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$\mathcal{H}_\infty$ state estimation for discrete-time neural networks with interval time-varying delays and probabilistic diverging disturbances

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ABSTRACT

This paper considers the problem of delay-dependent $\mathcal{H}_\infty$ state estimation for discrete-time neural networks with interval time-varying delays and probabilistic diverging disturbances. By constructing a newly augmented Lyapunov–Krasovskii functional, a less conservative criterion for the existence of the estimator of discrete-time neural networks without disturbances is introduced in Theorem 1 with the framework of linear matrix inequalities (LMIs). Based on the result of Theorem 1, a designing criterion of the estimator for a newly constructed error dynamic system with probabilistic diverging disturbances between original system and estimator will be proposed in Theorem 2. Two numerical examples are given to show the improvements over the existing ones and the effectiveness of the proposed idea.

1. Introduction

During the last few years, the stability [1–22], passivity [23–25], synchronization [26,27], $\mathcal{H}_\infty$ control [28–32], state estimation [33–35] and other problems are being put to treat in the various dynamic systems. Of this, one pay close attention to the $\mathcal{H}_\infty$ control and state estimation problems for the following two reasons. Also, the study for these problems is relatively more insufficient than others.

- $\mathcal{H}_\infty$ *control theory* has been used to minimize the effects of the external disturbances. It is the aim of this theory to design the controller such that the closed-loop system is internally stable and its $\mathcal{H}_\infty$-norm of the transfer function between the controlled output and the disturbances will not exceed a given $\mathcal{H}_\infty$ performance level γ.
- *State estimation problem* is important in both control theory and practice applications because the system states, particularly, in large scale systems, are not completely available in the system outputs in real applications.

Naturally, $\mathcal{H}_\infty$ state estimation problem was issued in the various dynamic systems [36–38]. The goal of this problem is to design an $\mathcal{H}_\infty$ state estimator to robustly stabilize the systems while guaranteeing a prescribed level of disturbance attenuation γ in the $\mathcal{H}_\infty$ sense for the systems with external disturbances. Within this framework, the estimator law will ensure an $\mathcal{H}_\infty$ performance for the systems in the face of the disturbances.

On the other hand, the neural networks are the network of mutual elements that behave like a biological neurons. These neurons can be mathematically described by difference or differential equations. Each single neuron has a simple structure. During a few decades, neural networks have been received a great deal of attentions due to their extensive applications such as pattern recognition, power systems, and other scientific areas [39–42]. Before handling this network, because most systems use computers, these days, we need to pay keen attention to the two following considerations.

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- Most systems use microprocessor or microcontrollers, which are called digital computer, with the necessary input/output hardware to implement the systems. A little more to say, the fundamental character of the digital computer is that it takes compute answers at *discrete steps*.
- *Time-delay* occurs due to the finite speed of information processing and/or amplifier switching in the implementation of the systems in various physical, industrial and engineering systems such as physical and biological systems, population dynamics, neural networks, networked control systems, and so on. It is well known that the time-delay often causes undesirable dynamic behaviors such as performance degradation and instability of the systems. Therefore, the study on stability analysis for systems with time-delay has been widely investigated. For more details, see the literature [1–22]. Moreover, the analysis for time-delay systems can be classified into two categories: namely, delay-dependent analysis and delay-independent analysis. Here, delay-dependent analysis has been paid more attention than delay-independent one because the sufficient conditions for delay-dependent analysis make use of the information on the size of time delay [3]. That is, the former is less conservative than the latter particularly when the size of time-delay is small.

Therefore, discrete-time modeling for neural networks with time-delay plays an important role in many fields of science and engineering applications [43,44].

In this regard, $\mathcal{H}_\infty$ state estimation is a prerequisite and important job to the applications of many practical systems with time-delay and disturbances. In [36,37], $\mathcal{H}_\infty$ state estimation analysis for the system with time-delay and the disturbances in only original system part has been investigated. Also, the same condition was considered in [31,32]. However, in the point of practical systems, it is worth to investigate the $\mathcal{H}_\infty$ state estimation problem for the delayed neural networks with the disturbances in both original system and state estimator parts. Unfortunately, to the best of authors' knowledge, this problem of $\mathcal{H}_\infty$ state estimation for discrete-time delayed neural networks with the probabilistic diverging disturbances between original system and state estimator parts has not been tackled in any other literature. In addition to this, some existing works such as [34,35] are dealt with the state-estimation for discrete-time delayed neural networks. However, H_∞ control scheme whose importance is mentioned before was not considered in [34,35]. Very recently, the $\mathcal{H}_\infty$ state-estimation for discrete-time delayed neural networks with randomly occurring quantizations and missing measurements is addressed in [45]. Therefore, the discrete-time neural network with the proposed $\mathcal{H}_\infty$ state-estimator has many advantages in being applied to various engineering problems such as optimization, pattern recognition, medical diagnosis and image and signal processing.

