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Image reconstruction algorithm from compressed sensing measurements by dictionary learning

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ABSTRACT

It is a challenge task to reconstruct images from compressed sensing measurement due to its implicit ill-posed property. In this paper, we propose an image reconstruction algorithm for compressed sensing image application based on the adaptive dictionary, which is learned from the reconstructed image itself. The sparsity level is enhanced since the sparse coding of overlapping image patches emphasizes the local image features; accordingly the quality of the reconstructed image is also improved. In addition, Batch-OMP algorithm, linearization technique and dynamic updating sparse coding algorithm are used to reduce the computational complexity of our proposed algorithm. Numerical experiments are conducted on several test images with a variety of sampling ratios. The results demonstrate that our proposed algorithm can efficiently reconstruct images from compressed sensing measurements and achieve more than 3 dB gain averagely over the current leading CS reconstruction algorithm.

1. Introduction

Reconstructing images from undersampled measurements has always been a topic of great interest because it constitutes a critical step in an emerging methodology in digital signal processing—compressed sensing (CS). The greatest challenge lies in the underdetermined linear system of the reconstruction process because the number of measurements may be significant fewer than the length of the original signal. To address this issue, the most effective method is to utilize prior knowledge or some constraints in the reconstruction process to offset the missing information.

Recent research on sparsity-promoting regularization for signal reconstruction from highly undersampled measurements has witnessed a fast increase. The CS theory also states that the signals/images can be recovered from undersampled measurements, which may even be far below the traditional Nyquist sampling rate [1,2]. This is possible provided that the underlying signal or image is sparse or compressible in some transform domain, and the sampling is incoherent, in an appropriate sense, with the transform. With such a framework, the CS reconstruction will be formalized as the following constrained optimization problem (1), which can be solved by some convex minimization or greedy algorithms in the transform domain.

$$\min_x F(x) = \frac{1}{2}A||x-y||_F^2 + \lambda\Phi(x) \tag{1}$$

where $A \in R^{m \times n}(m \ll n)$ is sensing measurement matrix, y is the measurement value, such as $y = Ax + \xi$, ξ is measurement noise, $\Phi(x)$ is sparse transform and λ is regularization parameter. More recently, CS theory has been applied to magnetic resonance imaging (MRI) demonstrating high quality reconstructions from a reduced set of measurements [3,4].

The sparsity of images in some transform domain, or equivalently, sparse representation in some set of atom signals known as dictionary, is key to accurate CS image reconstruction. One of the most commonly sparse representation methods is finite difference [5–8], which is more often known as total variation (TV) regularization for image recovery. The popularity of TV regularization is due to its fascinating properties such as simplicity, convexity and ability to preserve edges. However, as a non-adaptive and predefined transform, TV regularization favors solutions with sparse gradient, which is optimal in describing piecewise constant images and sometimes will cause undesired oil painting effects [9].

Recently, it has received much more attention that exploiting the inherent sparsity in the image by similarity property and nonlocal operation on corresponding overlapping image patches [10–13]. An important research topic is sparse representation method with dictionary learning (DL), which builds adaptive sparse transform from particular image instances for sparse approximation. Compared with the predefined sparse transforms, such as wavelet, DCT, and curvelet etc., the dictionary learning method eliminates the drawback of a fixed

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sparse transform because it is impossible for one predefined sparse transform to be universally optimal for all images.

Adaptive sparse transform or dictionary can sparsify images better since they are learnt for the particular image instance or class of images. Recent research on adaptive sparse transform has shown the promise in a variety of image processing applications such as image denoising, deblurring, and demosaicing [14,13], etc. The shift from global image sparsity to patch-based sparsity is appealing because patch based dictionaries can capture the local image features effectively, and can potentially eliminate the noise and aliasing artifacts in the reconstruction image from compressed sensing measurements. Patch based methods have been widely used in image denoising where using overlapping patches can develop an additional averaging effect that remove noises. In addition, a single image can be decomposed into sufficiently many overlapping patches which can be used to train an adaptive sparse dictionary.

Most existing DL algorithms adopt a two-step iterative method. Firstly the sparse approximations are calculated with fixed dictionary and the dictionary is subsequently updated based on the current sparse coefficients. One of the most widely used state-of-the-art DL algorithms is the K-SVD algorithm [13], which sparsely represents the image patches, rather than the whole image itself. The size of dictionary atom is the same as that of the image patches, for example, 8×8 image patches.

To reconstruct an image from its compressed sensing measurements with dictionary learning algorithm, Ravishankar et al. start from a rough estimate for the compressed sensing measurement image, then simultaneously update the sparse dictionary and sparse coefficients of all overlapped patches, and finally average all the reconstructed patches to estimate the image iteratively [15]. Liu et al. in [16] propose one gradient based dictionary learning method for image reconstruction which integrates TV and DL into one framework to achieve even sparser representation of an image, an efficient splitting Bregman method is developed to decouple the difference operation, dictionary and sparse coefficients variables. However, the periodic boundary condition is assumed and the 2D discrete Fourier transform is used in those algorithms in order to avoid the matrix inversion.

