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A global exponential robust stability criterion for interval delayed neural networks with variable delays

Chuandong Li[a,*], Xiaofeng Liao[a], Rong Zhang[b]

[a]College of Computer Science and Engineering, Chongqing University, 400030, PR China
[b]College of Economics and Business Administration, Chongqing University, 400030, PR China

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Abstract

The issue of exponential robust stability for interval delayed neural networks with variable delays is studied. An approach combining the Lyapunov–Krasovskii functional with the differential inequality and linear matrix inequality techniques is taken to investigate this problem. The proposed criterion for exponential stability generalizes and improves those reported recently in the literature. Two numerical examples are also presented to illustrate our results.

Keywords: Interval delayed neural networks (IDNN); Exponential robust stability; Variable delay; Lyapunov–Krasovskii functional; Linear matrix inequality (LMI)

1. Introduction

In the past decade, the issue on the stability of delayed neural networks (DNN) has received intensive attention, and a large number of stability criteria for DNN have been proposed [1,2,4–10,12,14–21,23,24,26–33]. However, the stability of a neural network may often be destroyed by its unavoidable uncertainty due to the existence of modeling errors, external disturbance and parameter fluctuation during the implementation on very-large-scale-integration chips. Thus, it is important to investigate the robust stability of the networks against such errors and fluctuation. In order to formulate this kind of DNN with uncertainty, Liao and Yu [19] have extended the model of DNN to the so-called interval DNN (IDNN). Now, several robust stability criteria for these systems have been derived [19,17,16,1,4,14,32]. However, those results reported in Refs. [19,17,16,1] required that the activation functions should be monotonic or that the parameters of systems should be limited strictly. In addition, to the best of our knowledge, there does not seem to be much (if any) study on the global exponential robust stability for IDNNs with time-varying delays via LMI approach so far. Moreover, to derive the stability conditions for the DNNs with invariable parameters and time-varying delays, the authors always assume that delay functions are differentiable and their derivatives are bounded (in general, less than 1). In practice, the switch speed in circuits and axonal transmission delays in neural networks are often not differentiable and even not continuous. Therefore, how to analyze the stability problem for IDNN with non-differentiable delays becomes attractive issue.

Motivated by the above reasons, we study the global robust stability for IDNN with the time delays in this paper. Several criteria for the global robust stability are proposed. Distinct from the previous investigations, the current study focuses on the global robust stability by using a combination of Lyapunov–Krasovskii functional [11] with the differential inequality and linear matrix inequality (LMI) technique [3]. The main advantages of the present approach include: (1) It leads to less conservation and less restriction, because we does not require that the delay functions are differentiable and at the same time we only require that the activation functions are Lipschitz-continuous. (2) It can be efficiently verified via solving

*Corresponding author. Tel.: +86 23 65103199.
E-mail addresses: cd_licqu@163.com, licd@cqu.edu.cn (C. Li).

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numerically the LMI using interior-point algorithms or just the LMI-toolbox in Matlab.

The rest of this paper is organized as follows: In the next section, the problem to be studied is formulated and some definitions, notations and lemmas are presented. Based on the Lyapunov–Krasovskii stability theorem [4], in combination with the differential inequality and LMI technique, a new exponential robust stability condition for IDNN with the time-varying delays is derived in Section 3. In Section 4, several numerical examples are given to demonstrate the effectiveness of our results. Finally, conclusions are drawn in Section 5.

2. Formulations and preliminaries

Consider the delayed neural networks described by the following functional differential equations:

$$\dot{u}_i(t) = -a_iu_i(t) + \sum_{j=1}^{n} w_{ij}g_j\big(u_j(t)\big) + \sum_{j=1}^{n} v_{ij}g_j\big(u_j\big(t-\tau_j(t)\big)\big) + I_i, \quad i = 1,2,\ldots,n \tag{1a}$$

or

$$\dot{u}(t) = -Au(t) + WG(u(t)) + VG(u(t-\tau(t))) + I, \tag{1b}$$

where $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^{\mathrm{T}}$ is the state vector of the neural networks, $A = diag(a_1, a_2, \ldots, a_n)$ is a diagonal matrix with positive entries, i.e., $a_i > 0$, $W = (w_{ij})_{n\times n}$, $V = (v_{ij})_{n\times n}$ are the connection weight matrix and delayed connection weight matrix, respectively, $G(u(t)) = \big[g_1(u_1(t)), g_2(u_2(t)), \ldots, g_n(u_n(t))\big]^{\mathrm{T}}$ denotes the neuron activation function vector, and $I = [I_1, I_2, \ldots, I_n]^{\mathrm{T}}$ is a constant external input vector. $\tau(t) = (\tau_1(t), \tau_2(t), \ldots, \tau_n(t)))^{\mathrm{T}}$ represents the time-varying axonal signal transmission delay vector with $0 \leqslant \tau_i(t) \leqslant \bar{\tau} < +\infty$.

Throughout this paper, we always assume that the activation functions are bounded and satisfy Lipschitz's condition:

(H) g_i $(i = 1,2,\ldots,n)$ is bounded on R, and there exist constants $M_i > 0$ such that, for any $x, y \in R$, $i = 1,2,\ldots,n$, $|g_i(x) - g_i(y)| \leqslant M_i|x-y|$.

