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Robust H_∞ containment control for second-order multi-agent systems with nonlinear dynamics in directed networks ☆

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ABSTRACT

This paper considers robust H_∞ containment control problem in directed networks for second-order multi-agent systems with inherent nonlinear dynamics. A distributed control protocol is proposed for each follower using the relative states among neighboring agents. The original problem under the proposed protocol is then converted into an H_∞ control problem by defining an appropriate controlled output function. Based on robust H_∞ control approach, sufficient conditions in terms of linear matrix inequalities (LMIs) are derived to ensure that all followers asymptotically converge to the convex hull spanned by the leaders with the prescribed H_∞ performance. Finally, a numerical example is provided to demonstrate the effectiveness of our theoretical results.

1. Introduction

Distributed cooperative control of multi-agent systems has received considerable attention in a variety of research fields including biology, physics, robotics, sensor networks, artificial intelligence and control engineering. Unlike traditional centralized control, distributed cooperative control has enormous advantages, such as easy implementation, low complexity, high robustness, and good scalability. One fundamental issue arising from distributed cooperative control of multi-agent systems is to design control protocols using only local information such that all agents reach an agreement on certain quantities of interest, known as consensus problem. The case of consensus with multiple leaders is called containment control, whose objective is to drive the followers into the convex hull spanned by the leaders. The study of containment control is motivated by numerous natural phenomena and potential applications, for instance, earth monitoring, stellar observation for satellite formation [1], removing hazardous materials for autonomous robots [2], and so on.

In the past decades, numerous interesting results have been obtained for the containment control problem from different perspectives. Containment control problems have been studied for multi-agent systems with single-integrator [3,4], double-integrator [5–8], general linear dynamics [9–11], and nonlinear dynamics [12,13]. Note that physical systems in practice are inherently nonlinear [14], and the above-mentioned methods in [12,13] can only deal with nonlinearities matched in the control input, Yu et al. addressed second-order consensus of nonlinear multi-agent systems under fixed network topology in [15]. Second-order leader-following consensus of nonlinear multi-agent systems was investigated in [16,17]. In addition, based on Lyapunov-based approach and adaptive control approach, second-order containment control problem for multiple agents with inherent nonlinear dynamics was considered in [18]. However, multi-agent systems are often subject to external disturbances in practical applications, which might degrade the system performance and even cause the network system to diverge or oscillate. For such cases, some researches have been carried out on the leaderless consensus by employing H_∞ control approach [19–22]. Besides, with the aid of sliding-mode control technique, finite-time containment control for first-order nonlinear multi-agent systems was considered in [23]. Since many practical individual systems, especially mechanical systems, are of second-order dynamics. Furthermore, the extension of consensus protocols from the first-order case to the second-order one is nontrivial. Therefore, it is more practical but challenging to investigate the containment control problem of second-order multi-agent systems with inherent nonlinear dynamics subject to external disturbances.

Motivated by the above analysis, in this paper, we employ H_∞ control approach to investigate the distributed containment control problem in directed networks of second-order nonlinear

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agents subject to external disturbances. A distributed control protocol is proposed for each follower using the relative states among neighboring agents. By defining an appropriate controlled output function, the original problem under the proposed protocol is then converted into an H_∞ control problem. Based on robust H_∞ control approach, sufficient conditions in terms of LMIs are derived to ensure that all followers asymptotically converge to the convex hull spanned by the leaders with the prescribed H_∞ performance. The main contribution of this paper is threefold: first, compared with the containment control approaches in [3–13], the method proposed in this paper can deal with external disturbances and nonlinearities unmatched in the control input. Second, in constant to [23], we consider H_∞ containment control for second-order nonlinear multi-agent systems. Third, the interaction topology among the followers is supposed to be directed, which is more common in reality. We address H_∞ containment control problem under a weak topology condition.

The remainder of this paper is organised as follows. In Section 2, some preliminaries are given, and the problem to be solved is formulated. In Section 3, H_∞ containment control problem for the second-order nonlinear multi-agent system is investigated. In Section 4, a numerical example is presented to illustrate the theoretical results. Finally, concluding remarks are given in Section 5.

