to the node is equal to zero, i.e., the in-degree of the node is balanced to other groups. Refs. [19,22] required the coupling matrix to be irreducible, while [20,23,24] and [21] assumed the coupling matrix to be symmetric and stochastic, respectively. Ref. [26] discussed the average group consensus for multi-agent systems with directed topology. By introducing double-tree-form transformation, [27] investigated group consensus of multi-agent systems with switching topologies and time delays, where the agents are described by single-integrator dynamics.

* Corresponding author.
*E-mail address:* lflxdm@pku.org.cn (D. Xie).

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Group consensus control for second-order dynamic multi-agent systems was investigated in [28,29]. The agents in [29] were described by nonlinear dynamics and group consensus problem was addressed through leader-following approach and pinning control. As for the aforementioned papers, most of them neglected the effect of time delays on the group consensus of MASs. In fact, time delay is ubiquitous in multi-agent systems due to the possible slow process of interactions agents. Control protocol without considering the effect of time delays may cause the instability of MASs. This motivates us to write this paper.

In this paper, we will establish the group consensus criteria of second-order multi-agent systems with time delays by seeking the maximal time-delay bound. The motivation of this work is to extend and complement the group consensus results in [27] from single-order MASs to second-order MASs with time-delays. As a comparison, [27] established the LMI-based group consensus criteria for single-order multi-agent system with time-delays by seeking a Lyapunov function and neglected the computation of the allowable upper bound of time delays. Moreover, the construction and choice of Lyapunov function will lead to much more conservative results. Ref. [28] just studied second-order multi-agent systems by matrix analysis and neglected the time delays between agents. Our main contributions can be summarized as follows. (I) The second-order group consensus problem is more complicated and challenging than that of the first-order case as its state vector includes not only the position vector but also the velocity vector. To analyze the system matrix, we establish two important lemmas (see Lemmas 1 and 3), which also shows that the extension of group consensus from single-integrator dynamics to second-order cases is not trivial. (II) We use Lyapunov first method [17] instead of Lyapunov second method in most existing literatures to establish the group consensus criteria, where the upper bound of time delays can be estimated by Eq. (8). This method avoids to seek the Lyapunov function and reduces the conservation of our group consensus criteria. (III) Hopf bifurcation theory is used as a different and efficient method to give the time delay bound. Compared with these two methods, it is easy to see that the latter method can provide a less conservative time delay bound, while the time delay bound is easier to describe and calculate by using the former method.

The remaining of this paper is organized as follows. In Section 2, we review some important knowledge in graph theory, formulate our system models and give some necessary lemmas. Main results are established in Sections 3. Section 4 gives numerical simulations to show the effectiveness of our results. Finally, conclusions are drawn in Section 5.

*Notations*: We use standard notations throughout this paper. $M^T$ is the transpose of the matrix $M$. $M^{-1}$ represents the inverse of the matrix $M$. $\otimes$ is the Kronecker product. $I_n$ denotes the identity matrix of dimension $n$. Define $\ell$:=$\{1,2,\ldots,n+m-2\}$. $\Lambda(M)$ denotes the eigenvalue set of the matrix $M$. $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ denote real and imaginary parts, respectively. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

## 2. Preliminaries and problem statement

In this section, some basic concepts, results about graph theory are introduced and some problem formulations, definitions, assumptions and supporting lemmas are given to obtain the main results of the paper.

### 2.1. Graph theory

Let $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ be a weighted directed graph of order $n+m$, where $\mathcal{V}=\{v_1,v_2,\ldots,v_{n+m}\}$ is the set of nodes, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the set of edges, and $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{(n+m)\times(n+m)}$ is a weighted adjacency matrix with real adjacency elements $a_{ij}$. The node indexes belong to a finite index set $\varphi$:=$\{1,2,\ldots,n+m\}$. An edge of $\mathcal{G}$ is denoted by $e_{ij}=(v_j,v_i)$. The adjacency elements associated with the edges of the graph are nonzero, i.e., $e_{ij}\in\mathcal{E}$ if and only if $a_{ij}\neq 0$. Moreover, we assume $a_{ii}=0$ for all $i\in\varphi$. The set of neighbors of node $v_i$ is denoted by $N_i=\{v_j\in\mathcal{V}:(v_j,v_i)\in\mathcal{E}\}$. The in-degree and out-degree of node $v_i$ are defined, respectively, as

$$deg_{in}(v_i)=\sum_{j=1}^{n+m}a_{ij},\quad deg_{out}(v_i)=\sum_{j=1}^{n+m}a_{ji}.$$

