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Reliable stabilization for memristor-based recurrent neural networks with time-varying delays

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ABSTRACT

In this paper, a general class of memristive recurrent neural networks with time-varying delays is considered. Based on the knowledge of memristor and recurrent neural networks (RNNs), a model of memristive based RNNs is established. After that the problem of reliable stabilization is studied by constructing a suitable Lyapunov–Krasovskii functional (LKF) and using linear matrix inequality (LMI) framework. By use of the Wirtinger-type inequality, sufficient conditions are presented for the existence of a reliable state feedback controller, which can guarantee the global asymptotic stability of the memristive RNNs. Finally, an example is given to illustrate the theoretical results via numerical simulations.

1. Introduction

Memristor was first proposed by Chua in 1971 [4]. He predicted that, in physical circuit models apart from the resistor, capacitor and inductor, there should be a fourth circuit element which is called the memristor (as a contraction of memory and resistor). Also, some inventions theoretically showed that the value of the memristor, called memristance, is the function of electric charge q given as $M(q) = d\phi/dq$, where ϕ denotes the magnetic flux. After that, in 2008, a group of scientists from Hewlett-Packard Lab used the theory to build a prototype of the memristor based on nanotechnology [26]. As mentioned in these theories, unlike the other three fundamental circuit elements, the memristor is a nonlinear one and its value is not unique. Moreover, this new circuit element shares many properties of resistors, and also shares the same unit of measurement (ohm). However, the new circuit element of memristor has got much attention of the researches due to its potential applications in many physical systems like nonlinear semiconductor devices, analog computation, circuits which mimic neuromorphic and biological systems, and programmable logic. Recently, oscillator circuits were designed using memristors [11]. As pointed in [31], new type of computers made by using memristor memory will have no boosting time, memristor based battery of cell phone could last one to two months rather than several days, etc.

On the other hand, it is noted that a memristor works like a real biological synapse, so it is useful to model a memristive based neural networks, which can approximately mimic real synapses and they may behave like the real object. The capability of memristors to remember and to work as analog devices allows them to assume any of the many values over a range, just as synapses do [22]. Many researchers in the field of memristors have suggested that this device has high potential for implementing artificial synapses, see [9,25,27]. Also, even brain-like computer can be made by using memristive synapses. Hu and Wang [10] studied the memristor based recurrent neural network by using the memristors instead of resistors. In the new model, the parameters change according to its state; i.e., it is a state-dependent switching. Further, many interesting results have been available in the literature, for example, dynamics analysis of a class of memristor-based recurrent networks with different time delays are discussed in [9,28]. A recurrent neural network model with time delays and its global uniform asymptotic stability is analyzed in [10]. The delay-dependent stability and strict $(Q, S, R)-$dissipativity analysis for cellular neural networks with distributed delay is investigated in [5] and the problem of reliable dissipative control for a singular Markovian system with actuator failure is investigated in [6]. The global asymptotic stability for a general class of memristor-based recurrent neural networks with time-varying delays has been investigated in [35]. Anti-synchronization control of a class of memristive recurrent neural networks was discussed in [31]. All the results in the

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literature are extracted from the theory of differential inclusions with discontinuous right-hand side as introduced by Filippov [7].

Moreover, it is well known that the RNNs are very important nonlinear circuit networks because of their wide range of applications in pattern recognition, associative memory, optimization problems, systems control, data compression, image processing, etc. However, all such applications depend on the dynamical behaviors, particularly, stability of networks. Therefore, stability is one of the main properties of RNNs [21]. In electronic implementation of analog networks, delay is usually time-varying due to the finite switching speed of amplifiers. It is also known that time delay is the main cause of instability and poor performance of networks. Therefore, it is important to study the stability of memristive RNNs with time-varying delays. Hence, the many interesting results for neural networks with time delays have been found in the literature, see [1,12,13,23]. The problem of robust stability for neutral type system with mixed delays and time-varying structured uncertainties has been discussed in [20]. The mean-square exponential stability of uncertain neural networks with time-varying delay and stochastic perturbation has been derived in [3]. The sampled-data synchronization for Markovian jump neural networks with time-varying delay and variable samplings is considered in [32]. Zhang et al. [33] studied the stability analysis for discrete-time recurrent neural networks with time-varying delays.

On the other hand, in recent years, the stabilization of neural networks has been appealing topic and got the attention of many researchers, see [14,29] and references therein. According to the related researches on memristive neural networks, it should be noted that, during the dynamical processes, the memristive neural networks exhibit state-dependent nonlinear switching behaviors due to abrupt changes at certain instants [30]. Therefore, the study of stabilization of memristive based RNNs is much more complex than the RNNs, because it is well known that a designed stable controller in non-hybrid systems may become unstable in hybrid systems due to the unsuitable switching rule. Wu and Zeng [30] proposed a succinct criterion for the exponential stabilization of memristive cellular neural networks. However, in practical applications, temporal failures may happen to actuators and subsequently the control signals cannot be successfully delivered [15]. Li et al. [16] focused on designing state-feedback and output-feedback sampled-data controllers to guarantee the resulting closed-loop system to be asymptotically stable with H_∞ disturbance attenuation level and suspension performance constraints. The problem of output-feedback H_∞ control for active quarter-car suspension systems with control delay has been proposed in [17]. The stabilization problem for Markovian stochastic jump systems against sensor fault, actuator fault and input disturbances is investigated in [18]. The presence of such a phenomenon may affect the performance of the system and even lead to instability of the controlled systems. So, it is of practical significance to design the reliable controllers in the presence of unexpected failures. Recently, considerable effort has been dedicated to the reliable control for more details, see [8,19,34]. Up to now, there are no results in the literature for problem of reliable stabilization of RNNs with time-varying delays. This paper is an attempt towards this goal.