Throughout this paper, let $\mathbb{R}^m$ be the m-dimensional real vector space, and $\mathbb{R}^{m\times n}$ be the $m\times n$ real matrix space. I_n and 0_n $(0_{m\times n})$ denote the $n\times n$ identity matrix and the $n\times n$ $(m\times n)$ zero matrix, respectively. In symmetric block matrices, $*$ is used to represent a term induced by symmetry. $\mathrm{diag}\{z_1,z_2,\ldots,z_n\}$ defines a diagonal matrix with diagonal entries being $z_1,z_2,\ldots,z_n$. $d_\Omega(x)$ denotes the Euclidean distance between a point x and a set Ω. $L_2[0,\infty)$ indicates the space of square integrable vector functions over $[0,\infty)$, and for $\forall v(t)\in L_2[0,\infty)$, its normalized energy is defined by $\|v(t)\|_2=(\int_0^\infty v^{\mathrm{T}}(t)v(t)dt)^{1/2}$. $\|\cdot\|$ and $\otimes$ represent the Euclidean norm and the Kronecker product, respectively.

2. Preliminaries and problem formulation

In this section, some basic concepts on algebraic graph theory (see [24] for details) and useful lemmas are briefly introduced, and then the problem formulation is presented.

2.1. Preliminaries

Directed graphs are used to model the information interaction among agents. Let $G(\mathcal{V},\varepsilon,\mathcal{A})$ be a weighted digraph of order N with the set of nodes $\mathcal{V}=\{s_1,s_2,\ldots,s_N\}$, the set of edges $\varepsilon\subseteq\mathcal{V}\times\mathcal{V}$, and a weighted adjacency matrix $\mathcal{A}=[a_{ij}]$. The node indexes belong to a finite index set $\mathcal{I}=\{1,2,\ldots,N\}$. An edge of G is denoted by $e_{ij}=(s_j,s_i)$, and the adjacency elements associated with the edges of graph are positive, i.e. $e_{ij}\in\varepsilon$ if and only if $a_{ij}>0$. Moreover, it is assumed that $a_{ii}=0$ for all $i\in\mathcal{I}$. In graph G, the node s_i represents the ith agent, and the edge (s_j,s_i) represents that the node s_i can access information from s_j but not necessarily vice versa. Then the set of neighbors of s_i is denoted by $N_i=\{s_j\in\mathcal{V}:(s_j,s_i)\in\varepsilon\}$. The Laplacian matrix associated with G is denoted as $L=[l_{ij}]$, where $l_{ii}=\sum_{j=1,j\neq i}^{N}a_{ij}$, and $l_{ij}=-a_{ij}, i\neq j$. A directed path in the graph G is a sequence of ordered edges of the form $(s_{i_1},s_{i_2}),(s_{i_2},s_{i_3}),\ldots$, where $i_j\in\mathcal{I}$ and $s_{i_j}\in\mathcal{V}$.

Suppose that a multi-agent system consists of M $(M<N)$ followers and $N-M$ leaders. An agent is called a leader if the agent has no neighbor, and a follower if the agent has at least one neighbor. Without loss of generality, we assume that the agents indexed by $1,2,\ldots,M$ are followers, and by $M+1,M+2,\cdots,N$ are leaders. We use $\mathcal{F}\triangleq\{1,2,\ldots,M\}$ and $\mathcal{R}\triangleq\{M+1,M+2,\ldots,N\}$ to denote the follower set and the leader set, respectively. Due to the fact that the leaders have no neighbors, the Laplacian matrix L associated with G can be partitioned as

$$L=\begin{bmatrix} L_1 & L_2 \\ 0_{(N-M)\times M} & 0_{(N-M)\times(N-M)} \end{bmatrix}, \tag{1}$$

where $L_1\in\mathbb{R}^{M\times M}$ and $L_2\in\mathbb{R}^{M\times(N-M)}$.

In the sequel, we assume that the communication graph G satisfies the following assumption.

Assumption 1. Suppose that for each follower, there exists at least one leader that has a directed path to that follower.