$L(\mathcal{G})=[l_{ij}]$ is the Laplacian matrix of topology $\mathcal{G}$, and is defined by

$$l_{ij}=\begin{cases}-a_{ij}, & j\neq i,\\ \sum_{k=1,k\neq i}^{n+m}a_{ik}, & j=i.\end{cases}$$

### 2.2. Problem formulations

Suppose that the network system consists of $n+m$ agents and each agent has the dynamics as follows:

$$\begin{cases}\dot{x}_i(t)=v_i(t),\\ \dot{v}_i(t)=u_i(t),\end{cases}\tag{1}$$

where $x_i\in\mathbb{R}$, $v_i\in\mathbb{R}$, $u_i\in\mathbb{R}$ are the position, velocity and control input of the $i$th agent, respectively.

Without loss of generality, this paper just considers the case that all the $n+m$ agents are divided into two groups, which can be easily generalized to multi-agent systems with more groups. Define $\varphi_1:=\{1,2,\ldots,n\}$, $\varphi_2$:=$\{n+1,n+2,\ldots,n+m\}$ and $\mathcal{V}_1$:=$\{v_1,v_2,\ldots,v_n\}$, $\mathcal{V}_2$:=$\{v_{n+1},v_{n+2},\ldots,v_{n+m}\}$ as the index sets and node sets to denote these two groups, respectively. Then, $\mathcal{V}=\mathcal{V}_1\bigcup\mathcal{V}_2$ and $\varphi$:=$\varphi_1\bigcup\varphi_2$. Moreover, define $N_{1i}=\{v_j\in\mathcal{V}_1:(v_j,v_i)\in\mathcal{E}\}$, $N_{2i}=\{v_j\in\mathcal{V}_2:(v_j,v_i)\in\mathcal{E}\}$ and $N_i$=$N_{1i}\bigcup N_{2i}$. It is easy to see that $N_{1i}$, $N_{2i}$, $N_i$ are the neighbor sets of node $v_i$ in $\mathcal{V}_1$, $\mathcal{V}_2$ and $\mathcal{V}$, respectively. According to the fact that agents may not be able to get instant information from each other because of coupling delays, this paper will take full account of the effect of time delays and adopt the following control protocol:

$$u_i(t)=\begin{cases}-2kv_i(t)+k\sum_{j\in N_{1i}}a_{ij}[x_j(t-\tau)-x_i(t-\tau)]+k\sum_{j\in N_{2i}}a_{ij}x_j(t-\tau), & i\in\varphi_1,\\ -2kv_i(t)+k\sum_{j\in N_{1i}}a_{ij}x_j(t-\tau)+k\sum_{j\in N_{2i}}a_{ij}[x_j(t-\tau)-x_i(t-\tau)], & i\in\varphi_2,\end{cases}\tag{2}$$

where $k>0$, $\tau>0$ is the time-delay constant, $a_{ij}\geq 0,\forall i,j\in\varphi_1$; $a_{ij}\geq 0,\forall i,j\in\varphi_2$; $a_{ij}\in\mathbb{R},\forall(i,j)\in I=\{(i,j):i\in\varphi_1,j\in\varphi_2\}\cup\{(i,j):j\in\varphi_1,i\in\varphi_2\}$. If $a_{ij}<0$, it can be considered that agent $j$ has a negative effect on agent $i$.

**Remark 1.** Protocol (2) can be regarded as a generalization of the control protocol in [27]. In protocol (2), information exchange exists not only between two agents in the same group but also in different groups. Besides, $k\sum_{j\in N_{1i}}a_{ij}[x_j(t-\tau)-x_i(t-\tau)]$ means that an agent gets relative information from the other agents in the same group. $k\sum_{j\in N_{2i}}a_{ij}x_j(t-\tau)$ means that an agent gets absolute information from the other agents in different group. The main difference between these two groups is that agents in different group have different consensus goal according to the task assignment.

**Remark 2.** The Laplacian matrix $L$ is not diagonally dominant as the weighted factors $a_{ij},(i,j)\in I$ in control protocol (2) are permitted to be negative in this paper. Thus, compared with the most existing literatures [1–17], the Laplacian matrix $L$ does not have the property that all the nonzero eigenvalues have positive real parts. Lemma 5 in [18] has proved that $L$ has a zero eigenvalue whose geometric multiplicity is at least two. Hence, the algebraic multiplicity of zero eigenvalue is at least two. In this paper, we give the following assumption: