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Finite-time synchronization of neutral complex networks with Markovian switching based on pinning controller ☆

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ABSTRACT

The finite-time synchronization problem is investigated for a class of neutral complex dynamical networks with Markovian switching, partly unknown transition rates and time-varying, mode-dependent delays. By utilizing the pinning control technique and constructing the appropriate stochastic Lyapunov–Krasovskii functional, several sufficient conditions are proposed to ensure the finite-time synchronization for the neutral complex dynamical networks with Markovian switching, on the basis of Kronecker product, inequality techniques and finite-time stability theorem. Finally, numerical examples and simulations are given to illustrate the feasibility and effectiveness of the proposed results.

1. Introduction

Since the pioneering work of Watts and Strogatz [1], complex dynamical networks have received extensive attention in many research fields. This is mainly due to their wide applications in many areas ranging from physics to biology, sociology and computer sciences, *etc*. As ubiquitous phenomenon in nature, synchronization has been one of the most demonstrating topics in complex networks and many relevant results have been derived for various network synchronization problems (see [2–14] and references therein). However, in reality, especially in some engineering fields, the networks are expected to achieve synchronization as quickly as possible. It can obtain more economic benefits if the synchronization objective is realized in a finite horizon. Based on this viewpoint, finite-time synchronization is very important. Besides, finite-time synchronization means the optimality in convergence time [15], and the finite-time control techniques have demonstrated better robustness or disturbance rejection properties [16]. Due to its advantages and important applications, finite-time synchronization is becoming one of the hot issues and many finite-time control techniques have been proposed to realize network synchronization, such as impulsive control [17], adaptive control [18,19], sliding mode control [20,21], and pinning control [22,23]. In fact, the pinning control technique differs from many other control strategies such as switching control or impulsive control, *i.e.*, the pinning control will not be disconnected or turned off throughout the entire control process after being put at a fraction of nodes of the network. From practical perspective, the technique is useful and important because it greatly reduces the number of controlled nodes for real-world complex networks in the presence of large nodes [24–27].

Additionally, in many real-world networks such as biological and communication networks, time delays are inevitable due to slow processing of information transmission among agents. So synchronization or finite-time synchronization problems for complex networks with time delays have gained increasing attention and considerable progress has been made, see, *e.g.*, [2–4,6–8,17] and references therein. However, in some practical applications, past change rate of the state variables affects the dynamics of nodes in the networks. This kind of complex dynamical network is termed as neutral complex dynamical network, which contains time delays both in its states and in the derivatives of its states. There are some results about the synchronization design problem for neutral complex networks [9–13]. Refs. [10,11] studied the synchronization control for a kind of master-response setup and further extended to the case of neutral-type neural networks with stochastic perturbation. Refs. [13,9] researched the synchronization problem for a class of complex networks with neutral-type coupling delays. Ref. [12] investigated the robust global

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exponential synchronization problem for an array of neutral-type neural networks. However, few results have been proposed for finite-time synchronization of neutral complex dynamical networks, compared with the rich results for general complex dynamical networks.

In another research field, Markovian jumping complex dynamical networks have received wide attention in the past few years. As a result of the high-order and large-scale networks, network mode switching is a universal phenomenon in complex dynamical network of the actual systems, and sometimes the network has finite modes that switch from one to another with certain transition rate, then such switching can be governed by a Markov process. The stability and synchronization problem of complex networks and neural networks with Markovian jump parameters and delays are investigated in [28–33]. Exponential synchronization problems have been studied for Markovian jumping complex networks with mixed time delays in [28,30,32], while [29] has considered the synchronization of discrete Markovian jumping complex networks with mixed mode-dependent time delays. Particularly, Refs. [31,33] considered the stability and synchronization problems for neutral Markovian jumping networks with partly unknown transition rates. Actually, the transition rates of Markovian jumping systems or networks are not exactly known in many practical cases. Then it is of great significance to consider partly unknown transition rates for Markovian jumping complex networks. However, there are few results on finite-time synchronization of neutral complex dynamical networks with Markovian switching and partly unknown transition rates.

Motivated by the above discussion, in this paper, we study the finite-time synchronization for a class of neutral complex dynamical networks with Markovian switching and partly unknown transition rates by pinning control. We design a series of suitable controllers on the set of pinned nodes. By constructing a new augmented stochastic Lyapunov functional, finite-time synchronization conditions are derived for neutral Markovian jumping complex dynamical networks with time-varying and mode-dependent delays on the basis of the finite-time stability theorem, inequality techniques and pinning control technique. Finally, numerical examples are given to verify the effectiveness of the theoretic result.

The main contributions of this paper can be summarized as follows: (i) the proposed results are applicable to the frequently changed network topology, which are closer to the practical complex systems or networks; (ii) with regard to neutral complex dynamical networks, a Markovian switching model with partially unknown transition rates is proposed, which can cover Markovian model with completely *unknown and known transition rates* as a special case. Besides, we do not need any knowledge of the unknown elements; (iii) finite-time synchronization of neutral complex networks with Markovian switching is less attended, and this paper is the first attempt to address it by constructing a novel stochastic Lyapunov functional. The feasible pinning controllers can be obtained by LMIs.

The rest of this paper is organized as follows. Section 2 presents the problem and preliminaries. Section 3 gives the main results including several finite-synchronization conditions for neutral Markovian jumping complex dynamical networks with time-varying and mode-dependent delays, which are then verified by numerical examples in Section 4. Section 5 concludes the paper.