Lemma 1 (*Ren and Cao [25]*). *Under Assumption 1, all eigenvalues of L_1 have positive real parts, $-L_1^{-1}L_2$ is nonnegative and each row of $-L_1^{-1}L_2$ has a sum equal to one.*

Lemma 2 (*Liu et al. [26]*). *For any vectors $x,y\in\mathbb{R}^m$ and positive definite matrix $Z\in\mathbb{R}^{m\times m}$, the following matrix inequality holds:*

$$2x^{\mathrm{T}}y\le x^{\mathrm{T}}Zx+y^{\mathrm{T}}Z^{-1}y.$$

Lemma 3 (*Jia [27]*). *For a given symmetric matrix S with the form $S=[S_{ij}], S_{11}\in\mathbb{R}^{r\times r}$, $S_{12}\in\mathbb{R}^{r\times(n-r)}$, $S_{22}\in\mathbb{R}^{(n-r)\times(n-r)}$. Then, $S<0$ if and only if $S_{11}<0, S_{22}-S_{12}^{\mathrm{T}}S_{11}^{-1}S_{12}<0$; or $S_{22}<0, S_{11}-S_{12}S_{22}^{-1}S_{12}^{\mathrm{T}}<0$.*

Definition 1 (*Rockafellar [28]*). A set $K\subset\mathbb{R}^m$ is said to be convex if for any $x,y\in K$ and $\theta\in[0,1]$, $(1-\theta)x+\theta y\in K$. The convex hull $\mathrm{Co}\{X\}$ of a finite set of points $X=\{x_1,x_2,\ldots,x_n\}$ is the minimal convex set containing all points in X. More specifically, $\mathrm{Co}\{X\}=\{\sum_{i=1}^{n}\alpha_i x_i|x_i\in X,\alpha_i\ge 0,\sum_{i=1}^{n}\alpha_i=1\}$.

2.2. Problem formulation

Consider the multi-agent system consisting of M $(M<N)$ followers and $N-M$ leaders. Each follower with nonlinear dynamics is represented as

$$\begin{aligned} \dot{p}_i(t)&=q_i(t),\\ \dot{q}_i(t)&=f(t,p_i(t),q_i(t))+u_i(t)+\omega_i(t),\quad i\in\mathcal{F}, \end{aligned} \tag{2}$$

where $p_i(t),q_i(t),u_i(t)\in\mathbb{R}^m$ are the position, velocity, and the control input (or protocol) of the ith follower, respectively, $\omega_i(t)\in\mathbb{R}^m$ is the external disturbance that belongs to $L_2[0,\infty)$, and $f:\mathbb{R}\times\mathbb{R}^m\times\mathbb{R}^m\to\mathbb{R}^m$ is the inherent nonlinear dynamics of the follower i.

The leaders of the concerned multi-agent system are described by

$$\begin{aligned} \dot{p}_i(t)&=q_i(t),\\ \dot{q}_i(t)&=f(t,p_i(t),q_i(t)),\quad i\in\mathcal{R}, \end{aligned} \tag{3}$$

where $p_i(t),q_i(t)\in\mathbb{R}^m$ are the position and velocity of the leader i, respectively. Suppose that the leaders share the same inherent nonlinear dynamics with all followers, and these inherent nonlinear dynamics satisfy the following assumption.

Assumption 2. Given $\eta_1,\eta_2,\ldots,\eta_{N-M}$ with $\sum_{i=1}^{N-M}\eta_i=1$, and $\eta_i\ge 0$, $i=1,2,\ldots,N-M$. There exist nonnegative constants ρ_1 and ρ_2 such that nonlinear function f satisfies

$$\left\|f(t,p,q)-\sum_{i=1}^{N-M}\eta_i f(t,r_i,s_i)\right\|\le\rho_1\left\|p-\sum_{i=1}^{N-M}\eta_i r_i\right\|+\rho_2\left\|q-\sum_{i=1}^{N-M}\eta_i s_i\right\|, \tag{4}$$

for any $p,q,r_i,s_i\in\mathbb{R}^m, i=1,2,\ldots,N-M$, and $t\ge 0$.

Remark 1. It is worth mentioning that the nonlinear function f in this paper is unknown. Since it is infeasible to introduce the term $-f$ in the control input u_i to cancel the unknown nonlinear term f, the nonlinear model in (2) cannot be converted to a (linear) double integrator. Moreover, for the single leader case, i.e. $N-M=1$, the