Remark 1. It is easy to see that the assumption (H) implies that the activation functions are continuous but not always monotonic. Also, we do not require that the time-varying delay be differentiable. Thus, our requirements for the activation functions and time-varying delay are weaker than those in Refs. [19,17,16,1,4].

In practice, the deviations and perturbations of the weights of the connections are bounded in general. Thus, we may intervalize the quantities mentioned above as follows:

$$\begin{aligned} A_I &= \{A = diag(a_i)_{n\times n} | \underline{a}_i \leqslant a_i \leqslant \bar{a}_i,\ i = 1,2,\ldots,n\}, \\ W_I &= \Big\{W = (w_{ij})_{n\times n} | \underline{w}_{ij} \leqslant w_{ij} \leqslant \bar{w}_{ij},\ i,j = 1,2,\ldots,n\Big\}, \\ V_I &= \{V = (v_{ij})_{n\times n} | \underline{v}_{ij} \leqslant v_{ij} \leqslant \bar{v}_{ij},\ i,j = 1,2,\ldots,n\}. \end{aligned} \tag{2}$$

Moreover, for notational convenience, we define, for $i,j = 1,2,\ldots,n$,

$$\underline{A} = diag(\underline{a}_i)_{n\times n}, \quad w_{ij}^{*} = \max\Big\{|\underline{w}_{ij}|, |\overline{w}_{ij}|\Big\}, \\ v_{ij}^{*} = \max\{|\underline{v}_{ij}|, |\bar{v}_{ij}|\} \tag{3}$$

and

$$b_i = \sum_{j=1}^{n}\left(w_{ij}^{*}\sum_{k=1}^{n} w_{kj}^{*}\right), \quad c_i = \sum_{j=1}^{n}\left(v_{ij}^{*}\sum_{k=1}^{n} v_{kj}^{*}\right), \\ B = diag(b_i)_{n\times n}, \quad C = diag(c_i)_{n\times n}. \tag{4}$$

Also, we use $P^{\mathrm{T}}, P^{-1}, \lambda_{M(m)}(P)$ to denote the transpose of, inverse of, and the maximum (minimum) eigenvalue of a square matrix P, respectively. The vector norm is taken to be Euclidian, denoted by $\|\cdot\|$. And we use $P > 0$ $(<0, \leqslant 0, \geqslant 0)$ to denote a symmetrical positive (negative, semi-negative, semi-positive) definite matrix P.

It is well known that bounded activation functions always guarantee the existence of an equilibrium point for system (1). For notational convenience, we will always shift an intended equilibrium point $u^* = (u_1^*, u_2^*, \ldots, u_n^*)^{\mathrm{T}} \in R^n$ of system (1) to the origin by letting $x(t) = u(t) - u^*$, which yields the following system:

$$\dot{x}(t) = -Ax(t) + WF(x(t)) + VF(x(t-\tau(t))), \tag{5}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}}$ is the state vector of the transformed system, and $F(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^{\mathrm{T}}$ denotes the activation function vector with $f_i(x_i(t)) = g_i(x_i(t) + u_i^*) - g_i(u_i^*)$, $i = 1,2,\ldots,n$.

Obviously, the equilibrium point u^* of system (1) with (H) is globally exponentially robustly stable if and only if the origin of system (5) is globally robustly exponentially stable. Thus in the sequel, we only consider global robustly stability of the trivial solution of system (5).

Before stating the main results, we first need the following preliminaries.

Definition 1 (*Zhang [31], Zhou et al. [33], Cao and Chen [4]*). The equilibrium point $u^* = (u_1^*, u_2^*, \ldots, u_n^*)^{\mathrm{T}} \in R^n$ is said to be globally exponentially stable, if there exist constants $\varepsilon > 0$ and $K > 0$ such that $\sum_{i=1}^{n}|u_i(t) - u_i^*| \leqslant K\|\phi - u^*\|e^{-\varepsilon t}$. Furthermore, system (1) with uncertainty is said to be globally exponentially robust stable if its unique equilibrium point $u^* \in R^n$ is globally exponentially stable for any $A \in A_I$, $W \in W_I$, $V \in V_I$.

Definition 2 (*Huang and Cao [12]*). For any continuous function $h: R \to R$, its Dini's time-derivative is defined as

$$\dot{h}(t) = \lim_{\theta\to 0^+} sup \frac{h(t+\theta) - h(t)}{\theta}.$$

Lemma 1 (*Sanchez and Perez [25]*). *Given any real matrices $\Sigma_1, \Sigma_2, \Sigma_3$ of appropriate dimensions and a scalar $\varepsilon > 0$ such that $0 < \Sigma_3 = \Sigma_3^{\mathrm{T}}$. Then, the following inequality holds*:

$$\Sigma_1^{\mathrm{T}}\Sigma_2 + \Sigma_2^{\mathrm{T}}\Sigma_1 \leqslant \varepsilon\Sigma_1^{\mathrm{T}}\Sigma_3\Sigma_1 + \varepsilon^{-1}\Sigma_2^{\mathrm{T}}\Sigma_3^{-1}\Sigma_2.$$