With this motivation mentioned above, in this paper, firstly, the problem to get an improved state estimation criterion for a class of discrete-time neural networks with interval time-varying delays is considered. Here, delay-dependent stability or stabilization of system with interval time-varying delays has been a focused topic of theoretical and practical importance [46] in very recent years. The system with interval time-varying delays means that the lower bounds of time-delay which guarantees the stability of system is not restricted to be zero. A typical example of dynamic systems with interval time-varying delays is networked control system. Secondly, a new model of discrete-time neural networks with probabilistic diverging disturbances between original system and state estimator parts is constructed and its $\mathcal{H}_\infty$ state estimation criterion is proposed for the first time. So, the gist of the main points can be summarized as follows:

- Due to the finite speeds of transmission as well as networked control systems, there are usually time-delay in communication. In analysis for system with time-delay, the most utilized index for checking the conservatism of sufficient conditions is to obtain maximum bounds of time-delay such that the concerned system ensures the sufficient conditions for various problems discussed above. In order to derivative less conservative results, the main approaches utilized in other literature to enhance the feasible region of stability criteria were free-weighting matrices techniques [7–9], zero equalities [7,9,31], delay-partitioning concept based methods [11–18] and [47–49].
- In the literature, the disturbances were constructed in only original system part. However, in the practical point of view, it is worth to investigate the $\mathcal{H}_\infty$ control problem for discrete-time neural networks with the disturbances in both original system and state estimator parts since disturbances can also exist in state estimator part. Thus, in this paper, a new model of discrete-time neural networks with probabilistic diverging disturbances between original system and state estimator parts is constructed and its $\mathcal{H}_\infty$ state estimation criterion is proposed. For the details, see Remarks 2–4.

To do this, by construction of a newly augmented Lyapunov–Krasovskii functional and utilization of reciprocally convex approach [6] with some added decision variables, a new state estimation criterion is derived in Theorem 1 with the LMI framework. The LMIs can be formulated as convex optimization algorithms which are amenable to computer solution [50]. Next, based on the results of Theorem 1, an $\mathcal{H}_\infty$ state estimation criterion for a class of discrete-time neural networks with probabilistic diverging disturbances between original system and state estimator parts will be introduced in Theorem 2. Finally, two numerical examples are included to show the effectiveness of the proposed methods.

Notation: $\mathbb{R}^n$ is the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ denotes the set of all $m\times n$ real matrices. $l_2[0,\infty)$ is the space of square integrable vector functions over $[0,\infty)$. For real symmetric matrices X and Y, $X>Y$ (resp., $X\ge Y$) means that the matrix $X-Y$ is positive (resp., nonnegative) definite. I_n, 0_n and $0_{m\cdot n}$ denote $n\times n$ identity matrix, $n\times n$ and $m\times n$ zero matrices, respectively. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator. $\|\cdot\|$ refers to the Euclidean vector norm or the induced matrix norm. $\mathtt{diag}\{\cdots\}$ denotes the block diagonal matrix. For square matrix X, $\mathtt{sym}\{X\}$ means the sum of X and its symmetric matrix X^T; i.e., $\mathtt{sym}\{X\}=X+X^T$. $X_{[f(t)]}\in\mathbb{R}^{m\times n}$ means that the elements of matrix $X_{[f(t)]}$ include the scalar value of $f(t)$; i.e., $X_{[f_0]}=X_{[f(t)=f_0]}$. $\Pr\{A\}$ means the occurrence probability of the event A.

2. Problem statements

Consider the following discrete-time neural networks with interval time-varying delays and disturbances:

$$y(k+1)=Ay(k)+W_0g(y(k))+W_1g(y(k-h(k)))+Bw(k)+b,$$