Notations: The following notations are used throughout the paper. $col\{\cdots\}$ *denotes a column of elements*. $\mathbb{R}^n$ denotes the n dimensional Euclidean space and $\mathbb{R}^{m\times n}$ is the set of all $m\times n$ matrices. $X<Y (X>Y)$, where X and Y are both symmetric matrices, means that $X-Y$ is negative (positive) definite. I is the identity matrix with proper dimensions. For a symmetric block matrix, we use $*$ to denote the terms introduced by symmetry. $\mathcal{E}$ stands for the mathematical expectation. $\Gamma V(x(t),r(t),t)$ *denotes the infinitesimal generator of* $V(x(t),r(t),t)$. $\|v\|$ is the Euclidean norm of vector v, $\|v\|=(v^Tv)^{1/2}$, while $\|A\|$ is the spectral norm of matrix A, $\|A\|=[\lambda_{\max}(A^TA)]^{1/2}$. $\lambda_{\max(\min)}(A)$ is the eigenvalue of matrix A with maximum (minimum) real part. The Kronecker product of matrices $P\in\mathbb{R}^{m\times n}$ and $Q\in\mathbb{R}^{p\times q}$ is a matrix in $\mathbb{R}^{mp\times nq}$ which is denoted as $P\otimes Q$. Let $\varsigma>0$ and $C([-\varsigma,0],\mathbb{R}^n)$ denote the family of continuous function φ, from $[-\varsigma,0]$ to $\mathbb{R}^n$ with the norm $|\varphi|=\sup_{-\varsigma\le\theta\le 0}\|\varphi(\theta)\|$. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problem statement and preliminaries

Given a complete probability space $\{\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\ge0},\mathbf{P}\}$ with a filtration $\{\mathcal{F}_t\}_{t\ge0}$ satisfying the usual conditions (*i.e.* it is increasing and right continuous while $\mathcal{F}_0$ contains all P-null sets), where Ω is the sample space, $\mathcal{F}$ is the algebra of events and $\mathbf{P}$ is the probability measure defined on $\mathcal{F}$. Let $\{r(t),t\ge0\}$ be a homogeneous and right-continuous Markov chain taking values in a finite state space $S=\{1,2,3,\ldots,M\}$ with a generator $\Pi=(\pi_{ij})_{M\times M}, i,j\in S$, which is given by

$$P(r(t+\Delta t)=j|r(t)=i)=\begin{cases}\pi_{ij}\Delta t+o(\Delta t), & i\neq j\\ 1+\pi_{ii}\Delta t+o(\Delta t), & i=j\end{cases}$$

where $\Delta t>0$, $\lim_{\Delta t\to0}o(\Delta t)/\Delta t=0$, $\pi_{ij}\ge0 (i,j\in S, i\neq j)$ is the transition rate from mode i to j and for any state or mode $i\in S$, it satisfies

$$\pi_{ii}=-\sum_{j=1,j\neq i}^{M}\pi_{ij}$$

Moreover, it is assumed that $r(t)$ is irreducible and available at time t. But the transition rates of the Markov process are partially unknown in this paper, which means that some elements in matrix $\Pi=(\pi_{ij})_{M\times M}$ are inaccessible. For instance, the neutral complex network with six operation modes, the jump rates matrix Π may be viewed as

$$\begin{bmatrix}\pi_{11}^0 & \pi_{12} & \pi_{13}^0 & \pi_{14}^0 & \pi_{15} & \pi_{16}^0\\ \pi_{21} & \pi_{22}^0 & \pi_{23}^0 & \pi_{24} & \pi_{25}^0 & \pi_{26}\\ \pi_{31}^0 & \pi_{32}^0 & \pi_{33} & \pi_{34} & \pi_{35}^0 & \pi_{36}^0\\ \pi_{41}^0 & \pi_{42} & \pi_{43}^0 & \pi_{44}^0 & \pi_{45}^0 & \pi_{46}\\ \pi_{51} & \pi_{52}^0 & \pi_{53}^0 & \pi_{54}^0 & \pi_{55} & \pi_{56}^0\\ \pi_{61} & \pi_{62}^0 & \pi_{63}^0 & \pi_{64} & \pi_{65} & \pi_{66}^0\end{bmatrix}$$

where $\pi_{ij}^0, i,j\in S$ represents the unknown element. For notational clarity, we denote $\mathcal{S}^i=\mathcal{S}_k^i\bigcup\mathcal{S}_u^i, \forall i\in S$ and

$$\mathcal{S}_k^i\triangleq\{j:\pi_{ij}\ \text{for}\ j\in S\},\quad \mathcal{S}_u^i\triangleq\{j:\pi_{ij}^0\ \text{for}\ j\in S\}$$

If $\mathcal{S}_k^i\neq\varnothing$, it is further described as

$$\mathcal{S}_k^i=\{k_1^i,k_2^i,\ldots,k_m^i\},\quad 1\le m\le M$$

where $k_j^i, (j=1,2,\ldots,m)$ represent the jth known element of the set $\mathcal{S}_k^i$ in the ith row of the transition rate matrix Π. It should be noted that if $\mathcal{S}_k^i=\varnothing$, $\mathcal{S}^i=\mathcal{S}_u^i$ which means that any information between the ith mode and the other $M-1$ modes is not accessible, then MJSs with M modes can be regarded as ones with $M-1$ modes.

The following neutral complex dynamical network consisting of N identical nodes with Markovian switching, interval mode-dependent and time-varying delays over the space $\{\Omega,\mathcal{F},\mathbf{P}\}$ is investigated in this paper:

$$\dot{x}_k(t)-C(r(t))\dot{x}_k(t-\tau(t,r(t)))=A(r(t))f_1(x_k(t))+B(r(t))f_2(x_k(t-d(t,r(t))))$$
$$+D(r(t))f_3(\dot{x}_k(t-\tau(t,r(t))))$$