Motivated by the above discussions, in this paper, a simplified mathematical model of memristors based RNNs with time-varying delays is considered and its global asymptotical stability is investigated. Our objective here is to design the reliable state feedback controller for the memristive RNNs. The main contribution of this paper is as follows: (1) The reliable stabilization problem is first time addressed for delayed memristive neural networks based on the framework of Filippov's solution. (2) The main objective of this work is to obtain a robust feedback controller with an appropriate gain control matrix to guarantee the asymptotic stability of the closed loop system. (3) The main results are derived by using the Lyapunov–Krasovskii stability theory and LMI technique together with the Wirtinger based inequality approach. (4) The sufficient conditions are derived for normal actuator operating case (without failure), fixed actuator failure and failure occurs in a range of interval (unknown or un-fixed failure).

The remainder of this paper is organized as follows. In Section 2, the model description and some preliminaries are presented. In Section 3, the stabilization problem of memristive neural networks is presented. In Section 4, a numerical example is provided to demonstrate the validity of the theoretical analysis and Section 5 concludes the investigation.

2. Problem formulation and preliminaries

2.1. Model description

In this subsection, we first describe the circuit of a general class of RNNs. From Kirchhoff's current law, the i-th subsystem a general class of RNNs is written as [29]

$$\dot{x}_i(t)=\begin{cases} -\left(\frac{1}{\mathcal{C}_i}\sum_{j=1}^{n}\left(\frac{1}{\mathcal{R}_{ij}}+\frac{1}{\mathcal{F}_{ij}}\right)\times \mathrm{sgin}_{ij}+\frac{1}{\mathbb{R}_i}\right)x_i(t)+\frac{1}{\mathcal{C}_i}\sum_{j=1}^{n}\frac{g_j(x_j(t))}{\mathcal{R}_{ij}}\times \mathrm{sgin}_{ij} \\ +\frac{1}{\mathcal{C}_i}\sum_{j=1}^{n}\frac{g_j(x_j(t-\tau_j(t))}{\mathcal{F}_{ij}}\times \mathrm{sgin}_{ij}+u_i(t), \quad t\ge 0,\ i=1,2,\ldots,n, \end{cases} \tag{1}$$

where $x_i(t)$ is the voltage of the capacitor $\mathcal{C}_i$; $\mathbb{R}_i$ represents the parallel-resistor corresponding to the capacitor $\mathcal{C}_i$; $\mathcal{R}_{ij}$ denotes the resistor through the feedback function $g_i(x_i(t))$ and $x_i(t)$, satisfying $g_i(0)=0$; $\mathcal{F}_{ij}$ denotes the resistor through the feedback function $g_i(x_i(t-\tau_i(t)))$ and $x_i(t)$; $u_i(t)$ denotes the appropriate control input; $\tau_i(t)$ denotes the time-varying transmission delay, satisfies $0\le \tau_i(t)\le \tau_i$, with $\dot{\tau}_i(t)\le \mu_i$, where τ_i, μ_i, $i=1,2,\ldots,n$, are constants and

$$\mathrm{sgin}_{ij}=\begin{cases}1, & i\neq j\\ -1, & i=j.\end{cases}$$

By replacing the resistors $\mathcal{R}_{ij}$ and $\mathcal{F}_{ij}$ in the primitive RNNs (1) with memristors, whose memductances $\mathcal{W}_{ij}$ and $\mathcal{M}_{ij}$, respectively, then the memristive RNNs can be constructed as follows:

$$\dot{x}_i(t)=-\sigma_i x_i(t)+\sum_{j=1}^{n}\left(\frac{\mathcal{W}_{ij}}{\mathcal{C}_i}\times \mathrm{sgin}_{ij}\times g_j(x_j(t)) +\frac{\mathcal{W}_{ij}}{\mathcal{C}_i}\times \mathrm{sgin}_{ij}\times g_j(x_j(t)-\tau_j(t))\right)+u_i(t), \tag{2}$$

or equivalently in other form

$$\dot{x}_i(t)=-\sigma_i x_i(t)+\sum_{j=1}^{n}\left(a_{ij}(x_i(t))g_j(x_j(t)) +b_{ij}(x_i(t))g_j(x_j(t-\tau_j(t)))\right)+u_i(t), \tag{3}$$

where $a_{ij}(x_i(t))$ and $b_{ij}(x_i(t))$ represent memristor-based weights, and

$$\sigma_i=\left(\frac{1}{\mathcal{C}_i}\sum_{j=1}^{n}(\mathcal{W}_{ij}+\mathcal{M}_{ij})\times \mathrm{sgin}_{ij}+\frac{1}{\mathbb{R}_i}\right);$$

$$a_{ij}(x_i(t))=\frac{\mathcal{W}_{ij}}{\mathcal{C}_i}\times \mathrm{sgin}_{ij};\quad b_{ij}(x_i(t))=\frac{\mathcal{M}_{ij}}{\mathcal{C}_i}\times \mathrm{sgin}_{ij}.$$

By considering the typical current–voltage characteristics of memristor, for convenience in this paper, we discuss a simple mathematical model of the memristor-based RNNs (3) as follows:

$$\dot{x}_i(t)=-\sigma_i x_i(t)+\sum_{j=1}^{n}\left(a_{ij}(x_i(t))g_j(x_j(t)) +b_{ij}(x_i(t))g_j(x_j(t-\tau_j(t)))\right)+u_i(t), \tag{4}$$