In addition, compressed sensing theory has also drawn quite an amount of attention in computation vision, such as palmprint recognition [17,18], face recognition [19] and image segmentation [20], etc. The basic idea of these methods is to exploit the underlying sparsity in the problem in order to improve the robustness, speed, or accuracy with which classification might be performed.

In this paper, we propose an image reconstruction algorithm which exploits adaptive patch-based dictionary to obtain substantially improved reconstruction performance for compressed sensing image application. Image denoising methods with sparse and redundant representation over learned dictionary have been shown to provide state-of-the-art results [21]. Image inpainting methods have also been studied using adaptive dictionary learned from the corrupted image [22]. Unlike the denoising and inpainting image recovery problems, the available partial information for image compressed sensing reconstruction application is in the compressed sensing domain, rather than in the original pixel domain, so the adaptive patch-based dictionary should be learned from the CS measurements data. Our proposed image reconstruction algorithm from compressed sensing measurements by dictionary learning (IRCSDL) mainly consists of two iterative steps, one is to learn the sparse dictionary from the reconstructed image itself and then use it to remove aliasing and noise, and the other is to reconstruct the image using the compressed sensing measurements and sparse representation in the first step.

Our proposed IRCSDL algorithm has three benefits. Firstly, IRCSDL reconstructs an image from its undersampled compressed sensing measurements by applying sparse coding to its patches, which can give rise to superior sparsity for every image instance thereby leading to substantially higher undersampling rates. Secondly, the introduction of adaptively learned dictionary alleviates the reconstruction artifacts and allows an image with complex structure to be recovered accurately due to the ability of patch-based dictionary to capture the local image features effectively. Thirdly, the linearization technique is used to avoid the matrix inversion and significantly reduce the computation complexity, which can be also applied to the other similar image inverse problems.

The rest of this paper is organized as follows. Section 2 states the prior work in sparse coding and dictionary learning. Our proposed model for CS image reconstruction based on adaptive dictionary learning is detailed in Section 3. Section 4 demonstrates the performance of the proposed algorithm on numerous examples with a variety of test images and undersampling ratios. Conclusion and future work are presented in Section 5.

2. Background and related work

In this section, we will review some classical dictionary learning models and sparse coding algorithms for image reconstruction in the context of CS. The following notational conventions are used throughout the paper. Let $A \in R^{m \times n} (m \ll n)$ be the sensing measurement matrix and let $||\cdot||_0$ count the nonzero number of its argument which is often approximated by $||\cdot||_1$ for tractable computation. Let $x \in R^n$ denote the vector representation of an image of size $\sqrt{n} \times \sqrt{n}$ to be reconstructed, $x_{ij} \in R^m$ is the vector representation of the square 2D image patch of size $\sqrt{m} \times \sqrt{m}$ pixels, indexed by its coordinates of top-left corner in the image. We use $D \in R^{m \times K}$ to represent the image patch-based dictionary, which has K atoms (columns), each corresponding to one $\sqrt{m} \times \sqrt{m}$ "element patch". It is assumed that each image patch x_{ij} can be approximated by a linear combination of dictionary atoms, such as $x_{ij} \approx Da_{ij}$, where $a_{ij} \in R^K$ is sparse, that is to say, $a||_{ij}||_0 \ll m$. Accordingly, we also say that a_{ij} is the sparse representation of x_{ij} with respect to the dictionary D. When the number of atoms is equal to the dimension of an image patch, $K = m$, the dictionary D is referred as a basis, otherwise, when $K > m$, it is said to be over-complete.

2.1. Dictionary learning

Dictionary learning aims to solve the following optimization problem:

$$\min_{D,\Gamma} \sum_{ij} ||R_{ij}x - Da_{ij}||_F^2 \quad \text{s.t.} \; ||a_{ij}||_0 \le T_0 \quad \forall \, i,j \tag{2}$$

where R_{ij} is an operator taking the (i,j) image patch from image x, T_0 represents the target of sparsity level, Γ denotes a set of sparse representation coefficients of image patches covering all the pixels of image x. This learning formulation is to minimize the total fitting error of image patches with respect to the dictionary D, subject to sparsity constraints.

Although the DL problem (2) is NP-hard, recently numerous algorithms have been proposed to solve it [23], the common idea of these algorithms is to alternate optimization for dictionary D and sparse representations Γ. In particular, the K-SVD algorithm has been widely used in many applications [13]. Firstly, the sparse coding is considered, where the dictionary D is assumed to be fixed, the above optimization problem is to search sparse representations for every image patch. The optimization problem can be decoupled for one image patch as following:

$$\min_{a_{ij}} ||x_{ij} - Da_{ij}||_F^2 \quad \text{s.t.} \; ||a_{ij}||_0 \le T_0 \tag{3}$$

This problem will be adequately addressed by the orthogonal matching pursuit (OMP) algorithms discussed in the next Section 2.2, and we have seen that if T_0 is small enough, the solution is a good approximation to the ideal one. Secondly, the dictionary